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Solving an aggregate production planning problem by using interactive fuzzy linear programming

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Abstract

This study utilizes an interactive Fuzzy Linear Programming (FLP) model for solving the Aggregate Production Planning (APP) problem in an uncertain environment. The uncertain conditions include uncertainties of customer demand, operation time, operation cost, and machine capacity. The proposed model tries to minimize the total costs of the APP plan. Through the concept of obtaining an optimal solution in different levels of the feasible degree (α), decision-makers can interact with the given goal until achieving an efficient compromised solution that presents the overall satisfaction level of Decision-Makers (DMs) based on the given goal values. The outcome of this approach provides more flexibility for DMs to achieve a satisfactory solution. Finally, the proposed approach is compared with other traditional approaches and the results are analyzed.

Keywords: Aggregate production planning, Possibilistic linear programming, Interactive fuzzy linear programming, Fuzzy set theory, Decision making

1. Introduction

Aggregate Production Planning (APP) is the intermediate-time capacity plan that identifies the cost minimization of production plans and human resources to fulfill market needs in the most effective way. Its purpose is to determine a suitable amount of production and the level of inventory in terms of aggregation. The time period ranging of APP is from 2 to 12, or even 18 months [1]. APP brings a connection between strategic management and operations management. In addition, APP operating strategies play a significant role in organizational integration and enterprise resource planning. The target making APP in manufacturing enterprise is to acquire the minimum cost and maximum profit by determining the quantity of produced products, the quantity of subcontracting products, levels of labor, etc., to fulfill the market demand [2].

In practice, some input data for APP problems are regularly imprecise owing to some information is incomplete or cannot be accurately obtained. In these circumstances, fuzzy logic can provide a form of reasoning that allows approximate human inference skills to be used as knowledge-based systems. Zadeh [3] developed the fuzzy logic theory. It brings a mathematical framework to apprehend the uncertainty related to the processes of human perception, such as reasoning and thinking. The theory of fuzzy sets has been extensively adopted in many fields (i.e. operations research, management science, artificial intelligence and control theory). By applying the theory of fuzzy sets, Fuzzy Mathematical Programming (FMP) is a well-known decision-making approach. Zimmermann [4] first proposed the theory of fuzzy sets into typical Linear Programming (LP) model that has both fuzzy objective and fuzzy constraints. An equivalent single-goal linear programming model by combining a linear membership function and the fuzzy decision-making method of Bellman & Zadeh [5] that is introduced in this study. Subsequently, some of fuzzy optimization approaches for handling APP problems in an ambiguous condition has been developed based on FMP. Moreover, Zadeh [3] introduced the possibility theory, which is in relation to the fuzzy set theory. The possibility distribution concept is defined as a vague limitation, which can work as a flexible constraint on the values that may be allocated to a variable.

The research also shows the significance of the possibility theory because most of the information about human decisions is understood to be possibilistic instead of being probabilistic as in nature. Uncertainties are common in aggregate production planning such as imprecise capacities, imprecise demands, imprecise operating times, imprecise costs, etc. The uncertainties of these types of data are not able to be completely depicted by frequency-based probability distributions. Thus, it is extremely necessary to use the theory of fuzzy sets and fuzzy optimization approaches for formulating APP problems.

Possibilistic Linear Programming (PLP) is a fuzzy optimization approach that could be modeled based on possibility distributions (triangular or trapezoidal distribution)**.** Tang et al. [6] introduced two kinds of PLP with common possibilistic distribution, which are Linear Programming (LP) problems with common possibilistic resources and common possibilistic objective coefficients by applying the concepts of the biggest and smallest of most likely value, the most pessimistic value and the most optimistic value. Hsu & Wang [7] used the PLP approach that are developed by Lai & Hwang [8] to deal with vague and imprecise data in APP problems in an Assemble-To-Order (ATO) system. Wang & Liang [9] studied APP problem in an uncertain environment. By using PLP to formulate for APP problem, which brings an efficient compromised solution of APP plan and overall satisfaction level of Decision-Maker (DM) with identified goal values. Liang [10] presented an interactive Possibilistic Linear Programming (*i*-PLP) approach to handle multiple imprecise objective and cost coefficients in an APP problem that considers multiple product and multiple time period based on triangular possibility distributions. The proposed *i*-PLP approach can provide a systematic framework that supports the process of decision-making for solving fuzzy multiple objective APP problems. This enables a Decision-Maker (DM) to interactively adjust fuzzy parameters as far as a set of satisfactory solutions is obtained. Sutthibutr & Chiadamrong [11] integrated a weighted additive method into PLP to solve an APP problem in an uncertain environment. They proposed a hybrid approach that is applied for solving most practical planning problems that are related to fuzzy parameters through an interactive process in making decisions. The results of the investigation can provide different compromise solutions which help the DM to select an efficient one based on their preferences with the highest satisfaction.

Fuzzy Linear Programming (FLP) is another fuzzy optimization approach that could be formulated by using subjective preference-based membership functions. Wang & Fang [12] studied an APP problem with some fuzzy parameters that consist of the product price, subcontracted cost, production quantity, workforce level, market demand, and the fuzzy satisfaction levels of objective functions. The proposed approach provided a systematic framework to interactively support DMs until satisfactory results are achieved. An aggregation operator was deployed at the final step to acquire the compromise solution of the proposed system. Madadi & Wong [13] extended the multiple objective fuzzy APP model to best serve businesses that aim to make the best utilization of their resources in an ambiguous condition whilst attempting to maintain an acceptable level of quality and customer service level at the same time. Iris & Cevikcan [2] presented a structure of mathematical programming for APP problems in an ambiguous data environment. After giving background information about Fuzzy Linear Programming (FLP) and the APP problem, the formulated FLP model for an APP problem that is solved on an illustrative example with the various values of α-cut. Chen & Huang [14] proposed a novel methodology for solving the APP problem in uncertain condition. After constructing the membership function by applying Zadeh's extension principle and fuzzy solutions, an equivalent mathematical parametric programming is formed to identify the lower and upper bound of the total cost with the different levels of α. Since the objective value is represented based on a membership function. Thus, the achieved solutions can be more information with more accuracy, which provides more opportunities to gain the optimal solution on the disaggregate plan.

The process of decision making can be better expressed based on the theory of fuzzy sets instead of using precise methods. Nevertheless, a person in charge of making decisions called the Decision-Maker (DM) does play a critical role in using the theory of fuzzy sets. Thus, an interactive process between DM and the processes of decision making is significant for dealing with practical issues [15]. The interactive Fuzzy Linear Programming (*i*-FLP) is considered as a problem-oriented and user-dependent method. This approach is a systematic and effective method to deal with linear problems because the feasible results will be provided for DM. The problem will be solved if the DM is satisfied with the obtained result. In contrast, if the DM is still not satisfied, an interactive process will continue until the DM can find a satisfactory solution [16].

Generally, the uncertainty of parameters imposes two key problems in the solving process of possibilistic fuzzy programming: handling the relationship between the fuzzy left-hand side as well as the right-hand side of constraints and finding the optimal value for the fuzzy objective function. These problems are associated with the process of ranking fuzzy numbers based on the interactive resolution method that is proposed by Jimenez et al. [17]. The method utilizes two primary ideas, which are optimality and feasibility for handling with ranking fuzzy objective functions and inequality relations in constraints, correspondingly. In addition, it also provides some benefits as follows:

- The method is computationally efficient for solving LP model since the linearity is maintained and does not raise the number of objective functions as well as inequality constraints. Thus, it can be applied to solve large scope FLP models.

- The method relies on a fuzzy relation to compare fuzzy numbers, while the other relevant methods just use comparison relations that commonly described as a fuzzy number that is larger or smaller than others. These relations do not provide any information about a likely violation of constraints (the concept of feasibility degree).
- This method could be applied for different kinds of fuzzy numbers (i.e. triangular, trapezoidal), and it also could be applied easily for linear or even nonlinear forms.
- The concepts of feasibility and optimality of this method permit DM to interactively make a compromise between the risk of constraints (feasibility degree) and the degree of achievement of DM's aspiration level.

The interactive method of Jimenez et al. [17] subjects to the definition of the expected interval and the expected value of fuzzy numbers, which are considered as strong mathematical concepts. These concepts were first developed by Arenas et al. [18], followed by Heilpern [19] and Jimenez [20].

Table 1 Summary of related studies on solving fuzzy APP models.

Articles	Place of fuzzy parameters	Defuzzification method	Type of Model	Solution technique
Iris & Cevikcan (2014) [2]	Objective function RHS of constraints	Zimmerman Method	FMILP	FLP
Hsu & Wang (2001) [11]	Objective function	Zimmerman Method	FMILP	PLP
Wang & Liang (2005) [13]	Objective function Both sides of constraints RHS of constraints LHS of constraints	Zimmerman Method Fuzzy ranking method Weighted average method	FMILP	PLP
Liang (2007) [14]	Objective function Both sides of constraints RHS of constraints LHS of constraints	Zimmerman Method Fuzzy ranking method Weighted average method	FMOMILP	PLP
Sutthibutr & Chiadamrong (2020) [15]	Objective function Both side of constraints RHS of constraints LHS of constraints	Zimmerman Method Fuzzy ranking method Weighted average method	FMILP	PLP
Wang & Fang (2001) [16]	Objective function RHS of constraints Both sides of constraints	Zimmerman Method	FMOMILP	FLP
Madadi & Wong (2014) [17]	Objective function RHS of constraints	Expected interval value α-Level of fuzzy sets	FMOMILP	FLP
Chen & Huang (2010) [18]	RHS of constraints	α-Level of fuzzy sets	FMILP	FLP
This study	Objective function Both side of constraints RHS of constraints LHS of constraints	Expected value Hybrid fuzzy ranking method (feasibility concept-violation of constraints)	FMOMILP	Interactive FLP

RHS: right-hand side, LHS: left-hand side, FMILP: Fuzzy mixed-integer linear programming, FMOMILP: Fuzzy multi-objective mixed-integer linear programming, FLP: Fuzzy linear programming, PLP: Possibilistic linear programming.

Most of the reviewed papers have studied on uncertainties in APP models. This has led us to pursue further approaches that can yield better solutions as well as can handle with the uncertainty more efficiently. Several approaches have been summarized in Table 1, and it can be seen that the feasibility concept (violation of constraints) of model is not considered in most studies. Taking the advantages of Jimenez's method that mentioned earlier, we propose Jimenez's method for solving the fuzzy APP model in this study. Noteworthy, Jimenez's method has recently been utilized as an efficient method to defuzzify imprecise data [21,22].

This study thus presents a form of the Fuzzy Linear Programming (FLP) model to cope with Aggregate Production Planning (APP) problems where these parameters such as production times and costs, machine capacity, and customer demand are considered as fuzzy numbers with triangular possibility distribution, which could represent the uncertainty of these parameters in practical production planning systems. By applying an interactive resolution method of Jimenez et al. [17]. It yields a systematic framework to assist in the process of decision-making, thereby allowing a DM to interactively adjust the membership functions of the objectives and constraints until the DM satisfies with the achieved solution. Then, the achieved result of the *i*-FLP model is compared to the results of the ideal optimal LP model and the typical PLP model, and its advantages are identified.

The remainder of this paper is arranged as follows. Fuzzy Linear Programming (FLP) model for an APP problem, a methodology for solving the fuzzy APP model with two-phase approach including the Possibilistic

Linear programming (PLP) approach and the interactive Fuzzy Linear Programming (*i*-FLP) approach, and a case study to validate the feasibility of using the *i*-FLP approach to practical APP problem are presented in Section 2. Subsequently, the results of each approach and a comparison between its solution are shown in Section 3. Lastly, the conclusion of the study is drawn in Section 4**.**

2. Materials and methods

2.1 Mathematical model

In this study, the Aggregate Production Planning (APP) problem is described. A company manufactures *P* types of products to satisfy the customer requirement during a medium-term planning period *T*. The APP problem is considered in an uncertain environment. Therefore, operation costs, labor level, machine capacity, and customer demand can vary for each planning period.

Indices:

p	Types of product, *p = 1, …, P*
t	Planning periods, *t = 1, …, T*

Parameters:

$\widetilde{D}_{pt}$	Fuzzy forecasted demand of product *p* in period *t* (units)
$\tilde{r}_{pt}$	Fuzzy cost of regular–time production for a unit of product *p* in period *t* ($/unit)
$\tilde{o}_{pt}$	Fuzzy cost of overtime production for a unit of product *p* in period *t* ($/unit)
$\tilde{s}_{pt}$	Fuzzy cost of subcontracting for a unit of product *p* in period *t* ($/unit)
$\tilde{\imath}_{pt}$	Fuzzy cost of inventory for a unit of product *p* in period *t* ($/unit)
$\tilde{b}_{pt}$	Fuzzy cost of backordering for a unit of product *p* in period *t* ($/unit)
$\tilde{h}_{pt}$	Fuzzy cost of hiring for one worker in period *t* ($/person-hour)
$\tilde{f}_t$	Fuzzy cost of firing for one worker in period *t* ($/person-hour)
$\widetilde{MaxL}_t$	Fuzzy maximum available level of labor in period *t* (person-hours)
$\widetilde{MaxM}_t$	Fuzzy maximum available capacity of machine in period *t* (machine-hours)
$\widetilde{MH}_{pt}$	Fuzzy machine hour usage for a unit of product *p* in period *t* (machine-hours/unit)
LH_{pt}	Labor hour usage for a unit of product *p* in period *t* (person-hours/unit)
$MaxW_t$	Maximum warehouse space available in period *t* (ft^2/unit)
WS_{pt}	Warehouse spaces for a unit of product *p* in period *t* (ft^2/unit)

Decision variables:

RQ_{pt}	Quantity of regular-time production product *p* in period *t* (units)
OQ_{pt}	Quantity of overtime production product *p* in period *t* (units)
SQ_{pt}	Quantity of subcontracted product *p* in period *t* (units)
IQ_{pt}	Quantity of inventory product *p* in period *t* (units)
BQ_{pt}	Quantity of backorder product *p* in period *t* (units)
H_t	Number of workers hired in period *t* (person-hours)
F_t	Number of workers fired in period *t* (person-hours)
$\tilde{Z}$	Total costs

Objective Function

The common objective function of an APP problem is to minimize the total costs. The total costs are the sum of the manufacturing cost, inventory cost, backordering cost, and costs of changing workforce levels over a period *T*. However, the coefficients of costs in the objective function can be imprecise due to some information being estimated, unobtainable or incomplete. Accordingly, the objective function is formulated in the following equation:

$$Min\ \tilde{Z} = \sum_{p=1}^{P} \sum_{t=1}^{T} (\tilde{r}_{pt}RQ_{pt} + \tilde{o}_{pt}OQ_{pt} + \tilde{s}_{pt}SQ_{pt} + \tilde{\imath}_{pt}IQ_{pt} + \tilde{b}_{pt}BQ_{pt}) + \sum_{t=1}^{T} \left(\tilde{h}_t H_t + \tilde{f}_t F_t\right) \qquad (1)$$

The production cost is shown in the first five terms. The production costs consist of regular-time production, overtime production, subcontracting, inventory, and backorder. The remaining portion indicates the costs of changing workforce levels, which are the costs of hiring and firing workers, where $\tilde{r}_{pt}$, $\tilde{o}_{pt}$, $\tilde{s}_{pt}$, $\tilde{\imath}_{pt}$, $\tilde{b}_{pt}$, $\tilde{h}_t$, and $\tilde{f}_t$ are uncertain parameters with the triangular possibility distribution.

Constraints

The minimization of the objective function is subject to the following constraints:

Carrying Inventory Constraint:

$$\widetilde{D}_{pt} = IQ_{pt-1} + BQ_{pt-1} + RQ_{pt} + OQ_{pt} + SQ_{pt} - IQ_{pt} + BQ_{pt} \qquad \forall P, \forall T \qquad (2)$$

The forecasted demand of a customer cannot be obtained exactly in the real-world. Therefore, $\widetilde{D}_{pt}$ denotes for fuzzy estimated demand of product *p* in period *t*. Equation (2) shows that the summation of amounts of regular and overtime production, inventory quantities, subcontracting quantities, and backordering quantities primarily must meet the amount of forecasted demand. The demand can be either met or backordered in a specific period, but a backorder in the following period must be fulfilled.

Labor Level Constraints:

$$\sum_{p=1}^{P} LH_{pt-1}\left(RQ_{pt-1} + OQ_{pt-1}\right) + H_t - F_t - \sum_{p=1}^{P} LH_{pt}\left(RQ_{pt} + OQ_{pt}\right) = 0 \qquad \forall T \qquad (3)$$

$$\sum_{p=1}^{P} LH_{pt}\left(RQ_{pt} + OQ_{pt}\right) \leq \widetilde{MaxL}_t \qquad \forall T \qquad (4)$$

where Equation (3) ensures that the labor level at the end of period *t-1* plus newly a number of hired workers, and minus a number of fired workers in period *t* must equal to the labor level in period *t*. Equation (4) shows that the levels of actual labor are limited by the maximum available labor level in period *t*. The maximum available labor levels can be inaccurate because of the uncertain conditions of supply, demand, and labor skills in the market.

Machine Capacity Constraint:

$$\sum_{p=1}^{P} \widetilde{MH}_{pt}\left(RQ_{pt} + OQ_{pt}\right) \leq \widetilde{MaxM}_t \qquad \forall T \qquad (5)$$

where $\widetilde{MH}_{pt}$ and $\widetilde{MaxM}_t$ are imprecise data of the machine hour usage for a unit of product *p*, and the maximum capacity of available machine in period *t*, respectively. Equation (5) is about the limitation of the machine capacity, where the machine hour usage for producing all the products in period *t* must not exceed the maximum capacity of available machine. Similarly, the maximum capacity of available machine can be fuzzy (in reality) as the available machine hours could be affected by the availability and working conditions of machines at each moment.

Warehouse Capacity Constraint:

$$\sum_{p=1}^{P} WS_{pt} IQ_{pt} \leq MaxW_t \qquad \forall T \qquad (6)$$

Equation (6) presents the limit of actual warehouse capacity in period *t*. The warehouse space for storing all the products in each period *t* must not surpass their respective maximum available warehouse space.

Non-negativity Constraint:

$$RQ_{pt}, OQ_{pt}, SQ_{pt}, IQ_{pt}, BQ_{pt}, H_t, F_t \geq 0 \qquad \forall P, \forall T \qquad (7)$$

2.2 Solution approaches

The APP model aforementioned is considered in an uncertain environment, some parameters in the APP model (i.e. operation times and costs, labor level, machine capacity, and customer demand) are described by fuzzy numbers, which can imitate the real-life. To cope with this problem, a given Fuzzy Linear Programming (FLP) model need to be transformed into a crisp Linear Programming (LP) model. To do that, a proposed methodology with the two-phase approach is implemented. In the first phase, Possibilistic Linear Programming (PLP) is used as a benchmark for comparison. The fuzzy APP model is then transformed into an equivalent auxiliary crisp model by applying the PLP approach that is introduced by Lai and Hwang [8]. In the second phase, an interactive fuzzy method is applied. There are two steps to implement this approach. The first step is to convert the fuzzy APP model into an equivalent auxiliary crisp model. The second step is to find a preferred compromise solution with the maximization of the DM's satisfaction by applying the interactive fuzzy methodology of Jimenez et al. [17]. Finally, a comparison between these two solution approaches is made.

2.2.1 Possibilistic linear programming (PLP) approach

The PLP can be applied to obtain the optimal solution in each scenario, subject to imprecise operation times and costs which are represented by the possibility distribution (triangular distribution).

2.2.1.1 Triangular possibility distribution of imprecise data

As shown in Figure 1 (A), the triangular (possibility) distribution of imprecise number $\widetilde{c_{pt}}$ is described by three prominent data points, which are the most optimistic point (c_{pt}^{o}), the most likely point (c_{pt}^{m}), and the most pessimistic point (c_{pt}^{p}).

The optimistic value (the lower bound value) is the value that can provide the best situation. It is an immensely low likelihood of belonging to the set of available values (at the point of possibility level = 0 if normalized).

- The most likely value (the modal value) is a value that provides the normal or general situation. It certainly is in the set of available values (at the point of possibility level = 1 if normalized).
- The pessimistic value (the upper bound value) is the value that can provide the worst situation. It is an

immensely low likelihood of belonging to the set of available values (at the point of possibility level = 0 if normalized).

2.2.1.2 Solving strategy for the imprecise objective function

All the given parameters of the objective function in the PLP model are based on the triangular possibility distribution. Geometrically, the imprecise objective function $\tilde{z}$ can be completely characterized by three prominent points; $(z^o, 0), (z^m, 1)$ and $(z^p, 0)$. Because the critical point of the vertical coordinates is fixed at either 0 or 1, the only item that should be considered is the horizontal coordinates. Therefore, the solving strategy for the imprecise objective functions aims to simultaneously minimize the most likely value of total costs, z^m, maximize the possibility of getting lower total costs, $(z^m - z^o)$, and minimize the possibility of getting higher total costs, $(z^p - z^m)$. These three objective functions guarantee that these three values are pushed toward the left, as seen in Figure 1 (B).

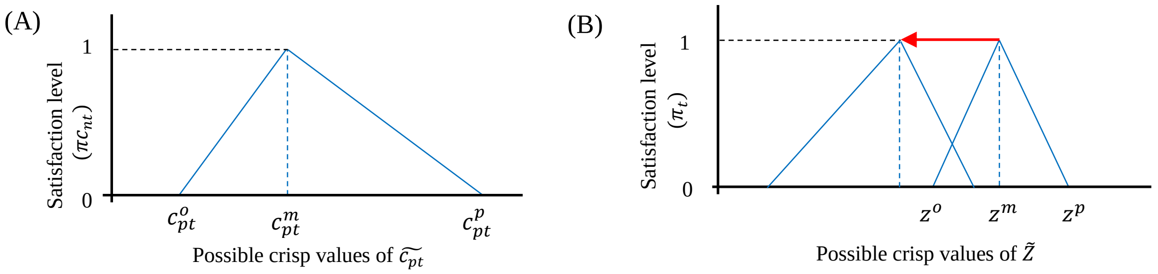

Figure 1 Triangular (possibility) distribution of $\widetilde{c_{pt}}$ (A) and strategy of cost minimization (B).

2.2.1.3 Handling the imprecise objective functions

As shown in Figure 1 (B), relied on the triangular (possibility) distribution, the original imprecise objective is replaced by three new crisps of multiple objective functions as follows:

Minimizing the most likely value of total costs

$$\begin{aligned} Min\ z_1 &= z^m \\ &= \sum_{p=1}^{P} \sum_{t=1}^{T} \left(r_{pt}^{m}RQ_{pt} + o_{pt}^{m}OQ_{pt} + s_{pt}^{m}SQ_{pt} + i_{pt}^{m}IQ_{pt} + b_{pt}^{m}BQ_{pt}\right) \\ &+ \sum_{t=1}^{T} \left(h_t^{m}H_t + f_t^{m}{}_{t}F_t\right) \end{aligned} \quad (8)$$

Maximizing the possibility of getting lower total costs

$$\begin{aligned} Max\ z_2 &= (z^m - z^o) \\ &= \sum_{p=1}^{P} \sum_{t=1}^{T} [(r_{pt}^{m} - r_{pt}^{o})RQ_{pt} + (o_{pt}^{m} - o_{pt}^{o})OQ_{pt} + (s_{pt}^{m} - s_{pt}^{o})SQ_{pt} \\ &+ (i_{pt}^{m} - i_{pt}^{o})IQ_{pt} + (b_{pt}^{m} - b_{pt}^{o})BQ_{pt} + \sum_{t=1}^{T} [(h_t^{m} - h_t^{o})H_t + (f_t^{m} - f_t^{o})F_t)] \end{aligned} \quad (9)$$

Minimizing the possibility of getting higher total costs

$$\begin{aligned} Min\ z_3 &= (z^p - z^m) \\ &= \sum_{p=1}^{P} \sum_{t=1}^{T} [(r_{pt}^{p} - r_{pt}^{m})RQ_{pt} + (o_{pt}^{p} - o_{pt}^{m})OQ_{pt} + (s_{pt}^{p} - s_{pt}^{m})SQ_{pt} \\ &+ (i_{pt}^{p} - i_{pt}^{m})IQ_{pt} + (b_{pt}^{p} - b_{pt}^{m})BQ_{pt}] + \sum_{t=1}^{T} [(h_t^{p} - h_t^{m})H_t + (f_t^{p} - f_t^{m})F_t)] \end{aligned} \quad (10)$$

2.2.1.4 Handling the imprecise constraints

Two sorts of constraints are presented in the model. These are fuzzy constraints (or soft constraints) and crisp constraints. Equation (3) and Equation (6) are crisp constraints that have no uncertainty related to limitation settings. The remaining constraints (Equation (2), Equation (4), and Equation (5)) are fuzzy constraints which have a specific degree of uncertainty. It is required to be converted into crisp constraints by using the defuzzification methods. Defuzzification is defined as a transformation process for converting imprecise data into crisp data. In this study, the fuzzy ranking method and weighted average method are used. The carrying inventory constraint (Equation (2)) and labor level constraint (Equation (4)) are defuzzified by the weighted average method. In contrast, the machine capacity constraint (Equation (5)) is defuzzified by the fuzzy ranking method. The weighted average method can defuzzify fuzzy number with the triangular distribution by assigning weights to the possible values (optimistic value, most likely value, and optimistic value). Here, customer demand is estimated by the knowledge and experience of the DMs. Thus, more crucial values will be assigned to higher weights.

Recalling Equation (2) of the original Fuzzy Linear Programming (FLP) model, consider the case in which the available resource (the right-hand side of constraint). The weighted average method is used for Equation (2) to convert imprecise demand ($\widetilde{D}_{pt}$) into crisp demands, presented as follows.

$$w_1^p D_{pt}^p + w_1^m D_{pt}^m + w_1^o D_{pt}^o = IQ_{pt-1} + BQ_{pt-1} + RQ_{pt} + OQ_{pt} + SQ_{pt} - IQ_{pt} + BQ_{pt} \quad (11)$$

where $w_1^p + w_2^m + w_3^o = 1$ and w_1^p, w_2^m, and w_3^o denote the weights of the most pessimistic, the most likely, and the most optimistic values of the imprecise demand, respectively. A suitable value of weights w_1^p, $w_2^m$and w_3^o can be specified subjectively by the knowledge and experience of DMs. In this study, w_1^p, w_2^m, and w_3^o are each set 33%.

Similarly, the imprecise maximum available labor level ($\widetilde{MaxL}_t$) in Equation 4, which is presented as follows:

$$\sum_{p=1}^{P} LH_{pt}(RQ_{pt} + OQ_{pt}) \le w_1^p MaxL_t^p + w_2^m MaxL_t^p + w_3^o MaxL_t^p \quad (12)$$

In addition, for the soft constraint that has fuzzy parameters both on the right-hand side and left-hand side, the fuzzy ranking method is used to defuzzify imprecise data. Equation (5) with machine hour usage per product ($\widetilde{MH}_{pt}$) on the right-hand side and maximum machine capacity available ($\widetilde{MaxM}_{pt}$) on the left-hand side, which is replaced with the three following equivalent auxiliary equations:

$$\sum_{p=1}^{P} MH_t^p(RQ_{pt} + OQ_{pt}) \le MaxM_t^p \quad \forall T \quad (13)$$

$$\sum_{p=1}^{P} MH_t^m(RQ_{pt} + OQ_{pt}) \le MaxM_t^m \quad \forall T \quad (14)$$

$$\sum_{p=1}^{P} MH_t^o(RQ_{pt} + OQ_{pt}) \le MaxM_t^o \quad \forall T \quad (15)$$

2.2.1.5 Solving the auxiliary multiple objective linear programming (MOLP) model

Furthermore, the auxiliary MOLP model could be transformed into an equivalent single-objective LP model from applying the fuzzy decision-making concept of Bellman and Zadeh [5], along with Zimmermann's linear programming [23]. The corresponding Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of each objective function are determined as follows, respectively.

$$\begin{aligned} z_1^{PIS} &= Min\ z^m & z_1^{NIS} &= Max\ z^m \\ z_2^{PIS} &= Max\ z^m - z^o & z_2^{NIS} &= Min\ z^m - z^o \\ z_3^{PIS} &= Min\ z^p - z^m & z_3^{NIS} &= Max\ z^p - z^m \end{aligned} \quad (16)$$

The values of PIS and NIS criteria can be obtained by using linear programming to solve the minimum and maximum solutions of each objective. Furthermore, the respective linear membership functions of each objective function are specified by:

$$f_1(z_1) = \begin{cases} 1 & , z_1 < z_1^{PIS} \\ \dfrac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}} & , z_1^{PIS} \le z_1 \le z_1^{NIS} \\ 0 & , z_1 > z_1^{NIS} \end{cases} \quad (17)$$

$$f_1(z_2) = \begin{cases} 1 & , z_2 < z_2^{PIS} \\ \dfrac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}} & , z_2^{NIS} \le z_2 \le z_2^{PIS} \\ 0 & , z_2 > z_2^{NIS} \end{cases} \quad (18)$$

$$f_1(z_3) = \begin{cases} 1 & , z_3 < z_3^{PIS} \\ \dfrac{z_3^{NIS} - z_3}{z_3^{NIS} - z_3^{PIS}} & , z_3^{PIS} \le z_3 \le z_3^{NIS} \\ 0 & , z_3 > z_3^{NIS} \end{cases} \quad (19)$$

2.2.1.6 Solving for the overall satisfaction by PLP

Lastly, aggregate fuzzy sets using the minimum operator of the fuzzy decision-making concepts, the final equivalent single-objective LP model for solving the APP problem is derived as follows:

$$\begin{aligned} & Max\ L \\ & \text{Subject to: } L \le f_j(z_j);\ j = 1, 2, 3 \\ & \qquad Equations\ (3), (6), (11) - (15) \\ & \qquad Equations\ (7);\ 0 \le L \le 1. \end{aligned} \quad (20)$$

where the auxiliary variable L denotes the overall satisfaction level of the decision-makers.

2.2.2 Interactive fuzzy linear programming (i-FLP)

The proposed APP model in this study considers these parameters to be imprecise numbers that can mimic the practical environment. By applying an interactive method [17], the FLP is able to be effectively transformed into an equivalent auxiliary crisp model. This approach allows the decision-makers (DMs) to use the optimal solution with several different degrees of feasibility.

2.2.2.1 Transforming the fuzzy linear programming model into the equivalent auxiliary crisp model

It is required to transform the FLP model into an equivalent auxiliary model to interpret the imprecise numbers in the fuzzy model. As a first step, a triangular fuzzy number $\tilde{c} = (c^{o}, c^{m}, c^{p})$ is defined by the DM, whereby c^{o}, c^{m}, c^{p} represent the optimistic, the most likely, and the pessimistic values of the triangular fuzzy number, correspondingly. The membership function $\mu_{\tilde{c}}(x)$ of $\tilde{c}$ is defined in the following equations:

$$\mu_{\tilde{c}}(x) = \begin{cases} f_c(x) = \dfrac{x - c^o}{c^m - c^o}, if\ c^o \le x \le c^m \\ 1 \quad if\ x = c^m \\ g_c(x) = \dfrac{c^p - x}{c^p - c^m}\ if\ c^m \le x \le c^p \\ 0\ otherwise \end{cases} \tag{21}$$

According to Heilpern [19], the expected interval of fuzzy number $\tilde{c}$, denoted $EI(\tilde{c})$, and the expected value of fuzzy number $\tilde{c}$, denoted $EV(\tilde{c})$ are defined as follows:

$$EI(\tilde{c}) = [E_1^c, E_2^c] = \left[\int_0^1 f_c^{-1}(x)dx, \int_0^1 g_c^{-1}(x)dx\right] = \left[\frac{1}{2}(c^o + c^m), \frac{1}{2}(c^m + c^p)\right] \tag{22}$$

$$EV(\tilde{c}) = \frac{E_1^c + E_2^c}{2} = \frac{c^o + 2c^m + c^p}{4} \tag{23}$$

Based on Jimenez et al. [17], if there are two fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree in which $\tilde{a}$ is larger than $\tilde{b}$ is given by:

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0 \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & \text{if } E_1^a - E_2^b > 0 \end{cases} \tag{24}$$

For $\mu_M(\tilde{a}, \tilde{b}) \ge \alpha$, $\tilde{a}$ is greater than, or equal to $\tilde{b}$ at least in a degree of α. When $\tilde{a}$ is indifferent to (equal to) $\tilde{b}$ in the degree of α, it is denoted $\frac{\alpha}{2} \le \mu_M(\tilde{a}, \tilde{b}) \le 1 - \frac{\alpha}{2}$ (Arenas et al. [18]). Hence, the following fuzzy mathematical programming model with the form:

$$\begin{aligned} &\text{Min } z = \tilde{c}^T x \\ &\text{s.t. } \tilde{a}_i x \ge \tilde{b}_i, i = 1, \dots, l \\ &\quad\ \tilde{a}_i x = \tilde{b}_i, i = k + 1, \dots, m \\ &\quad\ x \ge 0 \end{aligned} \tag{25}$$

where $\tilde{c}^T$ is a fuzzy vector, as mentioned by Arenas et al. [18]. With respect to Equations (24) - (25), constraints $\tilde{a}_i x \ge \tilde{b}_i$ and $\tilde{a}_i x = \tilde{b}_i$ are equivalent to the following formulations, respectively:

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \ge \alpha, i = 1, \dots, l \tag{26}$$

$$\frac{\alpha}{2} \le \frac{E_2^{a_i x}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \le 1 - \frac{\alpha}{2}, i = l + 1, \dots, m \tag{27}$$

Similarly, it can be proved that feasible solution x_0 among the feasible decision vector x is an α-acceptable optimal solution of Equation (25) just in case Equation (28) is satisfied:

$$\tilde{c}^T x \ \ge_{\frac{1}{2}} \tilde{c}^T x_0 \tag{28}$$

Finally, the complete equivalent crisp α-parametric model of Equation (25) can be derived by using expected value and hybrid fuzzy ranking method as follows:

$$\begin{aligned} &\text{Min } EV(\tilde{c})^T x \\ &[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]x \ge \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i}, i = 1, \dots, k \\ &\left[\left(1 - \frac{\alpha}{2}\right)E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}\right]x \ge \frac{\alpha}{2}E_2^{b_i} + \left(1 - \frac{\alpha}{2}\right)E_1^{b_i}, i = k + 1, \dots, n \\ &\left[\frac{\alpha}{2}E_2^{a_i} + \left(1 - \frac{\alpha}{2}\right)E_1^{a_i}\right]x \le \left(1 - \frac{\alpha}{2}\right)E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i}, i = k + 1, \dots, n \end{aligned} \tag{29}$$

An interactive procedure is presented in the next section to handle the equivalent crisp α-parametric model.

2.2.2.2 Interactive resolution approach

To obtain an optimal result that satisfies the aspirations of the DM, the DM has to compromise the two conflicting objectives, which is descending the value of objective function and improving the satisfaction degree of constraints. To deal with this problem, Jimenez et al. [17] proposed an interactive approach to obtain the optimal solution, to balance the two conflicting objectives. Let $x^0(\alpha_k)$ be the α_k-acceptable optimal solution, where $\alpha = \alpha_k$. Based on Equation (29), the respective fuzzy numbers of the objective value are calculated by $\tilde{z}^0(\alpha_k) = \tilde{c}^T x^0(\alpha_k)$. The discrete values of α_k in the set M are determined as follows:

$$M = \left\{\alpha_k = \alpha_0 + 0.1k | k = 0,1,\dots,\frac{1-\alpha_0}{0.1}\right\} \subset [0,1] \quad (30)$$

where α_0 is the minimum degree of the feasibility of the constraint that the DM can accept, and α is an optional value which is decided by the DM, $\alpha_0 \leq \alpha \leq 1$. Following Jindal & Sangwan [24], eleven scales of α are utilized to differentiate the number of linguistic labels of the decision maker in the fuzziness process as follows: (*α = 0)* unacceptable solution; (*α = 0.1)* basically unacceptable solution; (*α = 0.2)* mostly unacceptable solution; (*α = 0.3)* very unacceptable solution; (*α = 0.4)* quite unacceptable solution; (*α = 0.5)* neither acceptable solution nor unacceptable solution; (*α = 0.6)* quite acceptable solution; (*α = 0.7)* very acceptable solution; (*α = 0.8)* mostly acceptable solution; (*α = 0.9)* basically acceptable solution; (*α = 1)* absolutely acceptable solution. After observing all the different values of $\tilde{z}^0(\alpha_k)$, the DM decides a goal value $\underline{G}$ (the minimum value) and its tolerance threshold $\overline{G}$ (the maximum value), which are used to formulate the membership function of $\tilde{G}$ to assess the satisfaction level of DM for the value of objective function. The DM is satisfied if $z \leq \underline{G}$ while the DM is unsatisfied if $z \geq \overline{G}$. The membership function of $\tilde{G}$, and the satisfaction level of the fuzzy goal $\tilde{G}$ by each $\tilde{z}^0(\alpha_k)$ are, respectively, as follows:

$$\mu_{\tilde{G}}(z) = \begin{cases} 1 & z \leq \underline{G} \\ \dfrac{\overline{G} - z}{\overline{G} - \underline{G}} & \underline{G} < z < \overline{G} \\ 0 & z \geq \overline{G} \end{cases} \quad (31)$$

$$K_{\tilde{G}}\left(\tilde{z}^0(\alpha)\right) = \frac{\int_{-\infty}^{+\infty} \mu_{\tilde{z}^0(\alpha)}(z).\mu_{\tilde{G}}(z)dz}{\int_{-\infty}^{+\infty} \mu_{\tilde{z}^0(\alpha)}(z)\,dz} \quad (32)$$

where the denominator refers to the part under $\mu_{\tilde{z}^0(\alpha)}$. The numerator represents the possibility occurrence $\mu_{\tilde{z}^0(\alpha)}$ of each crisp objective value of z, which is weighted by its satisfaction level $\mu_{\tilde{G}}(z)$ of the goal value $\tilde{G}$. (as shown in Figure 2).

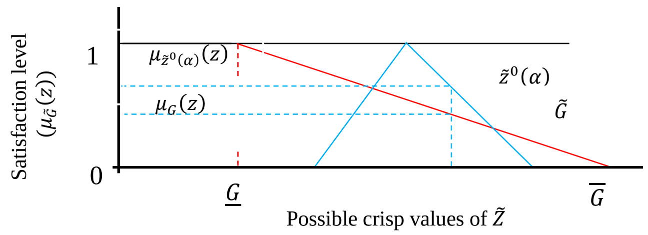

Figure 2 Occurrence possibility of crisp objective value *z* and its goal satisfaction level.

Finally, the balance level (or compromise) of each solution that respects to α_k, is calculated by:

$$\mu_{\tilde{D}}(x^0(\alpha_k)) = \alpha_k * K_{\tilde{G}}\left(\tilde{z}^0(\alpha_k)\right) \quad (33)$$

where * represents a *t*-norm that such as the algebraic product, the minimum, etc. The optimal solution x^* is the solution with the highest balancing level:

$$\mu_{\tilde{D}}(x^*) = \max_{\alpha_k \in M}\{\alpha_k * K_{\tilde{G}}(\tilde{z}^0(\alpha_k))\} \quad (34)$$

Following Equations (22) – (29), the proposed aggregate production planning model can be transformed entirely into an auxiliary crisp α- parametric model and then effectively solved as a LP problem.

2.3 Case study

The Aggregate Production Planning (APP) decision problem for a ball screw manufacturing plant is presented. The planning horizon of the APP decision is 4 months long, including January, February, March, and April. Two types of standard ball screws that are planned to be produced in the manufacturing plant, which are namely external recirculation (product 1) and internal recirculation (Product 2). Tables 2-3 are the production costs, forecast demand, and capacity data used in the model. The forecast demand, maximum workforce levels, maximum machine capacities, and production costs are presented as fuzzy numbers with the triangular possibility distribution from period to period. Other relevant data are described next:

- The initial carrying inventory level in the first month is 400 units of product 1 and 200 units of product 2. The ending carrying inventory level in the fourth month is 300 units of product 1 and 200 units of product 2.

- The initial labor level is 300 person/h. The fuzzy cost of hiring and firing are ($8, $10, $11) and ($2.0, $2.5, $3.2) per worker per hour, respectively.
- Hours of labor per unit for any months are fixed to 0.05 person–hours for Product 1 and 0.07 person/h for Product 2. Hours of machine usage per unit for any months are fuzzy with (0.09, 0.10, 0.11) and (0.07, 0.08, 0.09) machine–hours for Product 1 and Product 2, respectively. The required warehouse space per a unit of product is 2 ft^2 for Product 1 and 3 ft^2 for product 2.

Table 2 Related operating cost of both products.

Product	$\tilde{r}_{pt}$ ($/unit)	$\tilde{o}_{pt}$ ($/unit)	$\tilde{s}_{pt}$ ($/unit)	$\tilde{\iota}_{pt}$ ($/unit)	$\tilde{b}_{pt}$ ($/unit)
1	(17, 20, 22)	(26, 30, 33)	(22, 25, 27)	(0.27, 0.30, 0.32)	(35, 40, 44)
2	(8, 10, 11)	(12, 15, 17)	(10, 12, 13)	(0.13, 0.15, 0.16)	(16, 20, 23)

Table 3 Forecast demand of products, maximum labor, maximum machine, and warehouse space data.

Month	$\tilde{D}_{1t}$ (units)	$\tilde{D}_{2t}$ (units)	$\widetilde{MaxL}_t$ (person-h)	$\widetilde{MaxM}_t$ (machine-h)	$MaxW_t$ (ft^2)
1	(900, 1000, 1080)	(900, 1000, 1080)	(175, 300, 320)	(360, 400, 430)	10,000
2	(2750, 3000, 3200)	(450, 500, 540)	(175, 300, 320)	(450, 500, 540)	10,000
3	(4600, 5000, 5300)	(2750, 3000, 3200)	(175, 300, 320)	(540, 600, 650)	10,000
4	(1850, 2000, 2100)	(2300, 2500, 2650)	(175, 300, 320)	(450, 500, 540)	10,000

3. Results and discussion

3.1 Linear programming (LP) results

As aforementioned, the LP model can be used for finding the ideal solutions (i.e. maximizing the profit, minimizing the total costs) of the APP plan in specified conditions for every case. Here, the LP is also used to obtain the value of Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of three objective function (Z_1, Z_2, and Z_3). The most likely value of the possibility distribution of each fuzzy number is considered as an accurate number. Therefore, the results of the APP plan for the most likely case are described in Table 4.

Table 4 APP plan of LP results of most likely case.

Items		Month			
		1	2	3	4
Product 1	Regular–time production_RQ_{1t} (units)	600	3,000	5,000	2,300
	Overtime production_OQ_{1t} (units)	0	0	0	0
	Subcontracted_SQ_{1t} (units)	0	0	0	0
	Inventory_IQ_{1t} (units)	0	0	0	300
	Backordered_BQ_{1t} (units)	0	0	0	0
Product 2	Regular–time production_RQ_{2t} (units)	3,174	1,460	220	2,148
	Overtime production_OQ_{2t} (units)	0	0	0	0
	Subcontracted_SQ_{2t} (units)	0	0	0	0
	Inventory_IQ_{2t} (units)	2,374	3,333	552	200
	Backordered_BQ_{2t} (units)	0	0	0	0
Total hired workers_H_t (persons)		0	0	14	0
Total fired workers_F_t (persons)		48	0	0	0
Machine capacity (machine-hours)		314	417	518	402
Warehouse space (ft^2)		7,122	10,000	1,658	1,200
$Z_1 = Z^m$		$289,310.18			
$Z_2 = Z^m - Z^o$		$46,888.44			
$Z_3 = Z^p - Z^m$		$28,853.15			
Total cost (Z^m)		$289,310.18			

Based on Table 4, it is found that the quantities of regular-time production of Product 1 from the first month to the fourth month are 600, 300, 5000 and 2300 units respectively. It can be seen that product 1 has 300 units of ending inventory with no subcontracting units. In comparison with product 1, the quantities of ending inventory of products 2 have substantially raised, and subcontracting is also not required. Only fourteen new workers are employed in the third month, but the number of dismissed workers is 48 in the first month. It is required 7,122, 10,000, 1,658 and 1,200 ft2 of warehouse space to keep product 1 and product 2 from the first month to the fourth month. The total costs of this aggregate production planning are $289,310.18.

The Negative Ideal Solution (PIS) of the three new objective functions is ($265,219.3; $49,883.47; $26,449.56) and the Positive Ideal Solution (NIS) is ($307,681.64; $42,970.07; $30,687.37). The corresponding membership function of the three new objective functions are identified following to Eqs. (17), (18), and (19). Table 5 lists the multiple objective values for the three cases (optimistic, most likely, and pessimistic).

Table 5 Imprecise objective function values for three cases.

Imprecise objectives	Optimistic case	Most likely case	Pessimistic case
$Z_1 = Z^m$	$265,219.3	$289,310.18	$307,681.64
$Z_2 = Z^m - Z^o$	$42,970.07	$46,888.44	$49,883.47
$Z_3 = Z^p - Z^m$	$26,449.56	$28,853.15	$30,687.37
Total costs	$222,249.2	$289,310.18	$338,431.36

3.2 Possibilistic linear programming (PLP) results

The PLP approach can be used for practical APP in an uncertain environment. The PLP approach is employed to handle the simplified triangular possibility distribution for representing the imprecise objectives and related imprecise numbers. In this study, PLP is used to solve general imprecise APP problems through an interactive process with the DM. The PLP yields the overall satisfaction level of DM under the strategy of minimizing the most likely values, minimizing the possibility of achieving higher objective values, and maximizing the possibility of achieving lower objective values simultaneously. The PLP approach provides an efficient compromise solution. Table 6 presents the entire APP plan that is solved by the PLP approach.

According to Table 6, the produced quantities of product 1 in the regular-time production from the first month to the fourth month are 577, 2,985, 4,885, and 2,259 units, respectively. It can be seen that the quantities of ending inventory of product 1 are 31 and 300 units in the second and the fourth month, respectively. Product 1 also requires subcontracting with 6, 1, 5 units in the first, second, and fourth month, respectively. Product 2 shows 3,145, 1,425, 225, and 2,115 units of regular-time production from the first month to the fourth month. Product 2 also indicates that a higher level of ending inventory with 2,362, 3,295, 568, and 340 units from the first month to the fourth month, respectively. Subcontracting for product 2 is 2 and 116 units in the third month and the fourth month. The number of dismissed workers is 51 in the first month. Eleven new workers are employed in the third month and one is hired in the fourth month. It is required is 7,086, 9,947, 1,704, and 1,620 ft^2 of warehouse space to keep product 1 and product 2 from the first month to the fourth month, respectively. The results of maximizing the lower total costs, minimizing the most likely total costs, and minimizing the higher total costs are $241,325.56, $287,700.95, and $316,317.35, respectively. The overall satisfaction level of DM is 49.35%.

Table 6 PLP results.

Items		Month			
		1	2	3	4
Product 1	Regular–time production_RQ_{1t} (units)	577	2,985	4,885	2,259
	Overtime production_OQ_{1t} (units)	0	0	0	0
	Subcontracted_SQ_{1t} (units)	6	0	1	5
	Inventory_IQ_{1t} (units)	0	31	0	300
	Backordered_BQ_{1t} (units)	0	0	0	0
Product 2	Regular–time production_RQ_{2t} (units)	3,145	1,425	225	2,115
	Overtime production_OQ_{2t} (units)	0	0	0	0
	Subcontracted_SQ_{2t} (units)	0	0	2	116
	Inventory_IQ_{2t} (units)	2,362	3,295	568	340
	Backordered_BQ_{2t} (units)	0	0	0	0
Total hired workers_H_t (persons)		0	0	11	1
Total fired workers_F_t (persons)		51	0	0	0
Machine capacity (machine-hours)		309	413	507	395
Warehouse space (ft^2)		7,086	9,947	1,704	1,620
Overall satisfaction (L)		49.35%			
$Z_1 = Z^m$		$287,700.95			
$Z_2 = Z^m - Z^o$		$46,375.39			
$Z_3 = Z^p - Z^m$		$28,616.40			
Minimization of the most likely total costs		$287,700.95			
Maximization of the lower total costs		$241,325.56			
Minimization of the higher total costs		$316,317.35			

3.3 Interactive resolution results

The possibility distributions of objective function are evaluated for each discrete value of α_k in the set M = {0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}. The DM chooses α-acceptable optimal solutions (α = 0.2 as a minimum value). After calculating the different values of $Z(\alpha)$, the DM determines the value of $\underline{G}$ = 234,159.17 (the minimum value), which implies that the DM is satisfied with the objective value. In contrast, the DM specifies the value of $\overline{G}$ = 318,830.52 (the maximum value), which is the maximum possible cost that the DM can accept. The results are shown in Table 7.

Table 7 Optimal solutions with various values of α.

Feasibility degree (α)	Possibility distribution of objective value Z			The compatibility index of each solution $K_{\tilde{G}}\left(\tilde{z}^0(\alpha_k)\right)$	The degree of balance of each solution $\mu_{\tilde{D}}(x^0(\alpha_k))$
	Z^P	Z^m	Z^o		
0.2	307375.12	279442.30	234159.17	0.7198	0.2
0.3	308558.68	280519.20	235061.88	0.7050	0.3
0.4	309693.00	281552.50	235930.86	0.6982	0.4
0.5	310926.16	282671.15	236862.01	0.6814	0.5
0.6	312054.72	283701.05	237730.78	0.6661	0.6
0.7	313253.76	284790.90	238643.47	0.6496	0.6496
0.8	314732.68	286180.20	239889.29	0.6268	0.6268
0.9	316807.68	288145.20	241682.54	0.6075	0.6075
1	318830.52	290063.55	243437.66	0.5698	0.5698

Based on the results in Table 7, the bold values represent for the best degree of balance (0.6496). It is found that the value of α = 0.7 which is chosen to compute the optimal value of the total costs. The DM can easily adjust the goal values of $\underline{G}$ and its tolerance threshold $\overline{G}$ or alter the values of different feasibility degrees if the DM is not satisfied with the current achieved solution.From Table 8, it is found that the quantities of regular-time production of product 1 from the first month to the fourth month are 583, 3,180, 4,697 and 2,268 units respectively. It can also be seen that the quantities of ending inventory of product 1 from the first month to the fourth month are 1, 227, 1 and 300 units respectively, and has no subcontracting units. In comparison with product 1, the quantities of ending inventory units of product 2 has substantially raised and subcontracting is also not required. Only nine new workers are employed in the third month, but the number of discharge workers is 50 in the first month. It is required 7, 121, 10,000, 1,721, and 1,200 ft2 of warehouse space to keep product 1 and product 2 from the first month to the fourth month, respectively. The total costs of this aggregate production planning in optimistic, most likely, and pessimistic cases are $238,643.47, $284,790.90 and $313,253.76 respectively.

Table 8 Implementation of proposed APP plan.

Items		Month			
		1	2	3	4
Product 1	Regular–time production_RQ_{1t} (units)	583	3,180	4,697	2,268
	Overtime production_OQ_{1t} (units)	0	0	0	0
	Subcontracted_SQ_{1t} (units)	0	0	0	0
	Inventory_IQ_{1t} (units)	1	227	1	300
	Backordered_BQ_{1t} (units)	0	0	0	0
Product 2	Regular–time production_RQ_{2t} (units)	3,155	1,300	345	2,080
	Overtime production_OQ_{2t} (units)	0	0	0	0
	Subcontracted_SQ_{2t} (units)	0	0	0	9
	Inventory_IQ_{2t} (units)	2,373	3,182	573	200
	Backordered_BQ_{2t} (units)	0	0	0	0
Total hired workers_H_t (persons)		0	0	9	0
Total fired workers_F_t (persons)		50	0	0	0
Machine capacity (machine-hours)		322	426	515	398
Warehouse space (ft^2)		7,121	10,000	1,721	1,200
Total costs (Z)		Pessimistic case			$313,253.76
		Most likely case			$284,790.90
		Optimistic case			$238,643.47

3.4 Result comparison

Table 9 Result comparison.

Criteria	Linear Programming (LP) model		Possibilistic linear programming (PLP) model	Interactive fuzzy linear programming (*i*-FLP) model
	Z^{PIS}	Z^{NIS}		
Overall satisfaction	100%	0%	49.35%	53.78%
Minimization of the most likely total costs	$265,219.3	$307,681.6	$287,700.95	$284,790.90
Maximization of the possibility of getting lower total costs	$222,249.2	$257,798.2	$241,325.56	$238,634.47
Minimization of the possibility of getting higher total costs	$291,668.8	$338,369	$316,317.35	$313.253.76

Table 9 compares the i-FLP approach with the LP, and the PLP approaches. The overall results can be compared, with the objective values and overall satisfaction level from every single objective. From Table 10, it can be seen that the overall satisfaction level of the proposed approach reaches 53,78%. In comparison, the overall satisfaction of the typical PLP model is slightly less effective when only 49,35%. The LP model under the optimistic and pessimistic cases are instituted to be the PIS by 100% and the NIS by 0% overall satisfaction level. It means that a better result in terms of the optimal solution with reference to the overall satisfaction level of the proposed approach, and it is also found that the obtained results will be nearer to the ideal solutions of the LP model (as the ideal optimal solution does not consider any uncertainty). Additionally, the best values of the i-FLP method of all objective functions (the lowest cost) are able to approach or nearly approach the ideal optimal values (PIS) that are achieved by solving the LP model in the optimistic case (100% of the overall satisfaction). On the other hand, the worst values of all objective functions (the highest cost) are also better than the pessimistic values (NIS) that are solved by the LP model for the pessimistic case (0% of the overall satisfaction). This indicates that the achieved outcomes (even in an uncertain environment) nearly obtain the best optimistic values, and they are greater than the worst pessimistic values. Taking the advantages of the concepts of the i-FLP model, the proposed outcomes can easily indicate the most likely value, and the minimum and maximum possible values as well. From being known these values, the DMs can efficiently plan for their budget, taking necessary actions for any uncertainty in the future as a typical linear programming model cannot present such a possibility. Several significant characteristics distinguish between PLP and i-FLP, as shown in Table 10. Throughout the analysis and comparison, it is seen that the proposed approach (interactive fuzzy linear programming) produces better results compare to Possibilistic Linear Programming (PLP) approach. By applying the *i*-FLP to the Aggregate Production Planning (APP) problem, the costs of three cases (optimistic, most likely, and pessimistic cases) are $238,643.47, $284,790.90, $313,253.76 which are decreased compared to the PLP approach ($241,325.56, $287,700.95 and $316,317.35) respectively. In summary, the proposed approach (ì-FLP) is responsible for producing better results compared to the PLP approach. Hence, the proposed approach provides lower cost and provides a novel method for solving the Aggregate Production Planning (APP) problem in an uncertain environment.

Table 10 Distinguish between PLP and *i*-FLP.

Criteria	Possibilistic Linear Programming	Interactive Fuzzy Linear Programming
Defuzzification of the objective function	Uses the possibility and risk to obtain the lower and higher total costs	Uses the expected interval value to defuzzify the fuzzy cost structure
Defuzzification of the constraints	Requires the weight average method, Fuzzy ranking method	Requires the hybrid fuzzy ranking method
Possible results	Provides a range of possible total costs (pessimistic, most likely, optimistic)	Provides a possibility distribution of total costs (pessimistic, most likely, optimistic)
Fuzzy number	Uses only the triangular distribution to represent the fuzziness	Can be in any form (triangular, trapezoidal, and linear or nonlinear problem)
	Subject to weight allocation when defuzzifying fuzzy data	Subject to the level of α (feasible degree), integrating the expected value (EV) and expected interval (EI) when defuzzifying fuzzy data.

3.5 Managerial implications

According to the obtained results and comparison in this study, the introduced approach can yield some advantages as follows:

As compared to the deterministic LP model, the approach can help the DMs to optimize and realize the results in three possible cases of the situation including optimistic case, most likely case, and pessimistic case. Based on the obtained decision variables from these three cases, the planners or managers of company can identify the required levels of inventory, workforce, and machine capacity. Being well aware of this obtained information, they can prepare and take necessary actions about the company's budget and finance under any change in the future.

As compared to the PLP model with the defuzzification methods. The ranking method was used in the PLP model to defuzzify the fuzzy numbers by separating the constraints into different scenarios while the weighted average method just converts a fuzzy number to be a crisp number by assigning weights to possible values of fuzzy numbers. These methods do not provide any information about likely violation of constraints (feasibility concept). In contrast, having relied upon the fuzzy relation between fuzzy numbers, the defuzzification method of the proposed approach not only helps the fuzzy data to avoid being defuzzified earlier in the defuzzification process but also can seek the best fuzziness level of fuzzy constraints. This is one of the outstanding features of this proposed defuzzification method. The effectiveness of this method was also proven through obtaining better results for all three cases of the situation (the obtained results get closer to the ideal optimal results). In practice, it is difficult for the company to control the fuzziness levels of constraints such as workforce level and maximum machine capacity or even customer demand cannot be controlled. However, having known the optimal fuzziness level of these constraints will help the company in making effort to run its operation toward the obtained fuzziness level. For example, if the optimal fuzziness level of the maximum machine capacity is relatively on the right-hand side of the maximum available machine capacity. The company can spend more investing budget on buying more machines to enhance the machine capacity and vice versa. Thus, the justification of higher spending and gained benefits can be assessed by its worthiness.

The proposed approach could also be utilized easily for linear and nonlinear forms. Moreover, relied on historical data or subjective judgment, other forms of appropriate possibility distribution could also be generated and applied to solve the problem.

4. Conclusion

This study presents interactive Fuzzy Linear Programming (*i*-FLP) to support the decision-making process of a multi-product, multi-period Aggregate Production Planning (APP) problem. It considers the effects of uncertainty and incompleteness of data, which are significant issues in APP problems. It also yields alternative information on strategies for regular-time, overtime, inventory, subcontracting, backordering, and hiring and firing workers to cope with variations in forecast demand. Additionally, the approach also considers the actual limitations in labor, machinery, and warehouse capacity. The approach assists decision-makers (DMs) in the trade-off between two conflicting problems: obtaining the objective value and enhancing the satisfaction level of constraints. Once the satisfaction level of constraints is higher, the number of feasible solutions can be smaller, it causes the DM's choices to be restricted. Eventually, the optimal value of the objective can be worse. To validate and demonstrate the effectiveness of the proposed model and its solution, a study case is utilized to illustrate the feasibility of applying the proposed model. The outcomes also indicate that the proposed model can bring a better solution in terms of the actual total costs and the APP plan. Because both unbalanced and balanced efficient solutions may be obtained by this approach, DMs are given more flexibility to determine the most suitable plan that depends on objective conditions.

The limitation of this study is that there are only three fuzzy parameters (labor level, machine capacity, and customer demand) that are considered as uncertain or imprecise in the problem. With the proposed approach, more parameters could be considered to be fuzzy and in fact there is no limitation of the number of fuzzy parameters. Besides that, since the proposed APP model is only optimized based on the total costs of the plan, an APP model with multiple conflicting objectives and more constraints based on the business situations can be explored in further research. In addition, once APP models become very large and too complex to be solved by IBM ILOG CPLEX software (as it was used in this study), it is necessary to investigate the suitability of using metaheuristic algorithms such as Genetic Algorithm, Ant Colony, and so on for any possibility to obtain optimal results.

5. References

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