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Mathematical Problem-Solving Knowledge for Teaching in Context of Lesson Study: A Case Study of Mathematics In-service Teacher

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Abstract

This study aimed to investigate an in-service mathematics teacher's mathematical problem-solving knowledge for teaching in a Lesson Study context at the Theerakarn Banhong School, Lamphun Province, Thailand. The study employed a qualitative research methodology. The research used a case study approach of a mathematics in-service teacher who took charge as a member of the school's lesson study team which was composed of 1) mathematics in-service teachers in the school and 2) coaches from a university to collaboratively work as participatory research design and was used as a method for a data collection. Collaboratively work on the Lesson Study cycle in the Thailand context (Inprasitha, 2011; 2015; 2022) of the lesson study team was recorded in a form of a videotape as evidence for content analysis (Bengtsson, 2016). Then, this evidence was analyzed in accordance with Mathematical Problem-Solving Knowledge for Teaching (MPSKT) (Chapman, 2015). Research results showed that the mathematics in-service teacher performed his teaching practice which manifests the MPSKT in accordance with the Lesson Study; 1) Collaboratively Plan, the teacher reviewed students' how-to-learn and designed problem situations, identified students' difficulties, and anticipated students' ideas. Moreover, the teacher designed main and supporting materials to encourage students' problematic states. 2) Collaboratively Do, the teacher supported students to utilize their own how-to-learn to solve problems which related to their experiences and the teacher encouraged students to solve problems by themselves. 3) Collaboratively See, the teacher reflected on clusters of students' ideas and discussed each idea that was affected by the students' how-to-learn. Moreover, the teacher reflected on conditions used in problem situations that related to students' ideas, to enhance the problem situations for the next loop of the lesson study. MPSKT in each step of the Lesson Study was discussed. Additionally, the MPSKT supported his teaching practices for teaching mathematics through problem-solving.

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Introduction

Currently, teaching through problem-solving is a priority for mathematics teachers. The focus on mathematical problem-solving has increased the concern of teachers' knowledge which is known in prior research as Mathematical Problem-Polving Knowledge for Teaching (MPSKT) (Chapman, 2015). In other words, research concentration on teacher knowledge has increasingly emphasized the knowledge for teaching mathematics through problem-solving. Especially, teaching mathematics through problem-solving based on a collaboration of various teachers to discuss and negotiate what is an authentic-problematic condition, which could be a major concern in how teachers establish the mathematical problem-solving knowledge for their teaching practices.

A lesson study approach has been adopted in some schools in Thailand since 2002. The lesson study approach focus is to aid in improving teachers' practices incorporated with a teaching approach called Open Approach. The lesson study is a scientific activity for teachers who try to develop their theories for developing and sharing good practices. The product of the lesson study is not limited to what each participant learns from the classroom and the post-class reflective discussion. It also involves each participant reproducing the class materials with their developed theories of practice in each of their contexts (Isoda, 2010).

The research question for this study was how the in-service teacher performs mathematical problem-solving knowledge for teaching when incorporated with the Open Approach and the lesson study as a method to improve the teaching approach (Inprasitha, 2011; 2015; 2022).

Objective

This research aimed to investigate the in-service teacher's mathematical problem-solving knowledge for teaching mathematics and to discuss his teaching practice based on the lesson study cycle.

Literature Reviews

1. Mathematical Problem-Solving Knowledge for Teaching (MPSKT) (Chapman, 2015)

Research concentrated on teacher knowledge has increasingly emphasized the knowledge for teaching mathematics through problem-solving. Teachers who have the knowledge for teaching mathematics through problem-solving need to hold knowledge of and for teaching problem-solving. In other words, this kind of knowledge must be broader than general problem-solving abilities, or the teachers need to embrace the knowledge of mathematical problem-solving for themselves as problem solvers, and also support students in mathematics classrooms to enhance the student's development of problem-solving.

Chapman (2015) divided the MPSKT into 7 components as follows: 1) Mathematical problem-solving proficiency as understanding what is needed for successful mathematical problem-solving, 2) Mathematical problems as understanding the nature of meaningful problems and the impact of problem characteristics on learners, 3) Mathematical problem-solving as being proficient in problem-solving, and understanding mathematical problem-solving as a way of thinking and implications of students' different approaches, 4) Problem posing as understanding problem posing before, during and after problem-solving, 5) Students as mathematical problem solvers as understanding what a student knows, can do, and is disposed to do, such as students' difficulties with problem-solving, 6) Instructional practice for problem-solving as understanding how and what it means to help students to become better problem solvers, and 7) Affective factors and beliefs as understanding the nature and impact of productive and unproductive affective factors and beliefs on learning and teaching problem-solving. The researchers, then, adopted the MPSKT components as a conceptual framework for the study.

Chapman (2015) described the details related to MPSKT as a complex network of interdependent knowledge. Therefore, apprehension of this interdependent knowledge is significant to help the teachers to possess the MPSKT effectively. Consequently, this is a meaningful and effective way of supporting teacher's problem-solving proficiency in their teaching and supports the students in learning mathematics through problem-solving simultaneously. The MPSKT should also include the nature of problem-solving in which teachers should understand problem-solving not only as a process of solving problems but also as a way of thinking to teach problem-solving proficiency. Components of the MPSKT are described in a way relating to the research context in the topic of a theoretical framework. Chapman (2015) noted that the MPSKT-category-based perspective does not provide a complete representation of MPSKT. The importance of

the whole MPSKT is not only the knowledge but knowing how to apply it effectively. Therefore, the teacher who grasps the MPSKT in meaningful and usable method for the classroom should recognize diverse methods of applying problem-solving, which is central to supporting problem-solving proficiency in his/her teaching. In addition, teachers' knowledge of and for teaching problem-solving proficiency needs to be broader than their general ability in problem-solving.

2. Teaching through Problem-Solving (Takahashi, 2021)

Teaching mathematics is not easy for many teachers, especially when it comes to engaging students in problem-solving. Although the national standards and curricula emphasize the importance of nurturing students to think mathematically and solve problems on their proficiency, many teachers hesitate to provide opportunities for students to think by themselves. Teaching through problem-solving, therefore, is quite hard for mathematics teachers. Most teachers' experiences in learning mathematics as a student were based on a traditional mathematics classroom where the teacher demonstrated how to get answers and the students imitated the examples to complete exercises. Takahashi (2021) mentioned that teaching mathematics through problem-solving is the unique Japanese pedagogical approach in which teachers teach new mathematical concepts by providing students with compelling mathematical challenges to solve problems by themselves and discuss ideas occurring in classrooms together. Accordingly, researchers show that providing students with opportunities to solve novel problems by themselves can support them to achieve the demanding mathematical practice standards outlined by the Common Core State Standards. Problem-solving has been the major focus of school mathematics education research for decades. Researchers corroborate that it is better for students to learn new mathematical concepts by trying to solve problems by themselves rather than by just imitating teachers and even other students. Teaching mathematics through problem-solving is an advanced pedagogical approach.

Lessons for teaching through problem-solving are refined over time. This is due to the Japanese professional development practice in which teachers collaborate to develop a specific plan for a mathematics classroom, teach students based on this plan, and discuss its impacts on the students learning based on the lesson observation, called Jyugyou Kenkyuu or Lesson Study.

3. Lesson Study and Open Approach Innovations (Inprasitha, 2011; 2015; 2022)

The traditional teaching approach in Thailand emphasizes on product-oriented approach, which means the teacher needs only one correct answer from the students. Consequently, from a product-oriented approach, there is an endeavor to the paradigm shift of teaching practices to a product-process-oriented approach by introducing the innovations of Lesson Study and Open Approach since 2002. In other words, the teacher needs to go beyond the answer and importantly the teacher needs to encourage students to state reasons for answers. Additionally, the process of solutions and the reason behind these solutions should also be emphasized. This endeavor has been improving the teaching practices of teachers consecutively.

The Open Approach to teaching is composed of four phases; 1) Posing open-ended problems, 2) Students' self-learning, 3) Whole class discussion and comparison, and 4) Summarizing through connecting students' mathematical ideas that emerged in the classroom. This teaching approach emphasizes individual differences in students' thinking. Teachers try to collect their students' ideas to conduct a summarization in accordance with the student's ideas. In addition, the lesson study approach takes responsibility to provide a place for the teachers to reflect on those ideas which occurred in the mathematics classroom. The Lesson Study approach focuses on improving the collaborative working of teachers for improving teaching , and is composed of three steps; 1) Collaboratively Design Research Lesson (Co-Plan), 2) Collaboratively Observe Research Lesson (Co-Do), and 3) Collaboratively Reflect on Teaching Practice (Co-See). This collaborative work is conducted in a week or weekly cycle. These two innovations are incorporated

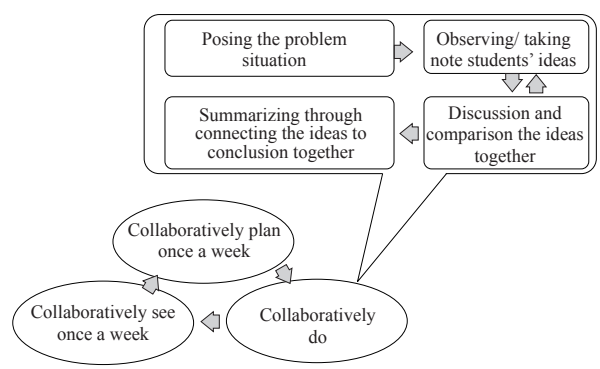

Figure 1 Weekly Cycle of Lesson Study incorporated with Open Approach (Adapted from Inprasitha, 2011; 2015; 2022)

in the second step of the Lesson Study or Collaboratively Do, called a Thailand Lesson Study incorporated Open Approach (TLSOA) (Inprasitha, 2022), as shown in Figure 1.

Conceptual Framework

The study employed the mathematical problem-solving knowledge for teaching (MPSKT) (Chapman, 2015) as a conceptual framework to investigate the MPSKT of the in-service teacher when collaborative working in the Lesson Study cycle as a way to improve teaching practice. The MPSKT was adapted to this research by contextualizing each element of MPSKT to this research context of Lesson Study and Open Approach.

1. Knowledge of Mathematical Problem-Solving Proficiency which is a comprehension of what students have learned or "how-to-learn" based on what students needed for mathematical problem-solving.

2. Knowledge of Mathematical Problems which is a comprehension of problem situations used for mathematical problem-solving, for example, purpose, structure, and subject matters of the problem situations.

3. Knowledge of Mathematical Problem-Solving which is a comprehension of mathematical problem-solving as a way of thinking to classify students' ideas from mathematical problem-solving. It also includes how to implicate students' unusual solutions or different approaches.

4. Knowledge of Problem Posing which is a comprehension of the significance of the teachers role in supporting students' understanding of mathematical problem-solving. For example, encouraging questions, and developing supporting materials used to extend students' ideas.

5. Knowledge of Students as Mathematical Problem Solvers which is a comprehension of students' difficulties when they encounter solving mathematical problems.

6. Knowledge of Teaching Practices for Problem-Solving which is a comprehension of the main materials for mathematical problem-solving. In addition, it could help students to recall their how-to-learn from previous lessons to encourage them to solve the problem situations.

7. Knowledge of Affective Factors and Beliefs which is a comprehension of students' problems (problematic) that affect mathematical problem-solving and nourish students' tolerance of solving problems.

Research Methodology

1. Research Context

A qualitative research methodology was employed in this study. A participative research design was adopted in which the researchers participated in the Lesson Study processes with the mathematics in-service teacher who served as a case study of the research.

The informant for this case study is based on the experience of a foreign teacher from the Philippines, who has worked for 5 years as a teacher in the mathematics program at Theerakarn Banhong School, Banhong District, Lamphun Province, Thailand. In the 2021 school year, the school launched the mathematics class for students in Grades 7-8 in the International Science Math and Technology Project. The case study and the researcher who has been a mathematics in-service teacher have taken responsibility for the mathematics teaching and learning of this project. The researcher from the school, then, asked for cooperation with coaches from the university who have experience related to the Lesson Study and Open Approach for more than 10 years. These groups of people are cooperating to work according to the Lesson Study cycle. In the 2022 school year, the group of researchers and teachers collaboratively worked in grades 7-9 mathematics class. The mathematics class takes 2 periods a week for teaching and learning mathematics in a way of providing opportunities for the students to think for themselves as a problem-solving classroom (Isoda, 2010).

To accomplish the research objective, the researchers decided to investigate a mathematics in-service teacher who was a member of a Lesson Study team at Theerakarn Banhong School. This team is included the researchers composed of 1) a mathematics in-service teacher in the school and 2) coaches from a university to collaboratively work on the case study. For 2 school years (WRITE THE YEARS), the school conducted teaching through online via the Google Meet caused by the COVID-19 pandemic. In this school year, the school returned to teaching on-site. In addition, the researchers from the Lampang Rajabhat University applied the Google Meet to Collaboratively Plan, Do, and See of the Lesson Study with the researcher in the school and the mathematics in-service teacher.

2. Research Participant

A mathematics in-service teacher previously taught English-Mathematics solely, but in the 2021 academic year, the school launched its inaugural mini-international class. He designed the teaching practice for

mathematics through collaborative planning of the lesson study. In addition, the students primary language was Thai and were being taught in English as the medium of instruction for mathematics.

The lesson study team collaboratively planned lessons by using Japanese textbooks and using them in the mathematics international class at the junior high school level. Additionally, the Lesson Study in the Thailand context (Inprasitha, 2011; 2015; 2022), is composed of 3 steps; 1) Collaboratively Design Research Lesson (Plan), 2) Collaboratively Observe Research Lesson (Do), and 3) Collaboratively Reflect on Teaching Practice (See), respectively. While the lesson study team collaboratively worked in accordance with the Lesson Study cycle via video conference by using the Google Meet platform, the researchers recorded videos as evidence for qualitative analysis of the in-service teachers' mathematical problem-solving knowledge for teaching.

3. Data Collection

A participatory research design was implemented to gain data about the mathematics in-service teacher's teaching practice using the Lesson Study and Open Approach. The data collection used video recording of the collaborative plan, do, and see in accordance with the Lesson Study steps.

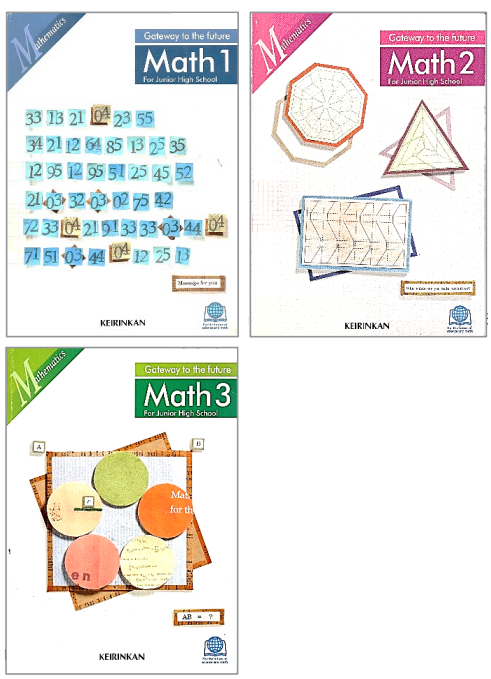

Figure 2 Keirinkan Junior High School Mathematics Textbooks (Reys, Reys, Shimizu, & Iwasaki, 2012)

4. Data Analysis

The data from the video recordings of teaching practices that followed the Lesson Study and Open Approach processes were analyzed using content analysis (Bengtsson, 2016). The data were further analyzed in accordance with the conceptual framework of the mathematics teacher's mathematical problem-solving knowledge for teaching (MPSKT) (Chapman, 2015).

Research Results

Results of the research showed that the in-service mathematics teacher performs the MPSKT in the steps of Lesson Study and Open Approach in accordance with the mathematics teacher's mathematical problem-solving knowledge for teaching (Chapman, 2015). The MPSKT of the research is described by each component of MPSKT in accordance with the steps of the Lesson Study as follows.

1. Knowledge of Mathematical Problem-Solving Proficiency

1.1) Collaboratively Plan The case study reviewed how-to-learn from previous lessons' subject matters and identified the goals of the present lessons by considering problem situations from the textbook pages.

1.2) Collaboratively Do The case study supported the students to use their how-to-learn from the previous lessons to understand the problem situations and solve the mathematical problems.

1.3) Collaboratively See The case study reflected on the students' how-to-learn or tools that the students used in solving the problems to establish the ideas that occurred in the classroom.

Figure 3 The mathematics in-service teacher and the researchers collaboratively considered students' how-to-learn from the previous lessons in step 1 of the Lesson Study

2. Knowledge of Mathematical Problems

2.1) Collaboratively Plan The case study discussed problem situations in textbooks. The topics of discussion referred to meaningful sentences that should be used in problem situations and directions. The sentences could support students to make sense of problem situations and instuction from their perspectives.

2.2) Collaboratively Do The case study developed problem situations based on student's experiences in order for students to relate to problem situations in an enhanced personal manner.

2.3) Collaboratively See The case study reflected on whether the students could make sense of the problem situations. Later, the case study considered enhancing problem situations for the next lessons and how to improve the same problem situations in the next year.

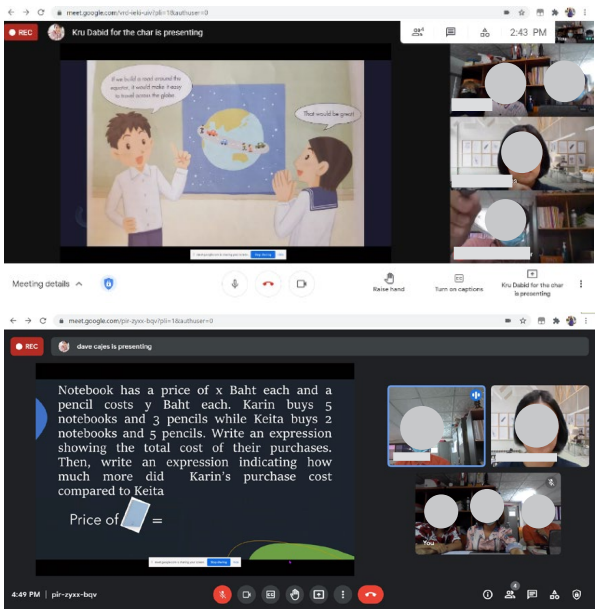

Figure 4 The mathematics in-service teacher cooperatively designed the problem situation with the researchers via an online meeting in step 1 of the Lesson Study

3. Knowledge of Mathematical Problem-Solving

3.1) Collaboratively Plan The case study identified students' difficulties and anticipated students' ideas by solving problem situations in the textbooks referring to the students' perspectives.

3.2) Collaboratively Do The case study provided the students with opportunities to solve the problems from their perspectives. Later, he collected and selected those students' ideas that occurred in the classroom to present to their peers and discuss with the whole class.

3.3) Collaboratively See The case study reflected on the student's mathematical ideas that occurred in the classroom and classified those ideas into groups for considering whether the ideas were in accordance to the subject matter and goals of the lessons.

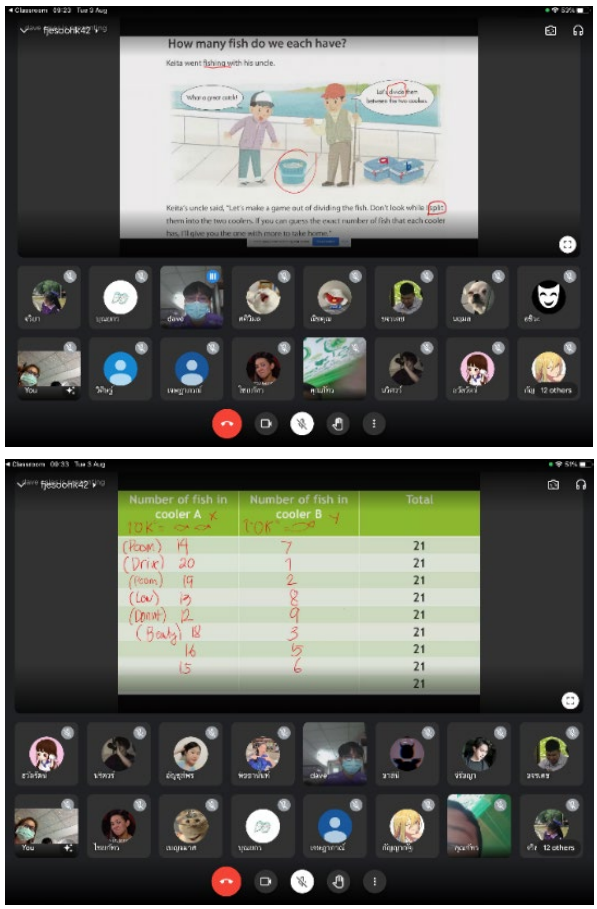

Figure 5 The mathematics in-service teacher collected the students' ideas from problem-solving of a simultaneous equation in step 2 of the Lesson study

4. Knowledge of Problem Posing

4.1) Collaboratively Plan The case study prepared questions to ask the students when they find obstacles with the problem situations and to show the ideas that could reach the goals of the lessons. Later, the case study arranged materials to support the students' ideas for other students to learn together.

4.2) Collaboratively Do The case study asked the question or adapted the supporting materials to

encourage the students to have strong reasons for their solutions or clear descriptions of their way of thinking.

4.3) Collaboratively See The case study reflected on how to improve the significant questions and supporting materials to enhance the students' ideas to become students' how-to-learn the lessons.

Figure 6 The mathematics in-service teacher prepared a picture of the glove to support the students to see the relationship between the equator and a 1-meter road above the globe in step 1 of the Lesson Study

5. Knowledge of Students as Mathematical Problem Solvers

5.1) Collaboratively Plan The case study considered the students' difficulties when they encountered the problem situations and then prepared the supporting questions or materials to encourage the students' ways of thinking.

5.2) Collaboratively Do The case study performed as he planned with his lesson study team. The team anticipated the difficulties of the students when they encountered problem situations and supported the students with the supporting questions or materials.

5.3) Collaboratively See The case study reflected on the student's difficulties that could be envisioned from the students' ideas. Therefore, he tried to use the students' ideas to plan and anticipate the difficulties and ideas that could occur in the classroom.

6. Knowledge of Teaching Practices for Problem-Solving

6.1) Collaboratively Plan The case study and his lesson study team designed materials that were used for presenting the problem situations and solving the problems. The materials were related to the textbooks and the lesson study team discussed whether they would be the same as textbooks by the goals of the lessons.

6.2) Collaboratively Do The case study carefully applied the main materials to pose the problem situations to the students and asked whether they understood the problem concisely.

6.3) Collaboratively See The case study reflected on the student's comprehension of the problem situations that related to the materials used to present the problem situations to the mathematics class.

7. Knowledge of Affective Factors and Beliefs

7.1) Collaboratively Plan The case study and the lesson study team discussed what kinds of conditions of the problem situations that support the students to generate the problematic state. Later, the entire lesson study team realized that the students were able to find methods to solve the problem in a productive way.

7.2) Collaboratively Do The case study presented the problem situations that related to the student's own experiences. This performance of the case study encouraged students' tolerance to solve the problems.

Figure 7 The mathematics in-service teacher supported the students to perceive their problem from the problem situation (problematic) in step 2 of the Lesson study

7.3) Collaboratively See The case study reflected on the problematic state of the problem situation was significant to the student's success in solving the problems.

Conclusion and Discussion

The performances of the mathematics in-service teacher or the case study in the context of the Lesson Study showed that he understood the MPSKT for his teaching practices in accordance with the Open Approach in the mathematic class and the Lesson Study as the improvement of the teaching approach in a weekly cycle. The case study could perform the MPSKT in every component of MPSKT through the Lesson Study steps with the lesson study team at Theerakarn Banhong School as described in the table below.

Table 1 The MPSKT of the Mathematics In-service Teacher in Lesson Study steps at Theerakarn Banhong School

Steps of Lesson Study	Mathematical Problem-Solving Knowledge for Teaching (MPSKT)
Collaboratively Plan	1) review how-to-learn from previous lessons 2) discuss problem situations in textbooks 3) identify students' difficulties and anticipate students' ideas 4) prepare questions to ask students and extend students' ideas with supporting materials 5) consider students' difficulties 6) design materials used for presenting problem situations 7) discuss what types of conditions of problem situations that generate problematic states
Collaboratively Do	1) support students to use how-to-learn from previous lessons 2) endeavor to support problems from problem situations 3) provide opportunities to solve problems from students' perspectives 4) ask questions or adapt supporting materials to encourage the students 5) offer support with follow-up questions or materials when students encounter difficulties 6) apply main materials to pose problem situations 7) present problem situations related to student's experiences
Collaboratively See	1) reflect on student's ideas that were affected by how-to-learn 2) reflect on whether students could make sense of the problem situations 3) reflect on student's mathematical ideas and classify those ideas as subject matters and lesson goals 4) reflect on how to improve significant questions and supporting materials 5) reflect on student's difficulties 6) reflect on student's comprehension of problem situations 7) reflect on problematic states of problem situations

The following discussion includes a description of the steps of the Lesson Study: Collaboratively Plan, Do, and See (Inprasitha, 2011; 2015; 2022) to discuss the MPSKT of each step of the Lesson Study used to run the mathematics class of the case study.

1. The mathematics in-service teacher performed the MPSKT in the Collaboratively Plan of the Lesson Study

The mathematics in-service teachers used the Japanese Textbook as the main material to consider the problem situations and objective of the lessons, the material design, students' ideas anticipation, and also planning how-to pose the problem situation, discuss and compare the student's ideas, and conclude based on the student's ideas. These actionsreflected the MPSKT that was possessed by the Japanese textbooks as Isoda (2010) stated that the junior high school textbook based on a problem-solving approach began to be published in the 1980s. In addition, Japanese textbooks are usually used as a curriculum sequence and employ the sequence of extension based on what the students have learned before (Isoda & Katagiri, 2012) or how-to-learn of the students.

2. The mathematics in-service teacher performed the MPSKT in the Collaboratively Do of the Lesson Study

The mathematics in-service teachers also performed throughout the Open Approach in the second step of the Lesson Study as Thailand Lesson Study and Open Approach (Inprasitha, 2022). The context of Lesson Study and Open Approach encourages the mathematics in-service teacher to be aware of MPSKT when he performed each step of Open Approach that encourage the students to have their problematic state and solve the problems by themselves (Isoda, 2010). In addition, teaching mathematics through problem-solving (TTP) (Takahashi, 2021) is similar to the teaching practice of the mathematics in-service teacher. In other words, this teaching practice could be classified into types of TTP Lessons; TTP lessons to develop conceptual and Procedural understanding, TTP lessons to expand understanding, and TTP lessons with multiple correct solutions.

3. The mathematics in-service teacher performed the MPSKT in the Collaboratively See of the Lesson Study

The mathematics in-service teachers reflected on the students' problematic state that would be depended on the students' experiences or real-world situations in which the students would have been confronted with in real life. This is related to Realistic Mathematics Education (Freudenthal, 1991) which explains realistic situations which serve as a source for the development of mathematical concepts, tools, and procedures. These realistic situations, moreover, function as a context in which students apply their mathematical knowledge,

and then gradually become more general and less context-specific knowledge and developed finally into formal knowledge that the student could apply in different contexts. Similarly, Gainsburg (2008) stressed the importance of real-world connections in teaching secondary mathematics. Moreover, K-12 mathematics teaching and learning have practically emphasized the importance of connecting school mathematics to real-world situations (National Council of Teachers of Mathematics, 2000; National Research Council, 1990).

Conclusion and Suggestions

The study of the mathematics in-service teacher's mathematical problem-solving knowledge for teaching in the Lesson Study context shows that the mathematics in-service teacher could perform the mathematical problem-solving knowledge for teaching according to the Lesson Study steps and Open Approach phases.

A recommendation for mathematics teachers is related to awareness of their teaching practices when they design a lesson plan, teach and/or observe a classroom, and reflect on teaching practice through problem-solving. In other words, the awareness is relevant to how to apply each component of MPSKT with subject matters of the lesson plans, also in phases of the learning and teaching through problem-solving which is challenging for mathematics teachers.

A suggestion for future research is the study about types of teaching through problem-solving (TTP) such as the following topics: 1) TTP Lessons to Develop Conceptual and Procedural Understanding, 2) TTP Lessons to Expand Understanding, and 3) TTP Lessons with Multiple Correct Solutions (Takahashi, 2021) as to determine whether teachers are following each TTP incorporated with MPSKT. In other words, the MPSKT of each type of TTP should be studied in-depth to provide information for teachers who will be able take precautions in their teaching practices through problem-solving.

In addition, MPSKT is more attended by mathematical teachers who need to develop problem-solving processes, and in the meantime develop processes of representing, communicating, reasoning, connecting, and creative thinking. In accordance with Foster, Wake, & Swan (2014) mentioned a conceptualization of mathematical processes by broadening mathematical knowledge for teaching (MKT) to include mathematical process knowledge (MKT) and pedagogical process knowledge (PPT) for teaching problem-solving that requires further study in future research.

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