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Hybrid analytical and simulation optimization approach for production and distribution supply chain planning

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Abstract

Supply chain planning consists of designing an optimal and feasible production and distribution plan for the whole supply chain. Traditionally, two common methods of optimization are analytical and simulation-based optimization, and each of them has pros and cons. In this paper, both methods are combined to consolidate the strengths of each, also known as the hybrid analytical and simulation approach. A case study of a multi-period, multi-echelon, and multi-product production and distribution problem that maximizes the whole supply chain's profit is introduced, to demonstrate the proposed hybrid approach. The analytical model is solved to find the optimal production-distribution plan, and then the plan is inputted into a simulation model, where uncertainties are incorporated. The proposed algorithm is then applied to identify a feasible plan that meets makespan limitation and service level requirements. Safety stock is incorporated to satisfy the service level requirements and maximize the supply chain's profit. This procedure continues iteratively until the production-distribution plan is feasible and optimized. The results show that the proposed approach can solve for a near or possibly optimal as well as feasible solution with relatively fast computational time.

Keywords: Production-distribution planning, Hybrid analytical-simulation optimization, Makespan, Safety stock

1. Introduction

A supply chain consists of activities involved in the production and distribution of products. A supply chain can generally be divided into two parts: production and distribution. Production consists of planning and control of the entire manufacturing process, such as production itself, material handling, scheduling, and inventory control. Distribution consists of processes that determine how products are retrieved and transported from the suppliers, manufacturer, or warehouse to customers, including the management of inventory retrieval, transportation, and final product delivery [1].

Supply chain optimization aims to plan and design the best production, storage locations, flow of materials among facilities, and transportation in the chain to either maximize the profit or minimize the costs. Two traditional optimization approaches are the analytical and simulation-based optimization methods. Analytical or mathematical model optimization can solve problems efficiently and quickly, but provides static information. In contrast, simulation-based optimization can solve problems under uncertainties and is capable of solving complex models. It is popular for solving most realistic problems but requires a long computational time and does not guarantee an optimal solution. Therefore, the hybrid analytical-simulation approach is proposed in this study, to be a useful way to reduce the computational time and provide realistic results. However, after the optimal production-distribution plan is achieved, the plan could be infeasible. For example, the optimal plan might suggest to produce all products in the first period for its low production cost. This creates too much workload in the first period and allowing the subsequent periods to be idle, which is unlikely to happen in real life because of the working time limit. The service level is one of the main requirements from customers that a company must consider. The optimal plan may suggest to ignore a customer completely as the shortage cost incurring from that customer is the least among all customers. Therefore, service level constraints must be introduced to ensure that all customers receive products no less than the minimum service level. It is imperative for the production-distribution plan to satisfy the service level requirement and complete before the working time limit imposed by the law or availability of labor. These two crucial requirements are addressed in this paper. First different types of optimization methods various methods of optimization have been introduced to solve problems in the supply chain, ranging from solving the optimal production plan to solving the optimal supply chain network. Beamon [1] provided a focused review of the literature in multi-stage and multi-shop supply chain modeling and suggested four categories of models: deterministic analytic, stochastic analytic, economic, and simulation. Deterministic analytic can be modeled simply and solved efficiently with mathematical formulation, however does not include uncertainty, while the stochastic analytic can consider uncertainty. Economic model is used as a framework for modeling the buyer-supplier relationship in a supply chain. Simulation is the use of software to imitate the behavior of a system that would otherwise be difficult to analyze in reality. In this study, we will focus on the deterministic analytic and simulation modelling.

Deterministic analytical optimization is one of the classic methods of optimization. Its ability to achieve a globally optimal solution makes it an ideal choice for optimization. However, its inability to incorporate uncertainties makes it difficult to represent real-life scenarios under various uncertainties. An example of analytical optimization is presented by Susarla & Karimi [2] where they solved for the optimal production and distribution plan that maximizes the profit of a pharmaceutical supply chain.

The simulation-based optimization model has been popular during the last decade with increasing computational power. Fu [3] indicated that current commercial software mainly combines heuristics and simulation in which a satisfactory solution can be obtained from working with the families of solutions. The biggest problem found, when applying simulation-based optimization, is that the stochastic nature of the systems is unaware. Thus, the efficiency of computation resources is not fully utilized. Variance reduction techniques are suggested in order to improve the convergence rate. Glover et al. [4] presented a simulation-based optimization model using a practical software system called OptQuest, which combines three metaheuristics to optimize decisions. Layeb [5] also used OptQuest to solve a scheduling problem in freight transportation. However, its long computational solving time has led to various modifications to lower the computational time. Chiadamrong & Kawtummachai [6] used a Genetic Algorithm (GA) to solve for the best transportation route and inventory position in the Thai sugar industry distribution system. The model solves for the minimal cost of sugar transport from the origin (mills) to the destination (seaports). The approach incorporates the uncertainties in the distribution network such as clients' demands, travel time, and order-picking time by using GA-based heuristics.

Second is the hybrid analytical-simulation methods to further increase the computational speed and get a better solution, many researchers started integrating the simulation model with the analytical model for achieving a reasonable computational time and solution. Acar et al. [7] proposed a hybrid method that uses mathematical optimization to solve for the optimal solution. Then, a simulation model of the analytical solution determines the impact of uncertainty on the objective function. The difference between the analytical and simulated objective function is included in the mathematical formulation. The process continues iteratively until a solution (with uncertainty impact from the simulation model) is found to be optimal in the current mathematical formulation. Thammatadatrakul & Chiadamrong [8] further modified Acar et al.'s [7] solving procedure. They used the modified hybrid method to find the optimal inventory control policy of a hybrid manufacturing–remanufacturing system. Chiadamrong & Piyathanavong [9] combined analytical and discrete event simulation models for optimizing an agricultural product supply chain network design. The proposed method solves the problem iteratively until the difference between the succeeding solutions satisfies the pre-determined termination criteria. The analytical model is used to solve for the best storage locations, and then the simulation-based optimization model improves the solution further under a stochastic environment with uncertainty arising from customer demand, plant production levels, and delivery lead time. The procedure ends when the solution cannot be further improved.

After the hybrid analytical and simulation method have sparked interest in the research area of optimization, many researchers started to apply this concept into various ways of optimization. Some studies have incorporated the utilization and capacities of the system as one of the criteria in determining the optimal solution. Ko et al. [10] proposed a hybrid optimization/simulation modeling approach for the design of a distribution network. GA-based heuristics was used to determine the dynamic distribution network. Then, the simulation model determines the best capacities of warehouses based on the level of service time. Suyabatmaz et al. [11] used Ko et al. [10] and Acar et al.'s [7] approach for solving the problem where the performance measures are related to the capacity utilization. Byrne & Bakir [12] proposed a hybrid analytical-simulation method to optimize the production planning in a multiperiod and multiproduct problem. Linear Programming (LP) was used to find the production level and inventory with inventory balance and capacity constraints. The LP solution is then inputted into a simulation model and checked for the capacity. The solving process stops when the capacity is feasible. Otherwise, the capacity in LP is adjusted and the solving process is repeated until the optimal and feasible solution is found. Lee & Kim [13] extended Byrne & Bakir's [12] study by increasing the complexity of the supply chain and modifying the capacity adjustment method. Lee & Kim [13] combined the analytic and simulation method to solve production-distribution problems in a supply chain. They optimized the problem analytically to obtain outputs that will be then be used as inputs in the next simulation procedure. Then, the output of the simulation, which is the makespan, was obtained and checked as to whether it met a certain criterion or not. If not, the capacity in the mathematical model was adjusted and recomputed, which is the beginning of a new iteration. This approach continues iteratively until the result from the simulation model meets the criterion and the procedure ends with the optimal production-distribution plan.

Many studies have also implemented the hybrid analytical-simulation optimization approach in real case studies with disparate constraints. Martins et al. [14] proposed a hybrid optimization and simulation approach for solving the wholesaler network redesign problem in the pharmaceutical business. The tradeoffs in the problem are the operational costs versus customer service levels. The tactical-strategic level problem is first solved by a mixed-integer linear programming (MILP) model. The capacity, number, and location of the warehouses are defined at this step. At the operational level, the solution from the MILP model is evaluated by the simulation model to evaluate the impact of the wholesaler's daily activities, and eventually the customer service level. de Keizer et al. [15] presented a hybrid optimization and simulation approach for finding the optimal network design for flowers under product quality requirement constraints. The mixed-integer linear programming (MILP) model is solved. Then, a discrete event simulation is used to check the feasibility of the plan given by the MILP, considering uncertainties in processing, supply, and transport. The initial MILP always suggests a plan with the lowest cost. However, the products will arrive at the retailer with too low a quality. A constraint on the approximated product quality is therefore added to the MILP model. The whole process is iteratively solved until a minimal cost solution is found that is feasible according to the simulation model. Table 1 summarizes different methods of optimization and various approaches of hybrid analytical-simulation optimization.

Table 1 Literature review of different methods of optimization.

Analytical optimization		Heuristic and simulation-based optimization		Hybrid optimization	
Authors	Area of interest	Authors	Area of interest	Authors	Area of interest
Susarla & Karimi (2012) [2]	Optimal production and distribution plan of pharmaceutical supply chain.	Fu (2002) [3]	Overview of simulation-based optimization.	Acar et al. (2009) [7]	Hybrid analytical-simulation method by determining the impact of uncertainty on the objective function.
		Glover et al. (1999) [4]	Introduction of simulation based-optimization tool called OptQuest.	Thammatadatrakul & Chiadamrong (2017) [8]	Optimal inventory control policy of a hybrid manufacturing-remanufacturing system.
		Layeb et al. (2018) [5]	Simulation-based optimization for scheduling freight transportation.	Chiadamrong & Piyathanavong (2017) [9]	Combination of analytical and simulation models for optimizing an agricultural product supply chain.
		Chiadamrong & Kawtummachai (2008) [6]	Optimal inventory position and transportation route in Thai sugar distribution system by Genetic Algorithm.	Ko et al. (2006) [10]	Hybrid analytical-simulation approach for the design of a distribution network.
				Suyabatmaz et al. (2014) [11]	Hybrid analytical-simulation approach for reverse logistic network design.

Analytical optimization		Heuristic and simulation-based optimization		Hybrid optimization	
Authors	Area of interest	Authors	Area of interest	Authors	Area of interest
				Byrne & Bakir (1999) [12]	Hybrid analytical-simulation approach by modifying the capacity adjustment method.
				Lee & Kim (2002) [13]	Hybrid analytical-simulation approach in production-distribution planning considering capacity constraints.
				Martins et al. (2017) [14]	Hybrid analytical-simulation approach in pharmaceutical business balancing the tradeoffs between cost and service level.
				de Keizer et al. (2015) [15]	Hybrid analytical-simulation approach in logistics network for perishable products under product quality constraints.

Most of the papers reviewed have shown that combining the analytical and simulation models can improve both the computational time and the solution. This has led us to pursue further approaches that can be implemented. The feasibility of a plan, including the working time limit in terms of limited makespan and possible required service level, has not been adequately considered in most studies. Therefore, the feasibility is considered in this study. As can be seen in most papers, the computational time is important, and potential methods to improve the computing speed are identified in this study.

2. Materials and methods

2.1 Case study

Supply chain models contain various structures, but generally, there are two distinguishing models, production and distribution models, which are the most vital parts. These two models can be linked together into one integrated model for a supply chain, which is called the production-distribution model. The production model consists of production plants where the manufacturing and transformation processes of materials occur. The distribution model consists of all locations that the products are stored and transported to, which commonly are upstream suppliers, distribution centers, warehouses, and downstream retailers.

This hypothetical problem is a multi-period, multi-echelon, and multi-product production and distribution problem. It is modified from Lee & Kim [13] to illustrate our hybrid analytical and simulation optimization approach. The flow in the supply chain in this study is illustrated in Figure 1. The production system consists of two shops, where Shop 1 produces N parts to be used for the production of M products in Shop 2. Each shop consists of 3 machine (MC), arranging in a flow line. Raw materials are inputted into the first machine of each shop, which are machine 1.1 and 2.1 (Figure 1). The arrows in Figure 1 shows the flow of production, starting from machine 1.1 and ending at machine 2.3. The quantity of products produced is determined by the demand from the retailers, where the products are sold, and revenue earned. It is assumed that products are sold at the end of each period, and the remaining unsold products are transferred to the next period, resulting in a holding cost.

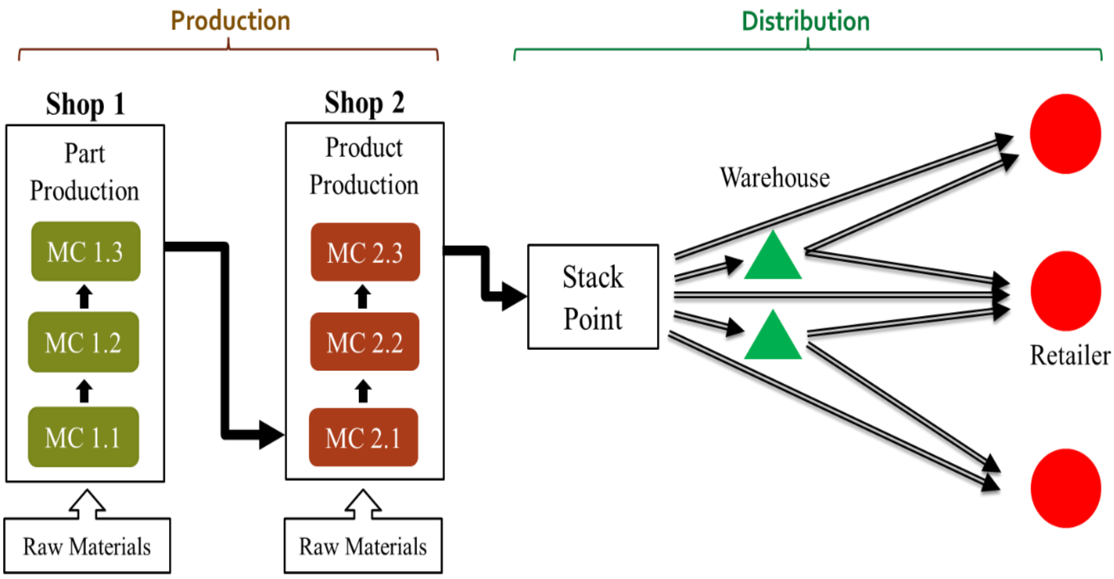

Figure 1 Production-distribution supply chain system in this study.

The distribution system consists of a stack point, warehouses, and retailers. The stack point is a location where the finished goods from production are initially stored before being sent to warehouses or retailers. Warehouses store the products before they are sent to retailers. Raw materials and parts are assumed to be stored in the production plant. It is assumed that backlogging is not permitted, so unsatisfied demand in the current period cannot be transferred to the next period and a shortage cost is incurred as lost sales. Products are directly transferred in a unit size from the stack point to warehouses or retailers. The initial inventory at every location is assumed to be zero. The goal is to maximize the supply chain profit, subject to various resource constraints. For demonstrative purpose, there are 3 periods in this study. Each period represents one working month. Each month is assumed to have 21,600 minutes of working time (30 days/month × 12 hours/day × 60 minutes/hour). Therefore, the makespan limit or the time until all products are produced and distributed must not exceed 21,600 minutes in each period. For other cases, this makespan limit may be the working hour limit imposed by law or the machine working capacity. The service level (*SL*), defined by Equation (1) is set to be at least 90% in this study for every product, retailer, and period.

$$SL = \frac{products\ sold}{demand} \tag{1}$$

Demand for each product at each retailer in each period is assumed to follow a normal distribution with a mean and a standard deviation (about 20% of the mean) as shown in Table 2. The number of units of raw materials or parts required to produce a unit of product or part is shown in Table 3. The Bill of Materials of both products is shown in Figure 2. The monetary unit in this study is the dollar ($).

Table 2 Mean and standard deviation of demand for product *j* at retailer *q* in period *t* (units).

Retailer (*q*)			1		2		3	
Product (*j*)			1	2	1	2	1	2
Mean Demand (D_{jqt})	Period (*t*)	1	14	12	14	16	14	12
		2	16	10	16	16	12	14
		3	14	14	14	14	12	14
Standard Deviation (sd_{jqt})	Period (*t*)	1	3	2	3	3	3	2
		2	3	2	3	3	2	3
		3	2	3	3	3	2	3

Table 3 Number of units of raw materials or parts required to produce one unit of product or part (units).

Part (*i*)		1	2	Product (*j*)		1	2	Product (*j*)		1	2
Material (*k*)	1	2	4	Material (*r*)	1	2	3	Part (*i*)	1	2	3
	2	3	2		2	2	2		2	3	4

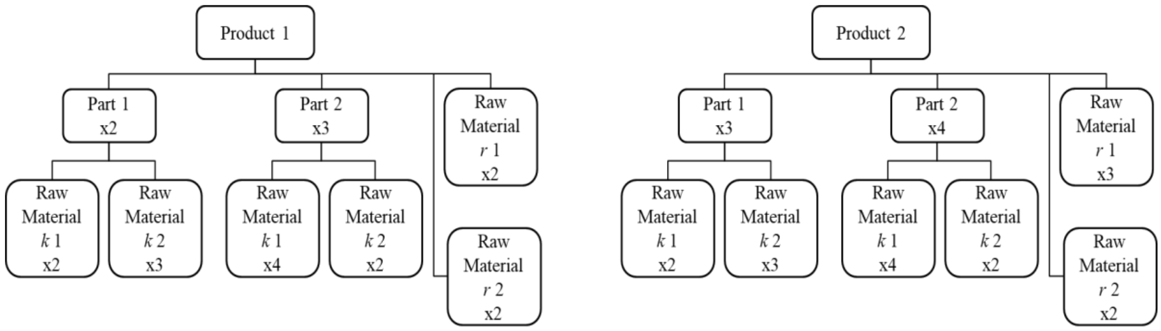

Figure 2 Bill of Materials for Products 1 and 2.

The production, holding, shortage, and distribution costs are shown in Tables 4 – 8. The transportation time from each origin to the destination is shown in Table 7. For example, transportation time from stack point to retailer 2 is 40 min and the cost is $25. The production time of each part and product is shown in Table 9. The holding capacity for each location in the supply chain is shown in Table 10.

Table 4 Production cost ($) of part *i*, product *j*, and raw materials *k* and *r* in each period *t*.

Part (*i*)		1	2	Product (*j*)	1	2	Raw Material (*k*)	1	2	Raw Material (*r*)	1	2
Period (*t*)	1	15	10		30	30		4	3		4	5
	2	15	10		30	30		5	4		6	6
	3	15	10		30	30		7	5		7	8

Table 5 Holding cost ($) of part *i* and raw materials *k* and *r* in each period *t* in the production plant.

Part (*i*)		1	2	Raw Material (*k*)	1	2	Raw Material (*r*)	1	2
Period (*t*)	1	12	10		5	5		5	5
	2	12	10		5	5		5	5
	3	12	10		5	5		5	5

Table 6 Holding cost ($) of product *j* at the stack point, warehouse *p*, and retailer *q*.

Product (*j*)			Stack Point	Warehouse (*p*)		Retailer (*q*)		
				1	2	1	2	3
1	Period (*t*)	1	10	20	15	30	20	30
		2	10	20	15	30	20	30
		3	10	20	15	30	20	30
2	Period (*t*)	1	10	15	20	40	50	40
		2	10	15	20	40	50	40
		3	10	15	20	40	50	40

Table 7 Transportation cost ($) and time (min) for all products.

Warehouse (*p*)		1	2	Retailer (*q*)	1	2	3	Retailer (*q*)		1	2	3
Transportation cost ($)	Stack	10	15	Stack	20	25	20	Warehouse (*p*)	1	20	20	15
									2	10	15	10
Transportation time (min)	Stack	80	90	Stack	50	40	50	Warehouse (*p*)	1	90	60	80
									2	80	70	90

Table 8 Shortage cost ($) of each product *j* at retailer *q* in period *t*.

Retailer (*q*)		1		2		3	
Product (*j*)		1	2	1	2	1	2
Period (*t*)	1	550	700	600	750	600	700
	2	550	700	600	750	600	700
	3	550	700	600	750	600	700

Table 9 Production time (min).

Shop 1					Shop 2				
		Machine (*u*)					Machine (*v*)		
		1	2	3			1	2	3
Part (*i*)	1	15	10	10	Product (*j*)	1	30	20	30
	2	15	15	5		2	30	30	20

Table 10 Holding capacity of each location in the distribution system (units).

		Stack Point	Warehouse (*p*)		Retailer (*q*)		
			1	2	1	2	3
Period (*t*)	1	10	1,000	1,000	20	20	20
	2	10	1,000	1,000	20	20	20
	3	10	1,000	1,000	20	20	20

2.2 Components in the hybrid analytical-simulation approach

The hybrid analytical-simulation approach starts with formulating and solving the analytical model by CPLEX optimization software. This model contains no uncertainty; therefore, it does not accurately reflect the reality where there might be some uncertainties involved. The decision variables or outputs from the solution are used as values of the inputs in the simulation model. The outputs from the simulation model are checked to see if a certain criterion is met. Otherwise, the mathematical model adjusts itself and continues the process iteratively until all criteria are met.

2.2.1 Analytical model

A Mixed-Integer Linear Programming (MILP) is used to solve the problem. The objective of the problem is to maximize the profit of the supply chain. The mathematical formulation and notation are presented as follows:
Indices:

t	period ($t = 1, 2, \ldots, T$)
i	part in Shop 1 ($i = 1, 2, \ldots, I$)
j	product in Shop 2 ($j = 1, 2, \ldots, J$)
u	machine in Shop 1 ($u = 1, 2, \ldots, U$)
v	machine in Shop 2 ($v = 1, 2, \ldots, V$)
k	raw material used for Shop 1 ($k = 1, 2, \ldots, K$)
r	raw material used for Shop 2 ($r = 1, 2, \ldots, R$)
p	warehouse ($p = 1, 2, \ldots, P$)
q	retailer ($q = 1, 2, \ldots, Q$)

Parameters:

D_{jqt}	demand for product j at retailer q in period t (units)
a_{ij}	units of part i required to produce one unit of product j (units)
d_{ki}	units of raw material k required to produce one unit of part i (units)
g_{rj}	units of raw material r required to produce one unit of product j (units)
ci_{it}	unit cost ($) of production for part i in period t
cj_{jt}	unit cost ($) of production for product j in period t
ck_{kt}	unit cost ($) of purchase for raw material k in period t
cr_{rt}	unit cost ($) of purchase for raw material r in period t
hi_{it}	unit holding cost ($) of part i in period t
hk_{kt}	unit holding cost ($) of raw material k in period t
hr_{rt}	unit holding cost ($) of raw material r in period t
HL_{jt}	unit holding cost ($) of product j at stack point in period t
HP_{jpt}	unit holding cost ($) of product j at warehouse p in period t
HQ_{jqt}	unit holding cost ($) of product j at retailer q in period t
SQQ_{jqt}	unit shortage cost ($) of product j at retailer q in period t
LPC_p	unit cost ($) of transporting any product from stack point to warehouse p
LQC_q	unit cost ($) of transporting any product from stack point to retailer q
PQC_{pq}	unit cost ($) of transporting any product from warehouse p to retailer q
LC_t	holding capacity of all products at stack point in period t (units)

PC_{pt} holding capacity of all products at warehouse *p* in period *t* (units)
QC_{qt} holding capacity of all products at retailer *q* in period *t* (units)
ai_{iu} processing time to produce a unit of part *i* on machine *u* (min)
aj_{jv} processing time to produce a unit of product *j* on machine *v* (min)
A_p time to transport any product from stack point to warehouse *p* (min)
B_q time to transport any product from stack point to retailer *q* (min)
C_{pq} time to transport any product from warehouse *p* to retailer *q* (min)
WLC_t workload capacity in period *t* (min)
$Price_j$ price of product *j* ($)
SS_{jq} safety stock of product *j* at retailer *q* (units)

Decision variables

X_{it} units of part *i* produced in Shop 1 in period *t* (units)
Y_{jt} units of product *j* produced in Shop 2 in period *t* (units)
Ii_{it} inventory of part *i* stored in the end of period *t* (units)
E_{kt} units of raw material *k* purchased in period *t* (units)
F_{rt} units of raw material *r* purchased in period *t* (units)
Ik_{kt} inventory of raw material *k* in period *t* (units)
Ir_{rt} inventory of raw material *r* in period *t* (units)
LP_{jpt} units of product *j* transported from stack point to warehouse *p* in period *t* (units)
LQ_{jqt} units of product *j* transported from stack point to retailer *q* in period *t* (units)
PQ_{jpqt} units of product *j* transported from warehouse *p* to retailer *q* in period *t* (units)
L_{jt} units of product *j* stored at stack point in period *t* (units)
P_{jpt} units of product *j* stored at warehouse *p* in period *t* (units)
Q_{jqt} units of product *j* stored at retailer *q* in period *t* (units)
$sold_{jqt}$ units of product *j* sold at retailer *q* in period *t* (units)
WL_t workload of system in period *t* (min)

Objective Function

$$Max\ Z = Profit$$
$$Profit = Revenue - (Production\ Cost + Holding\ Cost + Transportation\ Cost + Shortage\ Cost)$$

$$Profit = \sum_t^T \sum_q^Q \sum_j^J sold_{jqt} price_j - \sum_t^T \left\{ \begin{array}{c} \sum_i^I (ci_{it} X_{it} + hi_{it} \frac{Ii_{it} + Ii_{it-1}}{2}) \\ + \sum_j^J cj_{jt} Y_{jt} \\ + \sum_k^K \left(ck_{kt} E_{kt} + hk_{kt} \frac{Ik_{kt} + Ik_{kt-1}}{2} \right) \\ + \sum_r^R (cr_{rt} F_{rt} + hr_{rt} \frac{Ir_{rt} + Ir_{rt-1}}{2}) \\ + \sum_j^J HL_{jt} \frac{L_{jt} + L_{jt-1}}{2} \\ + \sum_j^J \sum_p^P HP_{jpt} \frac{P_{jpt} + P_{jpt-1}}{2} \\ + \sum_j^J \sum_q^Q HQ_{jqt} \frac{Q_{jqt} + Q_{jqt-1}}{2} \\ + \sum_j^J \sum_p^P LPC_p LP_{jpt} \\ + \sum_j^J \sum_q^Q LQC_q LQ_{jqt} \\ + \sum_j^J \sum_p^P \sum_q^Q PQC_{pq} PQ_{jpqt} \\ + \sum_j^J \sum_q^Q SQQ_{jqt} \left(D_{jqt} - sold_{jqt} \right) \end{array} \right\} \quad (2)$$

Subject to:

$$Ii_{it} = Ii_{it-1} + X_{it} - \sum_j^J a_{ij} Y_{jt}\ , \qquad \forall\ i, t \qquad (3)$$

$$Ir_{rt} = Ir_{rt-1} + F_{rt} - \sum_j^J g_{rj} Y_{jt}\ , \qquad \forall\ r, t \qquad (4)$$

$$Ik_{kt} = Ik_{kt-1} + E_{kt} - \sum_i^I d_{ki} X_{it}\ , \qquad \forall\ k, t \qquad (5)$$

$$L_{jt} = L_{jt-1} + Y_{jt} - \sum_p^P LP_{jpt} - \sum_q^Q LQ_{jqt}\ , \qquad \forall\ j, t \qquad (6)$$

$$P_{jpt} = P_{jpt-1} + LP_{jpt} - \sum_q^Q PQ_{jpqt}\ , \qquad \forall\ j, p, t \qquad (7)$$

$$Q_{jqt} = Q_{jqt-1} + \sum_p^P PQ_{jpqt} + LQ_{jqt} - sold_{jqt}\ , \qquad \forall\ j, q, t \qquad (8)$$

$$\sum_j^J L_{jt} \leq LC_t\ , \qquad \forall\ t \qquad (9)$$

$$\sum_j^J P_{jpt} \leq PC_{pt}\ , \qquad \forall\ p, t \qquad (10)$$

$$\sum_j^J Q_{jqt} \leq QC_{qt}\ , \qquad \forall\ q, t \qquad (11)$$

$$sold_{jqt} \leq D_{jqt}\ , \qquad \forall\ j, q, t \qquad (12)$$

$$WL_t = \sum_i^I \sum_u^U ai_{iu} X_{it} + \sum_j^J \sum_v^V aj_{jv} Y_{jt} + \sum_j^J \sum_p^P A_p LP_{jpt} + \sum_j^J \sum_q^Q B_q LQ_{jqt} + \sum_j^J \sum_p^P \sum_q^Q C_{pq} PQ_{jpqt}\ , \qquad \forall\ j, q, t \qquad (13)$$

$$WL_t \leq WLC_t\ , \qquad \forall\ t \qquad (14)$$

$$Q_{jqt} \geq SS_{jq}\ , \qquad \forall\ j, q, t \qquad (15)$$

$$X_{it}\ ,\ Ii_{it} \geq 0\ , \qquad \forall\ i, t \qquad (16)$$

$$Y_{jt} \geq 0\ , \qquad \forall\ j, t \qquad (17)$$

$$E_{kt}\ ,\ Ik_{kt} \geq 0\ , \qquad \forall\ k, t \qquad (18)$$

$$F_{rt}\ ,\ Ir_{rt} \geq 0\ , \qquad \forall\ r, t \qquad (19)$$

$$L_{jt} \geq 0\ , \qquad \forall\ j, t \qquad (20)$$

$$P_{jpt}\ ,\ LP_{jpt} \geq 0\ , \qquad \forall\ j, p, t \qquad (21)$$

$$Q_{jqt}\ ,\ LQ_{jqt} \geq 0\ , \qquad \forall\ j, q, t \qquad (22)$$

$$PQ_{jpqt} \geq 0\ , \qquad \forall\ j, p, q, t \qquad (23)$$

$$sold_{jqt} \geq 0\ , \qquad \forall\ j, q, t \qquad (24)$$

The objective of the analytical model is to find a production and distribution plan that maximizes the total profit of the supply chain under ideal conditions. The total profit of the supply chain is defined as the total revenue from all products sold at all retailers in all periods minus the total cost of the whole supply chain in all periods, as shown in Equation (2). The total cost consists of production and holding cost for all parts, products, and raw materials, transportation cost, and shortage cost at the retailers. Constraints (3) to (8) are the inventory balance constraints for their respective units of interest, ensuring that the product entering plus the inventory from the previous period equal the products leaving plus the inventory stored at the end of that period. Constraints (9) to (11) are the storage capacity of the stack point, warehouses, and retailers, respectively. Constraint (12) ensures that the sold products cannot be more than the demand in each period. Constraint (13) is the total workload of the system in each period. With a specified value of workload, this equation will determine the production and transportation quantity of the solution. Constraint (14) is the workload capacity constraint and is further discussed in the following section. Constraint (15) ensures that the inventory of product j at retailer q must be greater than the safety stock. Constraints (16) to (24) are the non-negativity constraints for all decision variables.

2.2.2 Simulation model

The simulation model is used to find the makespan, which is the total simulation run time. It is the time elapsed from the production of the first part in the first period to the last product delivered to the retailer in the last period. The mathematical formulation of the makespan is difficult because of the realistic nature of production, such as queueing and work that is done simultaneously. Makespan is usually mistaken for the workload, which is the total operational time for production and distribution calculated by the total number of units multiplied by the processing time. If it is assumed that there are no queues or simultaneous work in the system, the makespan is then equal to the workload. However, in reality, the system is more complicated. The actual makespan is too difficult to calculate. This problem is exacerbated by uncertainties such as machine breakdowns, as a machine breakdown increases the number of units waiting in the queue and possibly forces the following machines to be idle. Demand uncertainty is also incorporated into the simulation model following a normal distribution with a mean and a standard deviation, as shown in Table 2. To incorporate machine breakdown into the model, all machines are assumed to have an uptime that follows a normal distribution with a mean of 100 minutes and a standard deviation of 20 minutes. The downtime is also assumed to follow a normal distribution with a mean of 10 minutes and a standard deviation of 2 minutes. The simulation model is built in ARENA as shown in Figure 3, following the model configuration in Figure 1.

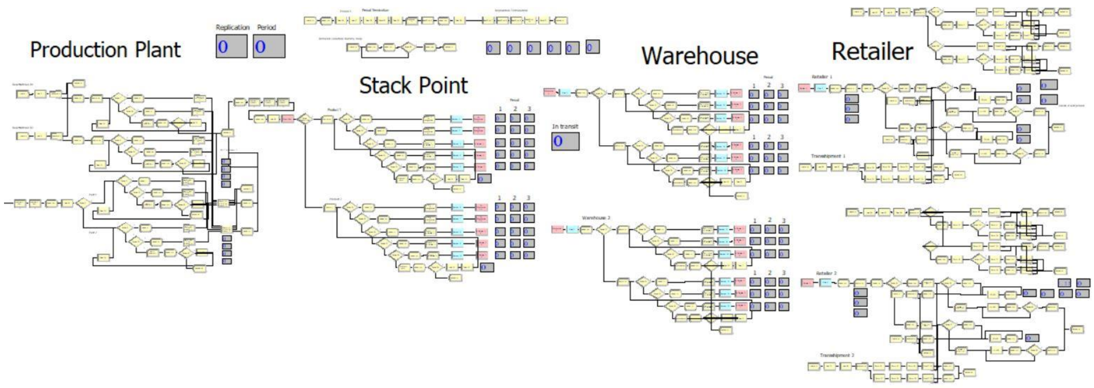

Figure 3 Production-distribution system modeled in ARENA software.

A total of 10 replications were simulated with the terminating condition between each replication. It was found that the 95% confidence interval of the objective value (profit of the chain) has a width of less than 5% of the mean.

2.3 Hybrid analytical-simulation approach

The proposed hybrid analytical-simulation approach is divided into two phases. Phase I is solved for a feasible plan that does not exceed the makespan limit and satisfies the service level requirements. Phase II is further solved for the best amount of safety stock by adding additional or decreasing excess safety stock (which increases the profit), resulting in the near optimal solution.

2.3.1 Phase I

The procedure of Phase I is illustrated in Figure 4. Phase I starts by calculating the initial safety stock and solving the analytical model. The result is an optimized ideal production-distribution plan without considering any uncertainties and makespan limits. With the safety stock as a preventive measure, it is assumed that the plan is feasible in terms of service level requirement. Phase II will further discuss the solution to the unsatisfied service level after the actual demand is realized. Having obtained the production-distribution plan, the plan is then inputted and simulated in the simulation model. This allows us to find the true makespan that otherwise would have been difficult to find in the analytical model. Uncertainties such as machine breakdowns, repair time, and demand fluctuation are also considered in the simulation model.

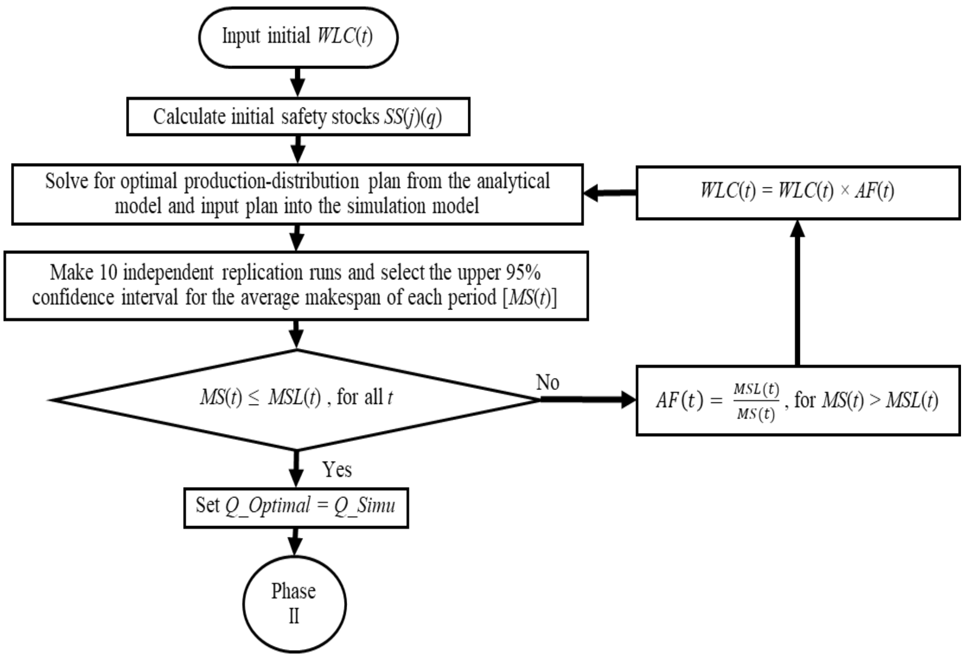

Figure 4 Phase I flowchart.

The makespan for each period is then checked if it exceeds the makespan limit. Any period that exceeds its makespan limit must have its production level cut down in that period or change modes of transportation, resulting in a shorter time but with a higher cost. Therefore, the workload capacity (WLC_t) for periods with a makespan exceeding the limit must be recalculated by Equation (25) and constraint (14) is updated in the analytical model. The adjustment of workload and mechanism combining the analytical and simulation model are made through the following formulation:

$$WLC_t = WLC_t AF_t \tag{25}$$

$$AF_t = \frac{MSL_t}{MS_t} \tag{26}$$

where $t = 1, 2, \ldots, T$ is the period. MSL_t is the makespan limit for each period t, which is set at 21,600 minutes for all periods in this case study. AF_t is the adjusting factor for each period t that exceeds the MSL_t in each period and can be calculated by Equation (26). The calculated AF_t is then used to calculate the WLC_t for the next iteration as seen in Equation (25). MS_t is the actual makespan found by the simulation model for each period t. The initial WLC_t must be set arbitrarily large to allow the analytical model to obtain the optimal solution. In this study, the initial WLC_t is set to 50,000 minutes. This adjustment mechanism of the workload capacity allows faster convergence to find an optimal and feasible solution. Periods with their makespan not exceeding the limit undergo no change in their WLC_t. The actual makespan in each period is found by simulating 10 independent replication runs and selecting the upper 95% confidence interval for the average makespan of each period. This ensures that a bad case scenario where the makespan may be higher than usual is employed for assurance. Each iteration is repeated until all periods satisfy the makespan limit.

The service level, defined by Equation (1), must be greater than 90%. Otherwise, the safety stock (SS_{jq}) should be added to satisfy the requirement. Adding safety stock in the analytical model will decrease the profit as the model does not consider demand uncertainty, incurring an additional holding cost. However, in the simulation model, increasing SS_{jq} can increase the profit as the shortage cost can be reduced with additional safety stock.
The initial SS_{jq} is calculated by Equations (27) – (29). y is first calculated by Equation (27) which is the density function of t-distribution from Chen [16] . However, y is in a standardized value. Therefore, y needs to be converted by Equation (28) into an observational value, x. SS_{jq} is the additional stock required to exceed the average demand and is calculated by Equation (29).

$$p = F(y|v) = p = \int_{-\infty}^{y} \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})} \frac{1}{\sqrt{v\pi}} \frac{1}{(1+\frac{t^2}{v})^{\frac{v+1}{2}}} dt \tag{27}$$

where $p = 0.90$ service level requirement
$v = n - 1$ degrees of freedom
$n = 10$ number of replications

then $y = \frac{x-\mu}{\sigma}$ t test statistic (28)
$SS = x - \mu$ safety stock (29)

x = total inventory required to achieve the 90% service level
$\mu_{jq} = \frac{\sum_t^T D_{jqt}}{T}$ average demand of each product j and retailer q
$\sigma_{jq} = \frac{\sqrt{\sum_t^T sd_{jqt}^2}}{T}$ average standard deviation of demand of each product j and retailer q

For example, 10 replications are run in the simulation model. Therefore, v is equal to 9. The goal is to find SS_{jq} that satisfies the 90% service level, so p is equal to 0.90. Product j at retailer q has a demand of 10 units in period 1, 12 units in period 2, and 8 units in period 3. The standard deviation of the demand (sd_{jqt}) is 2 units in period 1, 3 units in period 2, and 2 units in period 3. The average demand (μ) is therefore 10 units $[(10 + 12 + 8)/3]$. The average standard deviation (σ) is 1.37 units $[(\sqrt{2^2 + 3^2 + 2^2})/3]$. From Equation (27), y is equal to 1.38. Then, y is converted to observational units by Equation (28), which is equal to 11.89. This is rounded up to 12 units, which is the amount of stock required to fulfill the 90% service level. However, the analytical model produces an average demand of 10 units. Therefore, by Equation (29), the SS_{jq} is equal to 2 units.

The solution is now feasible in both makespan and service level requirements. The initial SS_{jq} is inputted into the analytical model and then the simulation model, to determine the current optimal profit (*Q_Optimal*) and service level (SL_{jqt}).

2.3.2 Phase II

Although the plan is now feasible, it may not yet be optimal. In this phase, the algorithm further fine-tunes or searches for the solution to further improve the profit. The algorithm in this phase is illustrated in Figure 5.

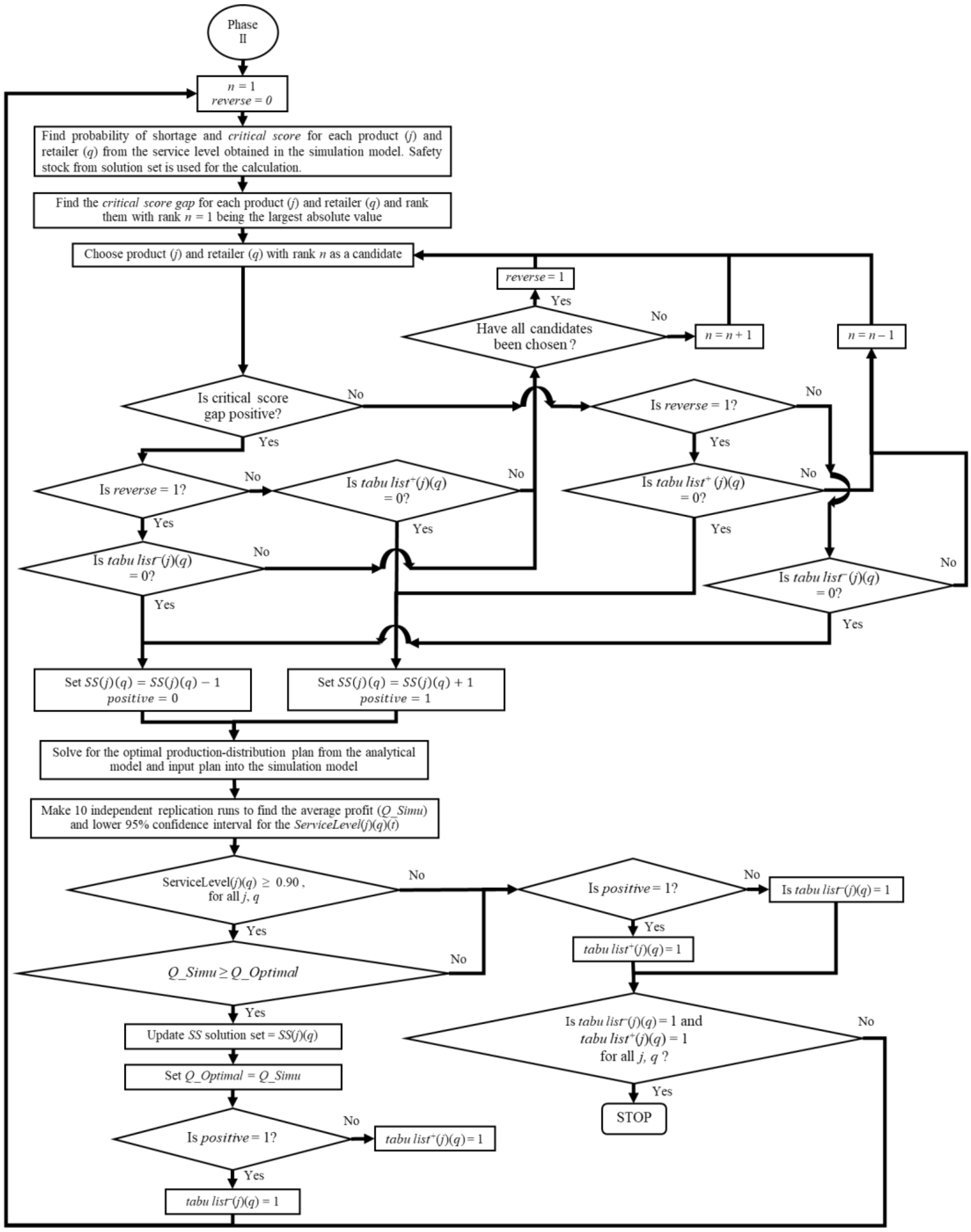

Figure 5 Phase II flowchart.

Phase II starts by finding the probability of shortage ($chanceshortage_{jq}$) by Equation (30). The critical score ($criticalscore_{jq}$) determines which product j at retailer q has the highest chance of increasing the profit. The calculation of the critical score is shown in Equation (31). All variables in Equation (31) have an impact on determining whether the safety stock should be increased or decreased. Products with a higher shortage cost can risk a huge penalty cost if products are out of stock. The inventory cost should also be considered, as products with a very high holding cost are less attractive to store (in large amounts). For example, product j at retailer q can have a very high shortage cost but a low probability of shortage. Therefore, additional safety stock is not needed for product j. Determining whether to increase or decrease the safety stock for product *j at* retailer q is important.

$$Chance\ shortage_{jq} = 1 - \frac{\sum_t^T Service\ Level_{jqt}}{T} \tag{30}$$

$$critical\ score_{jq} = \frac{shortage\ cost_{jq}}{inventory\ holding\ cost_{jq}} \times chance\ shortage_{jq} \tag{31}$$

$$critical\ score\ gap_{jq} = critical\ score_{jq} - \frac{\sum_j^J \sum_q^Q critical\ score_{jq}}{J \times Q} \tag{32}$$

The *critical score gap* is the difference of the critical score from the average critical score as shown in Equation (32), to determine which safety stock of product *j* at retailer *q* to increase or decrease first. One unit of safety stock is increased or decreased in each iteration to minimize the risk of missing the optimal solution. A positive *critical score gap* means that the safety stock of that product should be increased and a negative *critical score gap* means that the safety stock of that product should be decreased. Product *j* at retailer *q* with the highest absolute *critical score gap* will be considered first as a candidate. A positive value means increasing the safety stock and a negative value means decreasing the safety stock. After candidate SS_{jq} has been chosen, it is inputted into the analytical model and then simulated in the simulation model, to check whether the profit (*Q_Simu*) is increased and the service level is still feasible. If the profit is increased without affecting the service level feasibility, the SS_{jq} solution set and *Q_Optimal* are updated and the SS_{jq} candidate is added to the *tabu list*$^+$ (if SS_{jq} is decreased) or the *tabu list*$^-$ (if SS_{jq} is increased). SS_{jq} in the *tabu list*$^+$ will never be considered as a candidate for an increase. SS_{jq} in the *tabu list*$^-$ will never be considered as a candidate for a decrease. In Figure 5, *tabu list*$^+$(*j*)(*q*) = 1 means that product *j* at retailer *q* is in the *tabu list*$^+$. For example, if increasing SS_{jq} is shown to increase profit, it will not be reasonable to decrease that SS_{jq} in the following iterations. Therefore, the *tabu list*$^-$ will prevent this from happening. However, if the profit is not increased or the service level becomes infeasible, the SS_{jq} solution set will not be updated and the SS_{jq} candidate will be added to the *tabu list*$^+$ if SS_{jq} is increased or the *tabu list*$^-$ if SS_{jq} is decreased. This is the end of the first iteration.

The next iteration starts by recomputing the *critical score gap* if the SS_{jq} solution set has been updated. Otherwise, the previous iteration's *critical score gap* is used. After candidate SS_{jq} is chosen, if the SS_{jq} is in the *tabu list*$^+$ or *tabu list*$^-$ depending on whether the *critical score gap* is positive or negative, SS_{jq} with the next highest absolute *critical score gap* is selected as the candidate instead. For example, SS_{jq} of product *j* at retailer *q* has been chosen as a candidate based on having the highest absolute *critical score gap* that is originally positive. The SS_{jq} should be increased, but SS_{jq} is in the *tabu list*$^+$. In this case, the SS_{jq} with the second highest absolute *critical score gap* is selected as the candidate.

If all the SS_{jq} have been selected as candidates but cannot proceed further due to being in the *tabu list*, the procedure is reversed. Product *j* at retailer *q* with the lowest absolute *critical score gap* will be considered as a candidate. A positive value means decreasing the safety stock and a negative value means increasing the safety stock. This step allows a more thorough search for the potential optimal solution. For example, increasing SS_{jq} with a negative *critical score gap* can sometimes lead to a higher profit. If all the SS_{jq} are in both the *tabu list*$^+$ and the *tabu list*$^-$, the algorithm stops, as no candidates can be selected. The near or possibly optimal and feasible solution is then found.

3. Results and discussion

Visual Basic for Applications (VBA) is implemented to iteratively solve the analytical model coded in CPLEX and the simulation model coded in ARENA.

3.1 Phase I result

The makespan and workload capacity of each iteration in Phase I are shown in Table 11 and illustrated in Figure 6. The production plan and the profit for each iteration are shown in Table 12. The distribution plan for each iteration is shown in Table 13. From the results in Table 11, it was found that the initial solution (iteration 1) from the analytical model is optimal but infeasible, as the makespan in period 1 (26,768 min) exceeds the makespan limit of 21,600 min as shown in circle A. The model suggests the production of everything in the first period because of the low raw material cost, as shown in Table 12 (circle C for product 1). In the next iteration, the analytical model adjusts its workload constraint to reduce the production level in period 1 and push the production to other periods, as shown in circle D. This results in a balanced production throughout the timeline and the plan becomes feasible, as shown in Table 11 circle B. For the distribution plan as shown in Table 13, the model first suggests transporting the products to warehouses to be stored as the cost is lower (circle E), but later iterations suggest directly transporting the products to retailers as the transportation time is shorter (circle F).

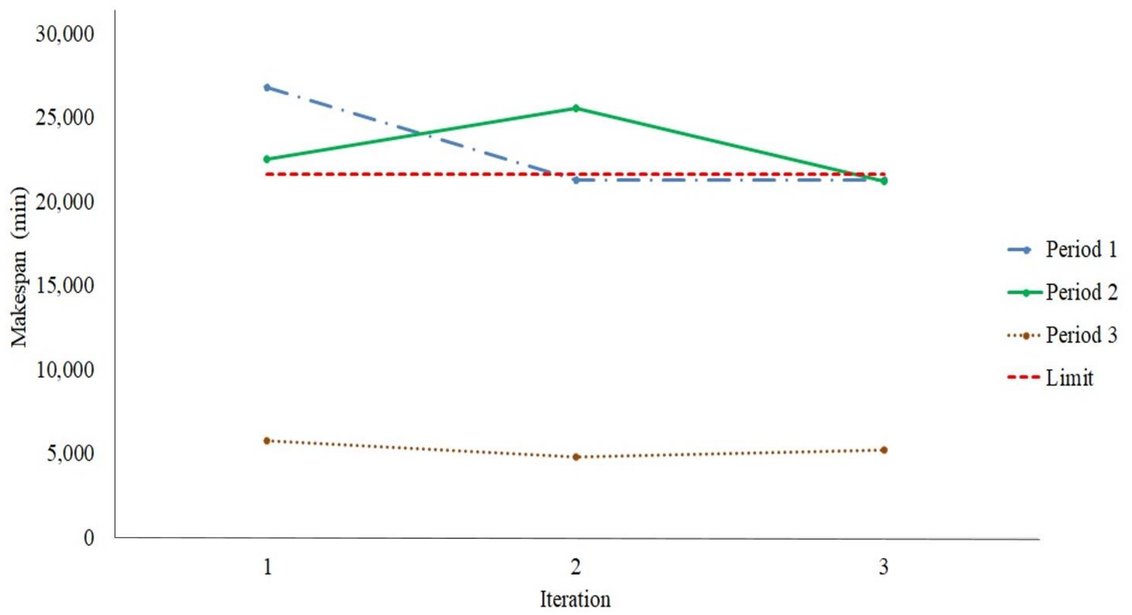

Figure 6 Makespan of each period *t* (MS_t).

Table 11 Workload capacity (WLCt) and makespan (MSt).

	Period (*t*)		Iteration 1	2	3	
Workload capacity (*WLC*) min	1		50,000	40,347	40,347	
	2		50,000	47,912	40,492	
	3		50,000	50,000	50,000	
Makespan (*MC*) min	1	A	26,768	21,311	21,296	B
	2		22,541	25,558	21,180	
	3		5,800	4,780	5,183	
Profit (*Q_MILP*) $			89,028	87,999	87,343	
Profit (*Q_SIMU*) $			86,433	85,542	85,294	

Table 12 Production levels and profit.

		Period (*t*)	Demand (units)		Iteration 1	2		3
Part (*i*) (units)	1	1	200		376	305		305
		2	204		291	344		300
		3	196		0	18		62
	2	1	280		520	426		425
		2	286		413	480		415
		3	274		0	27		93
Product (*j*) (units)	1	1	40		56	55		55
		2	42	C	75	67	D	45
		3	38		0	9		31
	2	1	40		88	65		65
		2	40		47	70		70
		3	40		0	0		0
Profit (*Q_MILP*) $					89,028	87,999		87,343
Profit (*Q_SIMU*) $					86,433	85,542		85,294

Table 13 Distribution plan.

			Iteration (*s*)		
Products (*j*)	Period (*t*)	Distribution routes	1	2	3
1	1	L – P	0	0	0
		L – Q	55	55	55
		P – Q	0	0	0
	2	L – P	38	19	0
		L – Q	38	48	45
		P – Q	0	0	0
	3	L – P	0	0	0
		L – Q	0	9	31
		P – Q	38	19	0
2	1	L – P	26	2	2
		L – Q	53	53	53
		P – Q	0	0	0
	2	L – P	6	30	30
		L – Q	40	40	40
		P – Q	0	0	0
	3	L – P	0	0	0
		L – Q	10	10	10
		P – Q	32	32	32
Profit (*Q_MILP*) $			89,028	87,999	87,343
Profit (*Q_SIMU*) $			86,433	85,542	85,294

Remarks: L = Stack point
P = Warehouse
Q = Retailer

From Equations (27)-(29), the initial safety stock (SS_{jq}) is calculated and shown in Table 14, where the results from excluding and including the initial safety stock are compared. The makespan has not changed much despite producing more products because of the change in the modes of transportation that reduces the makespan but increases the cost. However, by increasing the safety stock, the shortage cost is reduced, increasing the profit and eventually the service level.

Table 14 Makespan, minimum service level, and profit: before and after introducing SS_{jq}.

SS_{jq}	No safety stock	With initial safety stock
SS_{11}	0	4
SS_{12}	0	5
SS_{13}	0	4
SS_{21}	0	4
SS_{22}	0	5
SS_{23}	0	4
Makespan (min)		
Period 1	21,479	21,296
Period 2	21,392	21,180
Period 3	4,660	5,183
Minimum Service Level	78.12%	90.61%
Q_Simu ($)	77,687	85,294

3.2 Phase II result

After the makespan and service level are feasible for all periods, the algorithm checks whether the solution can be further improved by adding or removing the safety stock. The result of the safety stock solution set (SS_{jq}) of each iteration is shown in Table 16. The underlined value represents the SS_{jq} candidate selected in that iteration. *Q_Optimal* is the current optimal solution from the previous iterations. Phase II stops after all product *j* at retailer *q* are in the *tabu list*$^{+}$ and *tabu list*$^{-}$, as shown in Table 17 (iteration 13). In each iteration, the SS_{jq} that is added to either the *tabu list*$^{+}$ or the *tabu list*$^{-}$ is represented by the value 1. The approach of choosing a SS_{jq} candidate is demonstrated in Table 15. After the initial safety stock is calculated, an SS_{jq} candidate must be selected to decide whether to increase or decrease the safety stock for product *j* at retailer *q*. $chanceshortage_{jq}$ is calculated from Equation (30) and later used to calculate the *critical score gap* from Equation (32). The highest absolute *critical* *score gap*, which is SS_{21}, is selected as the candidate and should be increased because of its positive *critical score gap* value. The resulting impact of increasing SS_{21} is shown in Table 16 at iteration 2. As increasing SS_{21} increased the profit and satisfies the service level requirement, SS_{21} is added to the *tabu list*$^{-}$ as shown in Table 17 to never be chosen as a candidate to be decreased. From Table 16, the near or possibly optimal solution is found in iteration 7 with the profit equal to \$85,975. This solution has the highest profit (*Q_Simu* = *Q_Optimal*) that satisfies the service level requirement when all SS_{jq} are in both the *tabu list*$^{+}$ and the *tabu list*$^{-}$.

Table 15 Values used in selecting the SS_{jq} candidate in iteration 1.

SS_{jq}	SS_{11}	SS_{12}	SS_{13}	SS_{21}	SS_{22}	SS_{23}
Safety stock (units)	4	5	4	4	5	4
$chanceshortage_{jq}$	0.0123	0.0135	0.0222	0.0273	0.0221	0.0103
$criticalscore_{jq}$	135.32	242.31	266.77	382.20	265.26	154.41
$critical\ score\ gap_{jq}$	-105.73	1.26	25.72	141.16	24.22	-86.63

Table 16 Safety stock solution set of each iteration.

Iteration	SS_{11}	SS_{12}	SS_{13}	SS_{21}	SS_{22}	SS_{23}	*Q_SIMU* (\$)	*Q_Optimal* (\$)	Profit Increased?	Service Level satisfied?
1	4	5	4	4	5	4	85,294	–	–	Yes
2	4	5	4	5	5	4	85,774	85,294	Yes	Yes
3	3	5	4	5	5	4	85,775	85,774	Yes	No
4	4	5	5	5	5	4	85,824	85,774	Yes	Yes
5	4	5	5	5	6	4	85,814	85,824	No	Yes
6	4	6	5	5	5	4	85,840	85,824	Yes	Yes
7	4	6	5	5	5	3	85,975	85,840	Yes	Yes
8	4	6	6	5	5	3	85,864	85,975	No	Yes
9	4	6	5	6	5	3	85,828	85,975	No	Yes
10	4	6	5	5	5	2	85,619	85,975	No	Yes
11	5	6	5	5	5	3	85,856	85,975	No	Yes
12	4	7	5	5	5	3	85,872	85,975	No	Yes
13	4	6	5	5	4	3	85,956	85,975	No	No

Table 17 SS_{jq} in *tabu list*$^{+}$ and *tabu list*$^{-}$ on each iteration.

Iteration	*tabu list*$^{+}$						*tabu list*$^{-}$					
	SS_{11}	SS_{12}	SS_{13}	SS_{21}	SS_{22}	SS_{23}	SS_{11}	SS_{12}	SS_{13}	SS_{21}	SS_{22}	SS_{23}
1	–	–	–	–	–	–	–	–	–	–	–	–
2										1		
3							1			1		
4							1		1	1		
5					1		1		1	1		
6					1		1	1	1	1		
7					1	1	1	1	1	1		
8			1		1	1	1	1	1	1		
9			1	1	1	1	1	1	1	1		
10			1	1	1	1	1	1	1	1		1
11	1		1	1	1	1	1	1	1	1		1
12	1	1	1	1	1	1	1	1	1	1		1
13	1	1	1	1	1	1	1	1	1	1	1	1

3.3 Result comparison

A comparison of the solution and computational time among the proposed hybrid approach, the analytical model by CPLEX, and the simulation-based optimization model by OptQuest is shown in Table 18. OptQuest is a built-in optimization tool in ARENA. It is used as a benchmark of simulation-based optimization models for comparison. Further details of the advantages and disadvantages of OptQuest are discussed by Fu [3] and Glover et al. [4]. All three methods are run using an Intel Core i7-8750H CPU @ 2.20GHz and 8 GB of RAM.

Table 18 Comparison among the analytical model, simulation-based optimization model, and proposed hybrid approach.

	Analytical Model (CPLEX)	Simulation-based optimization model (OptQuest)	Hybrid approach
Profit ($)	98,770	77,807	85,975
Computational Time	3.86 sec	5 hr. 32 min	12 min 38 sec

From Table 18, the analytical model by CPLEX gives the best solution but is not feasible since it cannot incorporate uncertainties into the model. Since there are 96 decision variables to be determined in the solution, the simulation-based optimization by OptQuest requires a long computational time and yields a statistically worse solution than CPLEX but is feasible. With 10 replications, the profits obtained from the hybrid approach are significantly higher than the profits obtained from OptQuest under 95% confidence level (p-value < 0.01). Moreover, the computational time would be a lot longer if the model is extended to study longer periods, as it would require a higher number of decision variables. Therefore, the hybrid approach is shown to be superior to the other methods.

3.4 Managerial Implications

The proposed approach is useful for finding the production and distribution plan that maximizes the supply chain profit. With this approach, it is possible to find the total time needed for the production and distribution plan (makespan) and ensure that the plan is completed within the makespan limit. This approach considers the service level and is solved for the suitable amount of safety stock required to satisfy the minimum service level of the customers. Too high safety stock can lead to excessive inventory holding cost while too low may not satisfy the minimum service level. Phase II in the approach further improve the solution by searching for the best amount of safety stocks. The solution set consists of the optimal amount of production of each product in each period and its distribution plan as well as the best amount of safety stock required to satisfy the minimum service level as imposed by the retailers. By combining analytical and simulation models together, it is possible to find the near or possibly optimal solution with reasonable computational time compared to the simulation-based optimization alone. For a large-scale business, this approach's concept can be applied to the business model to significantly increase the profit of the supply chain within a reasonable solving time as required in the current market competition.

4. Conclusions

In this study, a hybrid analytical-simulation approach for supply chain optimization was proposed. The aim is to find an optimal production-distribution plan that is feasible in terms of meeting the makespan limit and service level requirement, which are often overlooked but can be crucial for the production level, labor, and machine capacity planning. From our literature review, no research paper considers both of these requirements and optimizing the production-distribution plan at the same time. The results show that the analytical model by itself cannot easily find the makespan, as queueing and uncertainties cannot simply be incorporated into the model. The service level is one of the main requirements from customers that a company must consider. Therefore, the safety stock must be introduced to increase the service level, and hence, increase the profit.

The proposed hybrid analytical-simulation approach is divided into two phases. Phase I is solved for a feasible plan that does not exceed the makespan limit and satisfies the service level requirements. Phase II is further solved for the best amount of safety stock by adding additional or decreasing excess safety stock (which increases the profit). Our proposed approach has a shorter solving time, compared to the simulation-based optimization by OptQuest, and can be easily applied in other cases that require near or possibly optimal and feasible plans under different types of uncertainty.

For larger problems with a higher number of decision variables, this approach may need some adjustments to reduce the computational time. For instance, if the number of safety stock required is large, the addition or subtraction of safety stock one unit at a time in Phase II might not be efficient. Therefore, the suitable amount of safety stock to be added or subtracted in each iteration must be considered in order to decrease the computational time. This is one limitation that should be considered based on the size of the problem. In addition, due to uncertainty of customer demand at the retailers, other techniques of product transshipments in the same echelon such as lateral transshipment among retailers can be introduced to balance the inventory among the retailers themselves. This can be further introduced in the model to improve its effectiveness.

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