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## Asset Pricing with Idiosyncratic Shocks

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### Abstract

This study shows the relationship between idiosyncratic shocks and expected returns on stock regarding theoretical and empirical results. The dynamic stochastic general equilibrium is derived from idiosyncratic stochastic productivity level in production function of heterogeneous firms to come up with a new asset pricing model. Given any state  $S$ , the main finding states that expected stock returns depends on the rate of time preference, depreciation rate, capital share, expected idiosyncratic productivity level at time  $t + 1$ , the percentage deviation of capital from steady state at time  $t + 1$ , and the percentage deviation of labor from steady state at time  $t + 1$ . In fact, expected idiosyncratic productivity level, expected capital, and expected labor are the determinant factors that affect on expected stock returns. Eventually, expected idiosyncratic stochastic productivity level is positively related to expected stock returns similar to expected labor. In contrast, expected capital has a negative effect on expected stock returns.

*Keywords:* Idiosyncratic productivity shock, Asset pricing, Dynamic general equilibrium model

*JEL classification:* G120

## Introduction

The capital market theory built on the mean-variance framework of Markowitz (1959) and extended by Sharpe (1964) and Lintner (1965), describes the substantial relationship between risk and expected returns. Still, there are several particular assumptions of the Capital Asset Pricing Model (CAPM) under perfect capital market. That is, investors would optimally hold a mean-variance portfolio. This means that they prefer portfolio with higher expected return given two investments with equal level of variance. Similarly, investors prefer portfolio with the lower variance given those with equal expected returns. In addition, it is assumed that all investors have homogeneous expectation, unlimited risk-free lending and borrowing, price taking, and there are neither transaction costs nor information costs. Therefore, the implication of the mean-variance efficiency portfolio shows that expected return on risky asset derives from return on risk-free asset and beta-adjusted market risk premium. This implies that beta measures systematic risk. Indeed, expected returns on any asset in equilibrium rely solely on systematic risk.

Systematic risk cannot be diversified away as it is associated with overall movement in the general market or economy, and is always referred to as the market risk. Therefore, such risk cannot be eliminated through portfolio diversification. This is a key fundamental of CAPM which demonstrates that systematic risk measures expected return of an individual stock. It seems useful implication for portfolio construction to invest in stock market because investors should consider only the sources of such risk, e.g. economic fluctuation, political turmoil, problem of public debts, oil shock. This means that most stocks should covary with market changes. The facts of stock value, however, indicate that some individual stocks move in the same direction with overall market but the others move in the opposite direction with it. As a result, the expected return on stock could not be explained by systematic risk alone. It can be concluded that there are other risks which are specific to an individual stock such as business risk or financial risk.

In other words, the risk associated with individual stocks is idiosyncratic risk. It can be diversified away by including a large number of stocks in portfolio. In this case, a rational investor should take not only on systematic risk, but also on idiosyncratic risk in order to obtain compensates. Additionally, the CAPM considers only systematic risk relative to expected returns on stock, disregards another risk especially idiosyncratic risk. It defined as diversified risk, does not figure in the traditional asset pricing model, the CAPM. It is because systematic risk is only priced in equilibrium. This particular characteristic is the same assertion as documented by Consumption-based Capital Asset Pricing Model (CCAPM) as in Lucas (1978) and Breeden (1979) or production-based asset pricing model in Cochrane (1991). Consequently, expected stock returns are determined solely by systematic risk since idiosyncratic risk can be eliminated through portfolio diversification.

Campbell et al. (2001) state that industry and idiosyncratic firm-level shocks are also the important components of individual stock returns as the following: (1) investors may fail to diversify in manner recommended by financial theory; (2) some investors who try to diversify do so by holding a portfolio of 20 or 30 stocks which all idiosyncratic risk is eliminated, but the adequacy of closely approximation depends on the level of idiosyncratic volatility in stocks; (3) arbitrageurs who trade to exploit the mispricing of an individual stock face risks that are related to idiosyncratic return volatility, not aggregate market volatility; (4) the events affect individual stocks, and the statistical significance of abnormal event-related returns determined by the volatility of individual stock returns relative to the market, and; (5) the price of an option on an individual stock depends on the total volatility, industry-level volatility, market volatility and idiosyncratic volatility.

In terms of theoretical papers in financial economics, almost all the asset pricing literatures have less examined in the role of idiosyncratic risk. It is surprisingly that there are a few studies on idiosyncratic risk especially the role of idiosyncratic and market risks. Even though there is very useful for portfolio construction to invest in stock market, idiosyncratic risk does not consider as a particular determining factor in expected stock returns. Such studies show the macroeconomic sources of risk that drive asset prices such as consumption, labor income, money demand, rate of inflation, habit formation, capital adjustment cost, output, debt and equity financing, etc. These variables are considered as the sources of systematic risk which affect expected returns.

Thus, the purpose of this study is to provide a preliminary investigation of the effect of idiosyncratic risk on expected stock returns in a dynamic general equilibrium asset pricing model. The proposed theoretical model is designed to reconcile an abstract model with the empirical findings in the literature above. In particular, previous studies have found that there are either positive, negative, or mixed relationship between idiosyncratic risk and expected stock returns. The model in this paper explores possibilities of various links between idiosyncratic risks on expected returns on stock.

The rest of the paper is organized as follows. Section 2 presents asset return's facts of Thailand. The stock returns and Treasury-bill returns, as well as the equity premium are shown in this section. Section 3 show the empirical results of expected stock returns in case of Thailand. Section 4 develops the asset pricing model which derived from a dynamic general equilibrium model, as in Lucas (1978). Asset price implication is presented in section 5. Section 6 concludes with the key findings and the final section discusses about such model with idiosyncratic productivity shock and policy implication as well.

## Asset Return Facts

In the case of Thailand, the key facts of asset returns are given in Table 1. The data comes from the daily data on the SET index and Treasury-bill between March 2001 and December 2011, a total of 2,560 trading-days each. The net daily SET index returns over the past one month are calculated by daily-closed SET index on day  $t$  minus daily-closed SET index on day  $t-20$ , and divided by daily-closed SET index on day  $t-20$ . The mean of stock returns for this period is 1.0813 percent per month with standard deviation of 6.8508. It has maximum returns of 24.6049 percent per month, and minimum returns of -35.5669 percent per month.

On the contrary, the returns on one- month Treasury-bill during the same period have an average of 0.1927 percent per month with standard deviation of 0.0927. The maximum returns on one-month Treasury-bill equal 0.405 percent per month, and the minimum returns equal 0.0646 percent per month. Consequently, there is an equity premium for Thai stock market of 0.8886 percent per month.

**Table 1** Stock Returns on SET Index and Treasury-bill

Variable	Obs.	Mean	Std. Dev.	Min	Max
SET index	2650	1.0813	6.8508	-35.5669	24.6049
1M T-bill	2650	0.1927	0.0927	0.0646	0.4051
3M T-bill	2650	0.1991	0.0915	0.0661	0.4062

Source: Srisuksai, P.(2012). Idiosyncratic Volatility and Expected Stock Returns: Evidence from Thailand. *Applied Economics Journal*, 19(2), 66-89.

Consistent with three-month Treasury-bill, it has average daily returns over the past one month of 0.1991 percent per month with standard deviation of 0.0915. It has maximum returns on three- month Treasury-bill of 0.4062 percent per month, and minimum returns of 0.0661 percent per month.

Figure 1 shows historical movement of SET index returns and one-month Treasury-bill returns. In fact, the daily stock returns over the past one month are more highly volatile than the daily returns on one-month Treasury-bill during the same period. This implies that stocks are riskier than Treasury-bills. This study will use these facts to develop an asset pricing model for capturing volatility, especially idiosyncratic volatility. The model is developed from the stochastic general equilibrium model. Empirical study also shows that a particular determinant which causes the returns on stock move relatively more volatile than risk free asset is idiosyncratic risk. It is because investor rewarded a premium for bearing this additional risk.

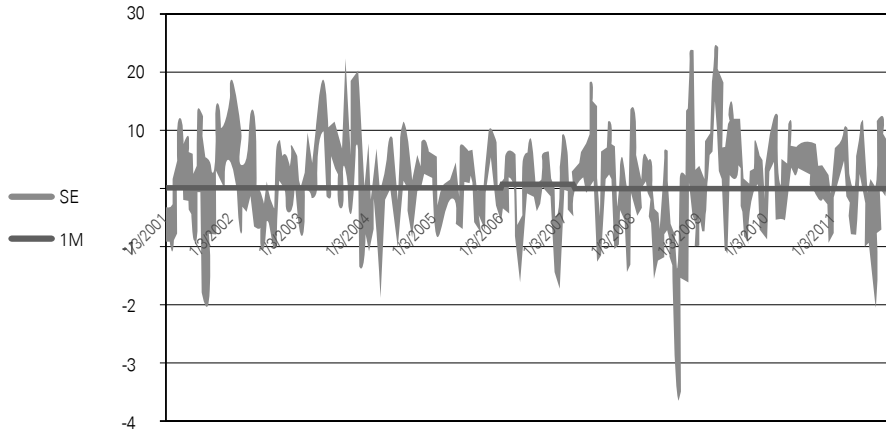


Figure 1 Asset Returns Facts: The return on the SET index and Treasury-bill between March 2001 and December 2011.

### Empirical Results of Thailand

Almost all empirical studies show that idiosyncratic volatility has a significantly positive impact on expected stock returns, especially on the NYSE, Amex, and Nasdaq. Srisuksai (2012) states that expected conditional idiosyncratic volatility (EIV) displays a strongly positive relationship to expected stock returns in the Stock Exchange of Thailand (SET) as show in Table 2. The size of coefficient is approximately 0.245. The positive relation is similar to Merton (1987), Ang et al. (2006; 2009) and Fu (2009).

Table 2 Pooled panel data regressions of Fama and MacBeth model for individual stocks in SET index

Model	Intercept	<i>EIV</i>	<i>EBETA</i>	<i>RS</i>	<i>VALUE</i>	<i>ILR</i>	<i>TURN</i>	$\bar{R}^2$
1	1.005***	(1.43×10 <sup>-33</sup> )***						0.001
2	-1.727***	(1.42×10 <sup>-33</sup> )***	2.462***					0.045
3	-1.658***	(1.42×10 <sup>-33</sup> )**	2.455***	-0.022***				0.046
4	-1.931***	(1.41×10 <sup>-33</sup> )***	2.549***	-0.060***	(7.36×10 <sup>-9</sup> )***			0.054
5	-1.931***	(1.44×10 <sup>-33</sup> )***	2.549***	-0.060***	(7.36×10 <sup>-9</sup> )***	-3.72×10 <sup>-6</sup>		0.054
6	-2.088***	(1.19×10 <sup>-33</sup> )***	2.292***	-0.131***	(4.16×10 <sup>-9</sup> )***	0.006***	1.130***	0.087

Denotes \*, \*\*, \*\*\* Statistical significance at 0.1, 0.05 and 0.01 respectively.

Source: Srisuksai, P. (2012). Idiosyncratic Volatility and Expected Stock Returns: Evidence from Thailand. Applied Economics Journal, 19 (2), 66-89. Note: This table reports the coefficients of the pooled panel data regressions using Fama and MacBeth (1973) model. The sample is the daily data from March 2001 to December 2011. Intercept is the constant stock return in each model.

This paper examines the role of conditional idiosyncratic volatility in six models using pooled panel, fixed effect, and random effect data regressions of Fama and MacBeth model for individual stocks and stock sectors in SET. The pooled and fixed effect panel data regressions come up with statistically positive effects of expected conditional idiosyncratic volatility on expected stock returns. In fact, it is significantly related to expected stock returns in all the models. The estimated coefficients from fixed effect panel data regressions on expected conditional idiosyncratic volatility are quite similar as show in Table 3, i.e. they vary little from 0.240 to 0.258.

Furthermore, the findings show that conditional idiosyncratic volatility plays a more important role than conditional market volatility in case of individual stocks. The average coefficient on conditional idiosyncratic volatility equals 0.245 which means that a change in 0.245% of conditional idiosyncratic volatility results in a change in 1% of stock returns in the next period. Moreover, the results of random effect panel data regressions are the same as the estimated coefficients from pooled and fixed panel data regressions. In other words, expected conditional idiosyncratic volatility has significantly positive effect on expected stock returns. Therefore, such impact could be explored in theoretical model in the next section.

**Table 3** Fixed effect panel data regressions of Fama and MacBeth model for individual stocks in SET50 index.

Model	<i>BETA</i>	<i>EIV</i>	<i>EBETA</i>	RS	VALUE	ILR	TURN	$\bar{R}^2$
1	-0.006	0.258***						0.75
2		0.241***	0.016**					0.75
3		0.240***	0.017**	-1.653***				0.75
4		0.242***	0.016*	-1.464***	$(8.15 \times 10^{-9})$ ***			0.76
5		0.242***	0.016*	-1.449***	$(8.15 \times 10^{-9})$ ***	-0.004***		0.76
6		0.243***	0.015*	-1.644***	$(4.53 \times 10^{-9})$ ***	-0.020***	2.110***	0.76

Note: This table reports the coefficients of the fixed effect panel data regressions using Fama and MacBeth (1973) model. The sample is the daily data from April 2001 to December 2009. Intercepts are not presented here due to an excess of data.

Denotes \*, \*\*, \*\*\* Statistical significance at 0.1, 0.05 and 0.01 respectively.

Source: Srisuksai, P. (2012). Idiosyncratic Volatility and Expected Stock Returns: Evidence from Thailand. *Applied Economics Journal*, 19 (2), 66-89.

## **The Model**

There are a few theoretical studies which consider the idiosyncratic risk so far. This fact motivates us to spell a decentralized economy out in order to explicitly explore asset pricing model. Thus, a purpose of this study is to examine a dynamic general equilibrium model which tries to show the effect of idiosyncratic risk on expected stock returns in the dynamic stochastic general equilibrium. In addition, this study also examines the role of an aggregate risk in asset pricing model. Therefore, the model designed to capture asset prices is an extension of Jermann and Quadrini (2009). It differs considerably from all asset pricing models in productivity shocks because such model is developed by introducing idiosyncratic shock into a production function.

There are two theoretical foundations of this underlying research. In fact, theoretical foundation of an asset pricing model with idiosyncratic risk is derived from a dynamic stochastic general equilibrium model. Another theoretical foundation is an empirical model for testing.

An asset pricing models in the past examine only the sources of systematic risk, for instance, labor income, money demand, inflation, habit formation, capital adjustment cost, debt, equity financing, and output. The assertions which account for such risk are stated in Alvarez and Jermann (2000), Belo (2010), Breeden (1979), Balvers and Huang (2007), Cochrane (1991, 1993, 1996), Constatinides and Duffie(1996), Gala (2005), Grishchenko( 2011), Jerman (1988, 2010), Kocherlakota(1996), Lettua (2003), Liu et al. (2009), Lucas (1978), Mehra and Prescott (1985), Rouwenhorst (1995), and Storesletten et al. (2001, 2007).

According to the empirical studies, CAPM is to be tested with data which in turn demonstrates a positive relationship between systematic risk and expected returns. As a result, idiosyncratic risk does not matter in determining expected returns. Nevertheless, later studies find out an influence of idiosyncratic risk. That is, an effect of such risk on expected returns is still not clear.

Systematic risk does matter in the CAPM. This particular characteristic is the same assertion as stated by other asset pricing models. The consumption-based capital asset pricing model (CCAPM) considers a consumption growth and an intertemporal marginal rate of substitution of consumptions as the important determinants, for example. Moreover, almost all of previous studies find the macroeconomic sources of risk that determine asset prices such as labor income, money demand, inflation, habit formation, capital adjustment cost, debt, equity financing, and output. Very few of asset pricing literatures have considered the role of idiosyncratic risk.

The well-known papers of Lucas (1978) and Breeden (1979) measure systematic risk in form of covariance of stock returns with stochastic discount factor, which is known as the CCAPM. This model implies that systematic risk can be explained by marginal rate of intertemporal substitution from utility function. Additionally, such model is developed further by Mehra and Prescott (1985) who appointed that expected returns depend solely on covariance of stock returns with consumption growth. That can be concluded that the systematic risk of expected returns can be described by consumption growth risk.

In addition to CCAPM, the asset pricing model is developed by taking the uncertainty into account of expected returns in the growth model. Storesletten et al. (2001, 2007) introduce idiosyncratic labor market risk in overlapping generation model. They state that idiosyncratic labor risk figures in determining expected returns, in turn, it can resolve the equity premium puzzle.

The aggregate productivity shock, on the other hand, is still considered as the main source of risky returns because it plays an important role in determining expected stock returns. Lettua (2003) indicates that the solution for asset pricing in a dynamic general equilibrium model. Technology shocks affect equity returns and real long-term bond returns through two channels: directly through the shock and indirectly through capital accumulation. The premiums of equity returns over the risk-free rate and real long-term bond are small and often negative when technology shock is permanent. The aggregate technology shock is an only one particular factor in Balvers and Huang (2007). Such aggregate shock is a key macroeconomic source of production-based asset pricing model conditional on the state of the economy which affects on asset returns. In other words, a pricing kernel is derived from production function arguments.

In addition, production-based asset pricing is derived by introducing some variables to account for expected stock returns and equity premium, such as habit formation preferences and capital adjustment cost (Jermann, 1998), endogenous solvency constraints (Alvarez and Jermann, 2000), capital adjustment cost and stochastic productivity (Jermann, 2010). In particular, Cochrane (1993) and Belo (2010) derive a stochastic discount factor in order to account for asset returns from equilibrium marginal rate of transformation instead of marginal rate of substitution for consumption. The stochastic discount factor depends on a growth rate in the price and a growth rate in the output of technology's goods.

An alternative approach of production-based asset pricing is first purposed by Cochrane (1991) who shows that investment returns is the same as stock returns. Cochrane (1996) then tests the investment-based asset pricing model. In fact, investment returns factors significantly price assets, and adjustment cost is useful in order to figure asset returns and investment returns. As a result,



this model is able to explain a wide spread in expected returns. Investment model performs substantially better than the standard consumption-based factor model. Consistent with Liu et al. (2009), such paper shows that stocks returns equal leveraged investment returns, which can be constructed from firm characteristics.

To some extent, production-based asset pricing model include the consumption side in order to demonstrate the economic fundamental behind the expected returns of growth stock and value stock as in Gala (2005). Such paper constructs a general equilibrium production economy with heterogeneous firms and irreversible investment. He documents that the dynamics of investors' demand for consumption insurance and irreversibility in firm investment play a key role in explaining value and size effects in stock returns.

In brief, most assertions as mentioned above focus on the effect of systematic risk on expected stock returns, which comes from the economic fundamental. Those conclusions are contrast with Merton (1987)'s paper which states that idiosyncratic risk has the predictive power within incomplete market. Such work shows that idiosyncratic risk is positively related to expected stock returns because of information cost and institutional factors. It is due to that investors do not fully diversify their portfolios under imperfect capital market, so idiosyncratic risk plays an important role. Still, later empirical studies document that idiosyncratic risk has a negative effect on expected stock returns.

As a result, there is a few studies of asset pricing model that provide the effect of idiosyncratic risk in determining stock returns in general equilibrium asset pricing model. In other words, such model does not account for an influence of idiosyncratic risk on stock returns yet. This is why this study tries to explore whether or not idiosyncratic risk affect on expected stock returns in the dynamic stochastic general equilibrium model. In addition, both idiosyncratic risk and aggregate productivity risk could be examined as the key determinants in determining expected stock returns similar to empirical research papers as performed before.

This economy model consists of two types of agent: infinitely-lived homogenous households and infinitely-lived heterogeneous firms. There are also two types of assets traded in this economy: real bonds and equity stocks. An infinitely representative household maximizes his expected life-time utility subject to budget constraint at each time. The infinitely heterogeneous firms maximize the expected present value of cash flow subject to their budget constraints. They differ in level of idiosyncratic productivity. All agents are in competitive market, and thus all prices are taken as given.

The homogenous households have to decide on how much consumption they will consume, how many bonds and stocks they will purchase at the beginning of period when they earn money from labor wages, bonds sold, stocks sold, and dividend payments at the end of period. Apart from consumption side, heterogeneous firms have to decide on how much they pay their debts, how much they pay dividends to the stock owners, how much they invest, and how many labors they hire. The funds available for this spending come from their outputs and future debts. At the end of period, bonds will be mature, and they are in zero net supply. In addition, equity stocks are in positive net supply in this economy. Therefore, such model economy then presents as the following and define the general equilibrium.

There are three subsections: households, firms, and equilibrium. The agents in each sector are optimizations together with the existence of markets. It leads to the equilibrium allocation in this economy.

### ***Households***

There are infinitely homogeneous households that will exist forever in this economy, so the economic behavior of the entire population can be modeled as a single representative household. An agent's endowment of time for each period has to divide into leisure,  $l_t$  and work  $h_t$ . For simplicity, such endowment is normalized to one, such that  $l_t + h_t = 1$ . Thus, household's preference is defined over stochastic sequences of consumption and leisure,

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, 1-h_t) \right\} ; \quad 0 < \beta < 1$$

where  $E_t(\bullet)$  is the expectation operator conditional on information available at time. The time  $t$  refers to time period from time  $t-1$  to  $t$ .  $c_t$  stands for consumption at time  $t$ .  $h_t$  stands for hours worked at time  $t$ .  $\beta$  is the subjective discount factor. The utility function is assumed to be twice continuously differentiable and strictly concave in both consumption and hours worked. This means that the first and second partial derivatives of utility function with respect to both arguments as follows:  $U_c > 0, U_h > 0, U_{cc} < 0, U_{hh} < 0$  and  $U_{cc}U_{hh} - (U_{ch})^2 > 0$ . Furthermore, household gets income from labor wage, bonds holding, equity stocks and dividend payments at time  $t$  to allocate for consumption, investment and lump-sum taxes financing on debt. There are two types of investment: holding bond issued by firms at time  $t+1$  and investing in equity stocks at time  $t+1$ . Thus, household's budget constraint can be written as

$$w_t h_t + \sum_i b_{it} + \sum_i s_{it} (d_{it} + q_{it}) = \sum_i \frac{b_{it+1}}{1+r_t} + \sum_i s_{it+1} q_{it} + c_t + T_t \quad (2)$$

where  $i$  represents firm  $i$ .  $w_t$  and  $r_t$  are wage rate and interest rate at time  $t$ .  $q_{it}$  is the price of equity stock  $i$  at time  $t$ .  $d_{it}$  represents the dividend payment received from firm  $i$  at time  $t$ .  $s_{it}$  represents the equity stocks for firm  $i$  at time  $t$ .  $b_{it}$  represents one-period bonds issued by firm  $i$  at time  $t$ .  $t_t$  are lump-sum taxes financing the tax benefits received by firms on debts, then  $T_t = \frac{B_{t+1}}{1+r_t(1-\pi)} - \frac{B_{t+1}}{1+r_t}$ , where  $\pi$  represents the tax benefit.

Taking all prices as given, a representative household will choose consumption, hours of work, investing in the next period bond, and investing in the next period equity stock to maximize expected discounted utility function (1) subject to a sequential budget constraint (2). This results in the optimality choices of the first-order conditions. However, the more details of computation of the first order conditions are given in Appendix. The solutions for household's optimization problem are the Euler equations as follows:

$$\left\{ \frac{U_h(c_t, 1-h_t)}{U_c(c_t, 1-h_t)} \right\} = w_t \quad (3)$$

$$\beta E_t \{ U_c(c_{t+1}, 1-h_{t+1})(1+r_t) \} = U_c(c_t, 1-h_t) \quad (4)$$

$$\beta E_t \left\{ U_c(c_{t+1}, 1-h_{t+1}) \left( \frac{d_{it+1} + q_{it+1}}{q_{it}} \right) \right\} = U_c(c_t, 1-h_t) \quad (5)$$

Equation 3 states that wage rate is equal to the expectation of ratio of marginal utility of hours of work to marginal utility of consumption. In other words, the wage rate is equivalent to the marginal rate of substitution between hours worked and consumption at the same time. Equation 4 shows that the risk-free asset returns equals to the marginal rate of intertemporal substitution between consumption at time  $t+1$  and consumption at time  $t$ . Additionally, the last Euler equation determines a stock price and a risky asset returns.

## Firms

This economy is also populated by infinitely heterogeneous firms which produce consumption goods. The outputs come from constant returns to scale of production functions that take labor  $h_{it}$  and capitals  $k_{it}$  as inputs. Capital depreciates at rate  $\delta$ . In particular, all firms face the uncertainties which consist of two types: aggregate stochastic productivity level  $z_t$  and idiosyncratic stochastic productivity level  $\varepsilon_{it}$ . Such idiosyncratic productivity makes all firms different in their levels of risk, so it becomes idiosyncratic stochastic risk. Thus, the production function of each firm has the following form:

$$F(z_t, \varepsilon_{it}, k_{it}, h_{it}) = (e^{\tilde{z}_t} \varepsilon_{it}) k_{it}^\theta h_{it}^{1-\theta} \quad (6)$$

where  $k_{it}$  is capital for firm  $i$  at time  $t$ .  $h_{it}$  is labor for firm  $i$  at time  $t$ .  $z_t$  is aggregate stochastic risk of all firms at time  $t$ .  $\varepsilon_{it}$  is idiosyncratic stochastic risk of firm  $i$  at time  $t$ .  $\theta$  is a capital share. Such aggregate stochastic risk is assumed further to follow a first-order autoregressive Markov process.

$$\begin{aligned} z_t &= \tilde{z} + \psi z_{t-1} + \nu_t & ; & & \nu_t \sim N(0, \sigma_\nu^2) \\ & & ; & & \mathbf{0} < \psi < \mathbf{1} \end{aligned} \quad (7)$$

where  $\nu_t$  is normally distributed with mean zero and constant variance, i.e.  $\nu_t \sim N(0, \sigma_\nu^2)$ . In addition, idiosyncratic stochastic risk also evolves according to a first-order autoregressive Markov process as the following.

$$\begin{aligned} \varepsilon_{it} &= \tilde{\varepsilon} + \tau \varepsilon_{it-1} + \mu_{it} & ; & & \mu_{it} \sim N(0, \sigma_\mu^2) \\ & & ; & & \mathbf{0} < \tau < \mathbf{1} \end{aligned} \quad (8)$$

where  $\mu_{it}$  is independently and identically distributed for firm at time with mean zero and constant variance, i.e.  $\mu_{it} \sim N(0, \sigma_\mu^2)$ .

More importantly, each firm can use debt in conjunction with output for investment, dividend payment, labor wage, and debt payment. Accordingly, the firm's budget constraint can be written as the following:

$$(1-\delta)k_{it} + F(z_t, \varepsilon_{it}, k_{it}, h_{it}) + \frac{b_{it+1}}{1+r_t} = b_{it} + d_{it} + k_{it+1} + w_t h_{it} \quad (9)$$

where  $b_{it}$  and  $b_{it+1}$  denotes debt in terms of bond issuing of firm  $i$  at time  $t$  and time  $t+1$ , respectively.  $d_{it}$  denotes dividend payment of firm  $i$  at time  $t$  to the ownership of equity stock. Furthermore, each firm operates business by maximizing his value of firm which equals to the present discounted value of cash flows. Consequently, the optimization problem can be written as recursive equation. That is, state variables for each firm are  $k_{it}$ ,  $\varepsilon_{it}$ ,  $z_t$ ,  $b_{it}$ ,  $H_t$ , and control variables are  $h_{it}$ ,  $k_{it+1}$ ,  $b_{it+1}$ ,  $d_{it}$ . Let's denote  $H_t$  as the summary of the next period information. Define the vector of state variables as  $X_{it} = (k_{it}, \varepsilon_{it}, z_t, b_{it}, H_t)$ . Thus, Bellman equation for optimal value of each firm is

$$V(X_{it}) = \max_{\{h_{it}, k_{it+1}, b_{it+1}, d_{it}\}} \{d_{it} + E_t(M_{t+1}V(X_{t+1}))\} \quad (10)$$

subject to

$$(1-\delta)k_{it} + (e^{\tilde{z}_t} \varepsilon_{it}) k_{it}^\theta h_{it}^{1-\theta} + \frac{b_{it+1}}{1+r_t} = b_{it} + d_{it} + k_{it+1} + w_t h_{it} \quad (11)$$

where  $M_{it+1}$  denotes the stochastic discount factor. In a competitive market, all prices are taken as given, and then each firm chooses current labor, the next capital, the next debt, and current dividend payment to maximize the present value of cash flow. Eventually, the efficiency conditions of firm come from the first- order conditions and Envelope conditions that are shown in Appendix. Such conditions take account of optimal choices of Euler equations as follows:

$$E_t \left\{ M_{it+1} \left[ (1-\delta) + \theta \left( e^{z_{it+1}} \varepsilon_{it+1} \right) k_{it+1}^{\theta-1} h_{it+1}^{1-\theta} \right] \right\} = 1 \quad (12)$$

$$E_t [M_{it+1}] = \frac{1}{1+r_t} \quad (13)$$

### Equilibrium

A solution for household and firm maximization problem as before must satisfy a recursive general equilibrium that defines as the following. The aggregate state variables in this economy are given by the aggregate capital  $K$ , aggregate stochastic productivity level  $z$ , aggregate bond  $B$ , and aggregate information  $H$ . That is,  $X = (K, z, B, H)$  and  $X_i = (k_i, \varepsilon_i, z, b_i, H)$  for firm  $i$ .

**Definition** A recursive general equilibrium for this decentralized economy is defined as a set of functions for (i) household's decision rules,  $c(X)$ ,  $h(X)$ ,  $b(X)$ , and  $s(X)$ ; (ii) firm's decision rules,  $h(X)$ ,  $k(X)$ ,  $b(X)$ , and  $d(X)$ ; (iii) a value function of firm  $V(X)$ ; (iv) price functions,  $w(X)$ ,  $r(X)$ ,  $q(X)$ ,  $M(X)$  such that household's decision rules satisfy the optimal conditions of equation 3, 4 and 5, and firm's decision rules satisfy the optimal conditions of equation 10 and 11. The resource constraints are also satisfied so that, at each time, all markets clear:

(i) The goods market:

$$C_t + I_t = Y_t$$

$$C_t + \sum_i (k_{it+1} - (1-\delta)k_{it}) = \sum_i \left( e^{z_i} \varepsilon_{it} \right) k_{it}^\theta h_{it}^{1-\theta} \quad (14)$$

where

$$\sum_i k_{it} = k_t$$

$$\sum_i h_{it} = h_t$$

(ii) The bond market:

$$\sum_i b_{it} = 0 \quad (15)$$

(iii) The stock market:

$$\sum_i s_{it} = 1 \quad (16)$$

In competitive market, all prices are taken as given. Therefore, the stochastic discount factor equals the intertemporal marginal rate of substitution between consumption at time  $t + 1$  and consumption at time  $t$ . This means that the rate at which agent is willing to substitute consumptions at time  $t + 1$  for consumption at time  $t$  equals the stochastic discount factor as follows:

$$M_{t+1} = \beta \left\{ \frac{U_c(c_{t+1}, 1-h_{t+1})}{U_c(c_t, 1-h_t)} \right\} \quad (17)$$

## Asset Price Implication

Asset price implication of this economy can be derived from Euler equation 5. This equation shows that the price of equity stocks at time  $t$  comes from the expected intertemporal marginal rate of substitution between consumption at time  $t + 1$  and consumption at  $t$  time as well as the next period dividend payout. This price can be rearranged in general form using forward iteration, so the price of equity stock  $i$  is

$$q_{it} = E_t \left\{ \sum_{j=1}^{\infty} \beta^j \frac{U_c(c_{t+j}, 1-h_{t+j})}{U_c(c_t, 1-h_t)} d_{it+j} \right\} \quad (18)$$

Denote  $R_{it+1}^s$  as the returns on stock  $i$  at time  $t + 1$ ; that is, returns on stock  $i$  at time  $t + 1$  can be defined as

$$R_{it+1}^s = \frac{q_{it+1} + d_{it+1}}{q_{it}} \quad (19)$$

Dividing both side of equation 5 by  $U_c(c_t, 1-h_t)$ , and substituting equation 19 into equation 5, Euler equation 5 then becomes

$$\beta E_t \left\{ \frac{U_c(c_{t+1}, 1-h_{t+1})}{U_c(c_t, 1-h_t)} R_{it+1}^s \right\} = 1 \quad (20)$$

Thus, by using equation 17 in Definition 2.1, such asset pricing equation can be written as

$$E_t \left[ M_{t+1} R_{it+1}^s \right] = 1 \quad (21)$$

Equation 21 implies that the expectation of the multiplication of stochastic discount factor and returns on stock is equal to one, which the same standard asset is pricing.

In addition, equation 12 can be rewritten as an asset pricing equation which shows the relationship between expected stock returns and idiosyncratic stochastic risk as the following.

Let's define

$$\Omega_{it+1} = \left[ (1-\delta) + \theta \left( e^{\varepsilon_{it+1}} \varepsilon_{it+1} \right) k_{it+1}^{\theta-1} h_{it+1}^{1-\theta} \right]$$

Substituting  $\Omega_{it=l}$  into equation 12, then such Euler equation becomes

$$E_t [M_{t+1} \Omega_{t+1}] = 1 \quad (22)$$

In other words, equation 22 can be written in form of probability on state  $s$  as

$$\sum_s \pi_s M_{t+1}(s) \Omega_{t+1}(s) = 1 \quad (23)$$

Define the right hand-side of equation 23 as the following.

$$\sum_s \pi_s M_{t+1}(s) \Omega_{t+1}(s) \equiv \sum_s P_s$$

Then, equation 23 becomes

$$\sum_s P_s = 1 \quad (24)$$

From the left hand-side of equation 21,  $E_t [M_{t+1} R_{it+1}^s]$  can be transformed into the probability on state  $s$  as

$$\sum_s \pi_s M_{t+1}(s) R_{it+1}^s(s) = 1 \quad (25)$$

Rearranging equation 25 by dividing and multiplying by  $\Omega_{it=l}(s)$ , so this equation can be written as

$$\sum_s \pi_s M_{t+1}(s) \Omega_{t+1}(s) \frac{R_{it+1}^s(s)}{\Omega_{t+1}(s)} = 1 \quad (26)$$

$$\sum_s P_s \frac{R_{it+1}^s(s)}{\Omega_{t+1}(s)} = 1 \quad (27)$$

$$E_p [R_{it+1}^s \Omega_{t+1}^{-1}] = 1 \quad (28)$$

Applying the Nikodym-Radon Theorem, the new probability measure is defined, instead of evaluating the uncertain prospect. With the underlying probability measures,  $\pi_s$ , new probability measure  $p_s = \pi_s M_{t+1}(s) \Omega_{t+1}(s)$  is used instead. As a result, equation 28 is the particular contribution of this study which explains that idiosyncratic stochastic risk has an effect on expected stock returns. In more detail, equation 28 can be instead solved in a way of an approximate analytical solution. Therefore, the method of log-linearization which first proposed by Campbell (1994) will apply to find the log-linear equation as the followings.

Let's define:  $\ln X_{it+1} = x_{it+1}$ , and  $x_i$  stands for a steady state value of  $x$ . In particular, equation 20 at steady state can be rewritten as follows:

$$\beta R_i^s = 1 \quad (29)$$

$$\ln R_i^s = \rho \quad (30)$$

where  $\rho$  is the rate of time preference.

At steady state,  $e^z = l$ , then equation 12 also becomes

$$1 - \delta + \theta \varepsilon_i k_i^{\theta-1} h_i^{1-\theta} = \frac{1}{\beta} \quad (31)$$

$$\theta \varepsilon_i k_i^{\theta-1} h_i^{1-\theta} = \rho + \delta \quad (32)$$

Let  $\hat{x}_t$  be a deviation from steady state at time  $t$ . The idiosyncratic productivity shock is assumed to be constant and equal to one at the steady state, i.e.,  $\varepsilon_i = l$ .

Consider  $R_{it+1}^s \Omega_{it+1}^l$  in equation 30, it can be rearranged and approximated in exponential term as follows:

$$\begin{aligned} \frac{R_{it+1}^s}{\left\{ (1-\delta) + \theta \left( e^{z_{t+1}} \varepsilon_{it+1} \right) k_{it+1}^{\theta-1} h_{it+1}^{1-\theta} \right\}} &= e^{r_{it+1}^s - \ln \left[ (1-\delta) + \theta \left( e^{z_{t+1}} \varepsilon_{it+1} \right) k_{it+1}^{\theta-1} h_{it+1}^{1-\theta} \right]} \\ &\cong 1 + (r_{it+1}^s - \rho) - \frac{1}{1 - \delta + \theta \varepsilon_i k_i^{\theta-1} h_i^{1-\theta}} \left\{ \begin{aligned} &\theta k_i^{\theta-1} h_i^{1-\theta} (\varepsilon_{it+1} - \varepsilon_i) \\ &+ \theta (\theta - 1) \varepsilon_i k_i^{\theta-2} h_i^{1-\theta} (k_{it+1} - k_i) \\ &+ \theta (1 - \theta) \varepsilon_i k_i^{\theta-1} h_i^{-\theta} (h_{it+1} - h_i) \end{aligned} \right\} \\ &\cong 1 + (r_{it+1}^s - \rho) - \frac{\theta \varepsilon_i k_i^{\theta-1} h_i^{1-\theta}}{1 + \rho} \cdot \varepsilon_{it+1} + \frac{\theta k_i^{\theta-1} h_i^{1-\theta}}{1 + \rho} \cdot \varepsilon_i \\ &\quad - \frac{(\theta - 1)(\rho + \delta)}{1 + \rho} \cdot \frac{k_{it+1} - k_i}{k_i} - \frac{(1 - \theta)(\rho + \delta)}{1 + \rho} \cdot \frac{h_{it+1} - h_i}{h_i} \\ &\cong 1 + r_{it+1}^s - \rho - \frac{\rho + \delta}{1 + \rho} \cdot \varepsilon_{it+1} + \frac{\rho + \delta}{1 + \rho} \\ &\quad - \frac{(\theta - 1)(\rho + \delta)}{1 + \rho} \cdot \hat{k}_{it+1} - \frac{(1 - \theta)(\rho + \delta)}{1 + \rho} \cdot \hat{h}_{it+1} \quad (33) \end{aligned}$$

Substituting result from equation 33 into equation 28, then we will obtain

$$1 + E_p r_{it+1}^s - \rho - \frac{\rho + \delta}{1 + \rho} \cdot E_p \varepsilon_{it+1} + \frac{\rho + \delta}{1 + \rho} - \frac{(\theta - 1)(\rho + \delta)}{1 + \rho} \cdot E_p \hat{k}_{it+1} - \frac{(1 - \theta)(\rho + \delta)}{1 + \rho} \cdot E_p \hat{h}_{it+1} = 1 \quad (34)$$

Rearranging equation 34, we obtain the log-linearized equation of expected stock returns as

$$E_p r_{it+1}^s = \rho + \frac{\rho + \delta}{1 + \rho} \left[ E_p \varepsilon_{it+1} + (\theta - 1) E_p \hat{k}_{it+1} + (1 - \theta) E_p \hat{h}_{it+1} - 1 \right] \quad (35)$$



Equation 35 is the main contribution in conjunction with equation 28 which show the relationship between expected returns on stock and idiosyncratic stochastic productivity level,  $\varepsilon_{it}$ . Since  $E_p \varepsilon_{it+1}$  is the conditional expectation which based on a different measure, its value can be different from  $E \varepsilon_{it+1}$ , which is equal to one. Equation 35 shows that the effect of idiosyncratic shock on the expected returns depends also on whether the conditional expectation is less than or greater than one. Consider the term  $E_p \varepsilon_{it+1} - 1$  in the bracket on the right hand side of equation 35, for all conditional expectation of idiosyncratic shock that has values less (greater) than one, the effect of idiosyncratic shock on the expected return is negative (positive). In case that  $E_p \varepsilon_{it+1}$  is exactly one, the idiosyncratic shock has no effect on the expected return.

## Conclusion

This study shows the relationship between idiosyncratic productivity level and expected returns on stock in forms of nonlinearity and linearity, respectively. This is the main finding we come up with new asset pricing model. The asset pricing model states that expected stock returns depends on the rate of time preference, depreciation rate, capital share, expected idiosyncratic productivity shock at time  $t + 1$ , the percentage deviation of capital from steady state at time  $t + 1$ , and the percentage deviation of labor from steady state at time  $t + 1$ . In other words, expected idiosyncratic productivity shock, expected capital, and expected labor affect on expected returns on stock since all the parameters are very small value. In addition, expected idiosyncratic productivity shock is positively related to expected stock returns similar to expected labor. Contrary to expected capital, it has a negative effect on expected stock returns since capital share  $\theta$  less than one. It is interestingly that the more capital used the less expected stock returns obtained. As a result, such equation takes idiosyncratic risk into account of expected stock returns as well as fundamental factors in production of firms.

So far, the asset pricing models do not consider idiosyncratic as an important factor in determining stock returns. It is because all of them assume that investors can construct efficient portfolio. This leads to elimination of diversified risk. Although there is a few studies try to examine the role of idiosyncratic risk, it does not develop model from the dynamic stochastic general equilibrium model. These results provide a new asset pricing models which derive from the dynamic stochastic general equilibrium model with idiosyncratic productivity shock. This theoretical finding demonstrates that, in equilibrium, idiosyncratic risk is an important determinant of expected stock returns. However, the effect of idiosyncratic shock on the expected returns depends also on whether the conditional expectation is less than or greater than one. This finding may help explaining the inconclusive evidences in empirical studies on this subject.

More importantly, idiosyncratic productivity level, which stands for idiosyncratic risk, has a positive effect on expected returns on stock. Such findings change the key determinants of asset pricing model from consumption to production factor and idiosyncratic productivity shock. Indeed, capital and labor affect on expected stock returns in different direction. The labor has a positive effect, but capital has a negative one. Still, this conclusion is contrary to Alvarez and Jermann (2000), Belo (2010), Breeden (1979), Balvers and Huang (2007), Cochrane (1991, 1993, 1996), Constatinides and Duffie (1996), Gala (2005), Grishchenko (2011), Jerman (1988, 2010), Kocherlakota (1996), Lettua (2003), Liu et al. (2009), Lucas (1978), Mehra and Prescott (1985), Rouwenhorst (1995), and Storesletten et al. (2001, 2007). As idiosyncratic component of expected returns is uncorrelated with the stochastic discount factor, results are consistent with empirical evidence for Thai stock market.

In more details, expected stock returns depends on the rate of time preference, depreciation rate, capital share, expected idiosyncratic productivity shock at time  $t + I$ , the percentage deviation of capital from steady state at time  $t + I$ , and the percentage deviation of labor from steady state at time  $t + I$ . In fact, expected idiosyncratic productivity shock, expected capital, and expected labor affect on expected returns on stock since all the parameters are very small value. The aggregate productivity level, however, does not show an effect on expected stock returns in our model.

## Discussion

The objective of this paper is to explore the asset pricing model which derives from the dynamic stochastic general equilibrium model with idiosyncratic productivity shock. It is quite similar contingent claims prices in complete market as stated in standard asset pricing model. That is, the preliminary result shows that the price of asset relies substantially on the stochastic discount factor. Once we carry out asset pricing to show expected returns on stock, idiosyncratic risk is an important determinant of expected stock returns in the last equation.

Such findings change the key determinants of asset price from consumption to production factor and idiosyncratic productivity shock. In terms of consumption, stochastic discount factor considerably determines stock returns which mean that the rate at which investor can give up consumption in time  $t + I$  in return for consumption in time  $t$  through buying and selling of stocks. This results in expected returns on stock for delay consumption.

Once we transform asset price model to expected stock returns model, idiosyncratic productivity level and production's factors affect expected returns on state  $s$ . Such productivity shock has positively predictive power. Even though it has similar procedure of consumption-based asset pricing model with contingent claims, the determined factors are very different. This result leads to the effect of idiosyncratic risk on expected stock return on state  $s$ . It is considerably different results from Alvarez and Jermann (2000), Belo (2010), Breeden (1979), Balvers and Huang (2007), Cochrane (1991, 1993, 1996), Constatinides and Duffie (1996), Gala (2005), Grishchenko (2011), Jerman (1988, 2010), Kocherlakota (1996), Lettua (2003), Liu et al. (2009), Lucas (1978), Mehra and Prescott (1985), Rouwenhorst (1995), and Storesletten et al. (2001, 2007). Idiosyncratic component of expected returns is uncorrelated with the stochastic discount factor.

Even though this study derives asset pricing model to show the effect of idiosyncratic risk on expected stock returns, such model does not test for whether it capture the data yet. It is because the limitation of data for testing such as labor used, capital used for each firm.

## Annex

### A. Solving household's maximization problem

Household's problem:

$$\max_{\{c_t, h_t, b_{it+1}, s_{it+1}\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, 1-h_t) \right\} ; \quad 0 < \beta < 1 \quad (1)$$

subject to

$$w_t h_t + \sum_i b_{it} + \sum_i s_{it} (d_{it} + q_{it}) = \sum_i \frac{b_{it+1}}{1+r_t} + \sum_i s_{it+1} q_{it} + c_t + T_t$$

Writing the problem in form of Lagrange function:

$$L = E_t \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, 1-h_t) \right\} \quad (2)$$

$$+ \eta_t \left\{ w_t h_t + \sum_i b_{it} + \sum_i s_{it} (d_{it} + q_{it}) - \sum_i \frac{b_{it+1}}{1+r_t} - \sum_i s_{it+1} q_{it} - c_t - T_t \right\}$$

To compute the first-order condition:

$$FOC_{c_t} : \beta^t U_c(c_t, 1-h_t) = \eta_t \quad (3)$$

$$FOC_{h_t} : \beta^t U_h(c_t, 1-h_t) = \eta_t w_t \quad (4)$$

$$FOC_{b_{it+1}} : \eta_{t+1} = \frac{\eta_t}{1+r_t} \quad (5)$$

$$FOC_{s_{it+1}} : \eta_{t+1} (d_{it+1} + q_{it+1}) = \eta_t q_{it} \quad (6)$$

To compute Euler equations, combining equation (3) with (4), we obtains

$$\left\{ \frac{U_h(c_t, 1-h_t)}{U_c(c_t, 1-h_t)} \right\} = w_t \quad (7)$$

Combining equation (3) with (5), yields

$$\beta E_t \{ U_c(c_{t+1}, 1-h_{t+1})(1+r_t) \} = U_c(c_t, 1-h_t) \quad (8)$$

Combining equation (3) with (6)

$$\beta E_t \left\{ U_c(c_{t+1}, 1-h_{t+1}) \frac{d_{it+1} + q_{it+1}}{q_{it}} \right\} = E_t U_c(c_t, 1-h_t) \quad (9)$$

Let's denote  $r_{it}$  as returns on stock  $k_{it}$  at time

$$\text{Define: } R_{it+1}^s = \frac{q_{it+1} + d_{it+1}}{q_{it}}$$

Substituting it into equation 9, this equation then becomes

$$\beta E_t \left\{ \frac{U_c(c_{t+1}, 1-h_{t+1})}{U_c(c_t, 1-h_t)} R_{it+1}^s \right\} = 1 \quad (10)$$

## B. Solving firm's maximization problem

Production function and constraints are as follows:

$$F(z_t, \varepsilon_{it}, k_{it}, h_{it}) = (e^{\tilde{z}_t} \varepsilon_{it}) k_{it}^\theta h_{it}^{1-\theta} \quad (11)$$

$$z_t = \tilde{z} + \psi z_{t-1} + v_t \quad ; \quad v_t \sim N(0, \sigma_v^2) \quad (12)$$

$$; \quad 0 < \psi < 1$$

$$\varepsilon_{it} = \tilde{\varepsilon} + \tau \varepsilon_{it-1} + \mu_{it} \quad ; \quad \mu_{it} \sim N(0, \sigma_\mu^2) \quad (13)$$

$$; \quad 0 < \tau < 1$$

$$(1-\delta)k_{it} + F(z_t, \varepsilon_{it}, k_{it}, h_{it}) + \frac{b_{it+1}}{1+r_t} = b_{it} + d_{it} + k_{it+1} + w_t h_{it} \quad (14)$$

The state variables are:  $X_{it} = (k_{it}, \varepsilon_{it}, z_t, b_{it}, H_t)$

The control variables are:  $h_{it}, k_{it+1}, b_{it+1}, d_{it}$

Writing the Bellman equation, we get

$$V(X_{it}) = \max_{\{h_{it}, k_{it+1}, b_{it+1}, d_{it}\}} \{d_{it} + E_t(M_{t+1}V(X_{t+1}))\} \quad (15)$$

subject to

$$(1-\delta)k_{it} + (e^{\tilde{\tau}_t} \varepsilon_{it}) k_{it}^\theta h_{it}^{1-\theta} + \frac{b_{it+1}}{1+r_t} = b_{it} + d_{it} + k_{it+1} + w_t h_{it}$$

Solving Bellman equation, then the maximization problem for firm is:

$$V(X_{it}) = \max_{\{h_{it}, k_{it+1}, b_{it+1}, d_{it}\}} \{d_{it} + E_t(M_{t+1}V(X_{t+1}))\} + \lambda_{it} \left\{ \begin{aligned} &(1-\delta)k_{it} + (e^{\tilde{\tau}_t} \varepsilon_{it}) k_{it}^\theta h_{it}^{1-\theta} \\ &+ \frac{b_{it+1}}{1+r_t} - b_{it} - d_{it} - k_{it+1} - w_t h_{it} \end{aligned} \right\}$$

The first-order conditions are:

$$FOC_{h_{it}} : (1-\theta) + (e^{\tilde{\tau}_t} \varepsilon_{it}) k_{it}^\theta h_{it}^{1-\theta} = w_t \quad (16)$$

$$FOC_{k_{it+1}} : E_t[M_{t+1}V_k(X_{t+1})] = \lambda_{it} \quad (17)$$

$$FOC_{b_{it+1}} : E_t[M_{t+1}V_b(X_{t+1})] + \frac{\lambda_{it}}{1+r_t} = 0 \quad (18)$$

$$FOC_{d_{it}} : 1 = \lambda_{it} \quad (19)$$

The envelope conditions are:

$$V_k(X_{it}) = \lambda_{it} \left[ (1-\delta) + \theta (e^{\tilde{\tau}_t} \varepsilon_{it}) k_{it}^{\theta-1} h_{it}^{1-\theta} \right] \quad (20)$$

$$V_b(X_{it}) = -\lambda_{it} \quad (21)$$

The Euler equations are as follows:

Combining  $FOC_{d_{it}}$  with  $FOC_{k_{it+1}}$  and  $V_k(X_{it})$ , we get

$$E_t[M_{t+1}V_k(X_{t+1})] = 1 \quad (22)$$

$$E_t \left\{ M_{t+1} \left[ (1-\delta) + \theta (e^{\tilde{\tau}_{t+1}} \varepsilon_{t+1}) k_{t+1}^{\theta-1} h_{t+1}^{1-\theta} \right] \right\} = 1 \quad (23)$$

Combining  $FOC_{d_{it}}$  with  $FOC_{b_{a+1}}$  and  $V_b(X_{it})$ , we get

From equation 19

$$I = \lambda_{it}$$

Substituting it into equation 18, we obtains

$$E_t \left[ M_{t+1} V_b(X_{t+1}) \right] + \frac{1}{1+r_t} = 0 \quad (24)$$

$$E_t [M_{t+1}] = \frac{1}{1+r_t} \quad (25)$$

Therefore, equation 23 and 25 are the particular Euler equations for derive further the relationship between idiosyncratic risk and expected returns which show the optimal choices.

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