Asia-Pacific Journal of Science and Technology: Volume: 28. Issue: 05. Article ID.: APST-28-05-03. Research Article

APST

Asia-Pacific Journal of Science and Technology

https://www.tci-thaijo.org/index.php/APST/index

Published by the Research and Graduate Studies Division,
Khon Kaen University, Thailand

Cosine similarity measures for Pythagorean fuzzy sets with applications in decision making

Mansi Bhatia[1], Hari D. Arora[1,*], Anjali Nathani[1] and Vijay Kumar[2]

[1]Department of Mathematics, Amity Institute of Applied Sciences, Amity University, Noida, India
[2]Department of Mathematics, Manav Rachna International Institute of Research & Studies, Faridabad, India
[*]Corresponding author: hdarora@amity.edu

Received 28 December 2021
Revised 16 September 2022
Accepted 17 September 2022

Abstract

Human life is full of uncertainties as they play a crucial role in several decision-making processes. Numerous approaches have been applied to deal with the ambiguous critical decision-making problems. Probably, the most recent approach in this is Pythagorean fuzzy sets (PFSs). These sets are an extension of intuitionistic fuzzy sets (IFSs) and are more powerful tool than PFS. The purpose of this article is to introduce some new cosine similarity measures by highlighting the standardized parameters that illustrate PFSs. Several similarity measures have been presented for PFS, however, many of these measures are ineffective in the sense that they have inherent shortcomings that restrict them from providing reliable and consistent results. The measures proposed are flexible and easy to use with a variety of decision making problems. A mathematical illustration has also been employed to check the reliability of the proposed similarity measures. Some real-life applications are also discussed and comparison of the results with the prevailing analogous similarity measures has been done to exhibit the efficacy of the suggested similarity measures.

Keywords: Fuzzy set, Intuitionistic fuzzy set, Medical diagnosis, Pattern recognition, Pythagorean fuzzy sets, Trigonometric similarity measure

1. Introduction

The classical set theory was extended to the theory of fuzzy sets by [1]. Fuzzy sets represent sets with a continuous degree of membership where classical set gives information about only inclusion and exclusion of an element or object. This concept was further extended to intuitionistic fuzzy sets (IFSs) by Atanassov [2] where degree of non-membership function was also included. The concept captivated the attention of numerous researchers due its applications in various fields like pattern recognition, medical diagnosis, decision making, image processing etc. Atanassov [3] introduced the concept of Interval valued intuitionistic fuzzy sets. De et al. [4] proposed the application of IFSs in medical diagnosis. Li et al. [5] further developed new similarity measures of IFSs and application to pattern recognitions and Li et al. [6] did a comparative analysis between similarity measures and IFSs related to pattern recognition.

Ye et al. [7] introduced cosine similarity measures for intuitionistic fuzzy sets and their applications. Rajarajeswari et al. [8] proposed Intuitionistic fuzzy multi similarity measure based on cotangent function in which multi membership and non-membership for the same element was considered. A new concept of distance measures in fuzzy sets was introduced which was used by many researchers in various fields. Zhou et al. [9] applied it in issues of decision-making incorporating risk preference. Further Dutta et al. [10] proposed an article on fuzzy decision making in medical diagnosis using an advanced distance measure on IFSs. Immaculate et al. [11] suggested some new similarity measures based on cotangent function for multi period medical diagnosis. Decision making is among one of the fields that have attracted many researchers. Garg et al. [12] correlated the theory of similarity measurement and intuitionistic fuzzy sets to find solutions of decision-making problems.

The latest concept in fuzzy sets is related to Pythagorean fuzzy sets (PFSs). It is the generalization of IFS and holds many applications in decision making. Yager et al. [13-16] introduced the concept of PFS where the membership $\xi(x)$ and non-membership function $\rho(x)$ satisfies $0 \leq \xi_A^2(x) + \rho_A^2(x) \leq 1$. Peng et al. [17] suggested Pythagorean fuzzy information measures and their application in clustering analysis, pattern recognition and medical diagnosis has been given. Verma et al. [18] applied PFS on graph theory and examined the construction of Pythagorean fuzzy graphs. Ejegwa et al. [19] extended normalized Euclidean distance to PFS using distance similarity measures. Wei et al. [20] proposed similarity measures based on the cosine function and their application. However, Xuan [21] offered exponential similarity measures for PFS and their applications to pattern recognition and decision-making process. Ejegwa et al. [22] introduced PFS and its application in career placements based on academic excellence using the maximum minimum composition which helps the applicants to discover the suitable career for them. Furthermore, Zhang et al. [23] proposed new similarity measures on PFS and their Applications. Ejegwa et al. [24] discussed new similarity measures for Pythagorean fuzzy sets with applications in influential issues. Hussain et al [25] developed similarity measures of PFSs with TOPSIS method. Agheli et al. [26] proposed similarity measure for PFSs and application on multiple-criteria decision making (MCDM). Further Zhang et al [27] discussed some similarity measures of interval-valued intuitionistic fuzzy sets with their application in pattern recognition. Sharma et al. [28] developed some properties related to Intuitionistic fuzzy trigonometric distance and similarity. Tian et al. [29] applied new similarity measure based on cotangent function for medical purpose. Further, He et al. [30] introduced a new distance measure of PFSs based on matrix and its application in medical diagnosis. PFS is used extensively with MADM which refers to making decisions by prioritizing a set of attributes. PFS is a more powerful than IFS since it helps in cases where IFS fails. Molodstov et al. [31] introduced the concept of soft sets to deal with uncertain objects. Zhang [32] discussed the concept of bipolar fuzzy sets for cerebral decisions which provided a unified approach to fuzziness. Mahmood et al. [33] combined both the concepts and discussed bipolar valued soft sets. Riaz et al. [34] suggested linear Diophantine fuzzy sets which is a new way to deal with fuzzy uncertainities. Peng et al. [35] suggested measure based on multi parametric similarity measure using MCDM. Yager et al. [36] in 2017 introduced q- rung orthopair fuzzy sets which were extension of IFS and PFS. Later spherical fuzzy sets were introduced which are extensions of picture fuzzy sets and PFSs. Mahmood et al [37-38] discussed T- spherical fuzzy sets and later suggested GRA spherical linguistic fuzzy sets. Qiaz et al. [39] introduced Pythagorean fuzzy Dombi aggregation operators. Janani et al [40] proposed complex Pythagorean fuzzy Einstein aggregation operators. Verma et al. [41] discussed two trigonometric measures based on cosine and cotangent function and applied them to MCDM. Later Peng et al. [42] developed new similarity and distance measures and discussed their applications in pattern recognition. Barukab et al. [43] proposed new entropy based on spherical fuzzy environment. Abdullah et al. [44,45] discussed spherical fuzzy distance measures. Further, Ashraf et al. [46,47] suggested spherical logarithmic aggregation operators and logarithmic hybrid aggregation operators. Rafiq et al. [48] proposed cosine similarity measure of spherical fuzzy sets. Ashraf et al. [49-51] proposed different aggregation operators with applications in decision making. Batool et al. [52] suggested Pythagorean hesitant decision making based on entropy. Ashraf et al. [53] discussed fuzzy decision support modelling for land agriculture on sine valued neutrosophic information and later Jin et al. [54] suggested its application for hydrogen plant.

PFS is a powerful tool which can be applied in many real-life problems to solve complex decision making problems. Since very less work has been done with cosine similarity measure our proposed measure will prove to be a useful tool which can be combined with various decision-making problems to get the answer to multifarious issues. With the passing of time people have been exposed to many options and choosing the best out of available is becoming tougher. This measure when combined with MCDM methods will prove to be an easy and flexible option.

In this paper similarity measures based for PFSs based on cosine trigonometric functions are discussed. To successfully carry out this, the paper is designed as follows: Section 2 is based on Preliminaries; In section 3 cosine similarity measures are proposed. Section 4 discusses their application in medical diagnosis and pattern recognition. In section 5 comparative analysis of the measures is done to ensure the efficacy of the measures with the measures proposed by different authors. Section 6 concludes the work done.

2. Materials and methods

2.1 Preliminaries

Before starting our work, we will firstly discuss some of the notions on Fuzzy sets, IFS and PFS that are required for better understanding of the concept [1].

Let F be a fuzzy set X then F= $\{x, \xi_F(x) \mid x \in X\}$ where $\xi_F(x): X \to [0, 1]$ Where $\xi_F(x)$ is the degree of membership of F [2]. For intuitionistic fuzzy set F in X can be defined as F= $\{x, \xi_F(x), \rho_F(x) \mid x \in X\}$ where $\xi_F(x): X \to [0, 1]$ and $\rho_F(x): X \to [0,1]$ where $\xi_F(x)$ is the degree of membership and $\rho_F(x)$ is the degree of non-membership such that $0 \leq \xi_F(x) + \rho_F(x) \leq 1$ [7]. Let E= $\{x, \xi_E(x), \rho_E(x) \mid x \in X\}$, F= $\{x, \xi_F(x), \rho_F(x) \mid x \in X\}$ are two IFS in $X = \{x_1, x_2 \ldots, x_n\}$ then

$$IF^1(E,F)= \frac{1}{n}\sum_{j=1}^{n}\left(\frac{\xi_{E(x_i)}\xi_{F(x_i)}+\rho_{E(x_i)}\rho_{F(x_i)}}{\sqrt{\xi_E^2(x_i)+\rho_E^2(x_i)}\sqrt{\xi_F^2(x_i)+\rho_F^2(x_i)}}\right) \quad (1)$$

Ye [7] proposed Cosine similarity measure between two Intuitionistic Fuzzy sets E and F as

$$IF^2(E,F)=\frac{1}{n}\sum_{i=1}^{n}\left[cot\left\{\frac{\pi}{2}\left(\left|\xi_{E(x_i)}-\xi_{F(x_i)}\right| \vee \left|\rho_{E(x_i)}-\rho_{F(x_i)}\right| \vee \left|\pi_{E(x_i)}-\pi_{F(x_i)}\right|\right)\right\}\right] \quad (2)$$

where " $\vee$ " denotes the maximum operation

$$IF^3(E,F)= \frac{1}{n}\sum_{i=1}^{n}\left[cot\left\{\frac{\pi}{4}\left(\left|\xi_{E(x_i)}-\xi_{F(x_i)}\right| + \left|\rho_{E(x_i)}-\rho_{F(x_i)}\right| + \left|\pi_{E(x_i)}-\pi_{F(x_i)}\right|\right)\right\}\right] \quad (3)$$

Tian [29] proposed cotangent similarity measure between IFS E and F as

$$IF^4(E, F) =\frac{1}{n}\sum_{i=1}^{n}\left[cot\left\{\frac{\pi}{4}+\frac{\pi}{4}\left(\left|\xi_{E(x_i)}-\xi_{F(x_i)}\right| \vee \left|\rho_{E(x_i)}-\rho_{F(x_i)}\right|\right)\right\}\right] \quad (4)$$

F= {<x, ξ_F (x), ρ_F (x) | x ∈ X} where ξ_F (x): X → [0, 1] and ρ_F (x): X → [0, 1] where ξ_F (x) is the degree of membership and ρ_F (x) is the degree of non-membership such that $0 \leq \xi^2_F(x) + \rho_F^2(x) \leq 1$ [13]. $\pi_F^2(x) = 1- \xi_F^2(x) - \rho_F^2(x)$ where $\pi_F(x)$ is called hesitancy or uncertainty of Pythagorean Fuzzy Set F.

2.2 Similarity measures based on PFSs

In this section we will discuss some existing similarity measures in the concerned field.

2.2.1 Existing similarity measure

Let E = { $(x_i , \xi_E(x_i) , \rho_E(x_i) \mid x_i \in X)$} and F = { $(x_i , \xi_F(x_i) , \rho_F(x_i) \mid x_i \in X)$} be the two PFSs in X = { $x_1, x_2 ,.., x_n$ } then similarity measure is given as [17]

$$Csim^1 (E, F) = 1- \frac{1}{2X}\sum_{x\in X}\left(|\xi_E^2(x_i)-\xi_F^2(x_i)| + |\rho_E^2(x_i)-\rho_F^2(x_i)| + (|\pi_E^2(x_i)-\pi_F^2(x_i)|)\right) \quad (5)$$

For E and F ∈ PFS (X) such that X = {$x_1, x_2, \dots, x_n$} then similarity measures are given as [20]

$$Csim^2 (E, F) =\frac{1}{n}\sum_{j=1}^{n}\left[cos\left\{\frac{\pi}{2}(|\xi_E^2(x_i)-\xi_F^2(x_i)| \vee |\rho_E^2(x_i)-\rho_E^2(x_i)|)\right\}\right] \quad (6)$$

$$Csim^3 (E, F) = \frac{1}{n}\sum_{j=1}^{n}\left[cos\left\{\frac{\pi}{4}(|\xi_E^2(x_i)-\xi_F^2(x_i)| + |\rho_E^2(x_i)-\rho_E^2(x_i)|)\right\}\right] \quad (7)$$

$$Csim^4 (E, F) =\frac{1}{n}\sum_{j=1}^{n}\left[cos\left\{\frac{\pi}{2}(|\xi_E^2(x_i)-\xi_F^2(x_i)| \vee |\rho_E^2(x_i)-\rho_E^2(x_i)| \vee |\pi_E^2(x_i)-\pi_F^2(x_i)|)\right\}\right] \quad (8)$$

$$Csim^5 (E, F) =\frac{1}{n}\sum_{j=1}^{n}\left[cos\left\{\frac{\pi}{4}(|\xi_E^2(x_i)-\xi_F^2(x_i)| \vee |\rho_E^2(x_i)-\rho_E^2(x_i)| + |\pi_E^2(x_i)-\pi_F^2(x_i)|)\right\}\right] \quad (9)$$

For E and F ∈ PFS (X) such that X = {$x_1, x_2, \dots x_n$} then the exponential similarity measure is given as [23]

$$Csim^6(E, F) =\frac{1}{n}\sum_{j=1}^{n}[2^{1-(|\xi_E^2(x_i)-\xi_F^2(x_i)| \vee |\rho_E^2(x_i)-\rho_F^2(x_i)|)-1}] \quad (10)$$

Let E ={ $(x_i, \xi_E(x_i), \rho_E(x_i) \mid x_i \in X)$} and F = { $(x_i , \xi_F(x_i) , \rho_F(x_i) \mid x_i \in X)$} be the two PFSs in X = { $x_1 , x_2 , \dots , x_n$ } then similarity measure can be formulated as [24]

$$Csim^7(E, F) = 1-\frac{1}{4n}\sum_{i=1}^{n}[|\xi_E(x_i)-\xi_F(x_i)| + |\xi_E(x_i)-\rho_E(x_i)| + |\xi_E(x_i)-\pi_E(x_i)| - |\xi_F(x_i)-\rho_F(x_i))| - |\xi_F(x_i)-\pi_F(x_i)|] \quad (11)$$

Let E ={ $(x_i, \xi_E(x_i), \rho_E(x_i), \pi_E(x_i) \mid x_i \in X)$} and F = { $(x_i , \xi_F(x_i) , \rho_F(x_i) , \pi_F(x_i) \mid x_i \in X)$} be the two PFSs in X = { $x_1 , x_2 , \dots , x_n$ } then generalized cosine similarity measure can be formulated as [41]

$$\text{Csim}^8(E, F) = \frac{1}{n}\sum_{i=1}^{n} \cos\left[\left(\frac{|\xi_E^2(x_i)-\xi_F^2(x_i)|^\alpha + |\rho_E^2(x_i)-\rho_F^2(x_i)|^\alpha + |\pi_E^2(x_i)-\pi_F^2(x_i)|^\alpha}{2}\right)^{1/\alpha} \frac{\pi}{2}\right], \alpha \geq 1 \quad (12)$$

3. Results

3.1 Proposed cosine similarity measures

Four cosine similarity measures based on PFS, and their properties have been proposed in this section. Let E = { (x_i , $\xi_E(x_i)$, $\rho_E(x_i)$ | $x_i \in X$)} and F = { (x_i , $\xi_F(x_i)$, $\rho_F(x_i)$ | $x_i \in X$)} be the two PFSs in X = {x_1 , x_2 , …, x_n}. Then we propose a PFS (E, F) as

$$\text{Csim}^9(E, F) = \frac{1}{2n}\sum_{i=1}^{n}\left(\cos\frac{\pi}{2}|\xi_E^2(x_i) - \xi_F^2(x_i)| + \cos\frac{\pi}{2}|\rho_E^2(xi) - \rho_F^2(xi)\right) \quad (13)$$

when hesitancy function is considered then similarity measure can be defined as

$$\text{Csim}^{10}(E,F)= \frac{1}{3n}\sum_{i=1}^{n} \cos\left(\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)|\right) + \cos\left(\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)|\right) + \cos\left(\frac{\pi}{2}|\pi_E^2(x_1) - \pi_F^2(x_1)|\right) \quad (14)$$

In many situations weights are being taken into consideration. Therefore, weighted cosine similarity measures are proposed as

$$\text{Csim}^{11}(E, F) = \frac{1}{2n}\sum_{i=1}^{n} wi\left(\cos\frac{\pi}{2}|\xi_E^2(x_i) - \xi_F^2(x_i)| + \cos\frac{\pi}{2}|\rho_E^2(xi) - \rho_F^2(xi)\right) \quad (15)$$

$$\text{Csim}^{12}(E, F) = \frac{1}{3n}\sum_{i=1}^{n} wi\left[cos\left(\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)|\right) + \cos\left(\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)|\right) + \cos\left(\frac{\pi}{2}|\pi_E^2(x_1) - \pi_F^2(x_1)|\right)\right] \quad (16)$$

For this Csim^9 (E, F), E and F in X the cosine similarity measure will have to satisfy following four axioms:
1. $0 \leq \text{Csim}^9(E, F) \leq 1$
2. $\text{Csim}^9(E, F) = 1 \Leftrightarrow E = F$.
3. $\text{Csim}^9(E, F) = \text{Csim}^9(F, E)$
4. If G is a PFS in X and $E \subseteq F \subseteq G$, then $\text{Csim}^9(E, G) \leq \text{Csim}^9(E, F)$ and $\text{Csim}^9(E, G) \leq \text{Csim}^9(F, G)$

Proof:
1. $0 \leq \text{Csim}^9(E, F) \leq 1$
Since cos lies between [0,1]. Csim^9 (E, F) also lies between [0, 1]. Thus,
$\Rightarrow 0 \leq \left(\cos\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)| + \cos\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)\right) \leq 2$
$\Rightarrow 0 \leq \frac{1}{2}\left(\cos\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)| + \cos\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)\right) \leq 1$
$\Rightarrow 0 \leq \frac{1}{2n}\left(\cos\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)| + \cos\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)\right) \leq 1$
$\Rightarrow 0 \leq \text{Csim}^9(E, F) \leq 1$
2. $\text{Csim}^9(E, F) = 1 \Leftrightarrow E=F$.
$\Leftrightarrow \frac{1}{2n}\sum_{i=1}^{n}\left(\cos\frac{\pi}{2}|\xi_E^2(x_i) - \xi_F^2(x_i)| + \cos\frac{\pi}{2}|\rho_E^2(xi) - \rho_F^2(xi)\right) = 1$
$\Leftrightarrow \cos\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)| + \cos\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)| = 2 \Leftrightarrow |\xi_E^2(x_1) - \xi_F^2(x_1)| = 0 \ \& \ |\rho_E^2(x_1) - \rho_F^2(x_1)| = 0$
$\Leftrightarrow \xi_E(x_1) = \xi_F(x_1) \ \& \ \rho_E(x_1) = \rho_F(x_1) \Leftrightarrow E = F$
3. $\text{Csim}^9(E, F) = \text{Csim}^9(F, E)$
Proof is obvious.
4. If G is a PFS in X and $E \subseteq F \subseteq G$, then $\text{Csim}^9(E, G) \leq \text{Csim}^9(E, F)$ and $\text{Csim}^9(E, G) \leq \text{Csim}^9(F, G)$
Proof. For Csim^9 (E, F): If $E \subseteq F \subseteq G$, then for $x_i \in X$, we have $0 \leq \xi_E(x_1) \leq \xi_F(x_1) \leq \xi_G(x_1) \leq 1$ and $1 \geq \rho_E(x_1) \geq \rho_F(x_1) \geq \rho_G(x_1) \geq 0$ $\Rightarrow 0 \leq \xi_E^2(x_1) \leq \xi_F^2(x_1) \leq \xi_G^2(x_1) \leq 1$ and $1 \geq \rho_E^2(x_1) \geq \rho_F^2(x_1) \geq \rho_G^2(x_1) \geq 0$
$\Rightarrow |\xi_E^2(x_1) - \xi_F^2(x_1)| \leq |\xi_E^2(x_1) - \xi_G^2(x_1)|$, $|\xi_F^2(x_1) - \xi_G^2(x_1)| \leq |\xi_E^2(x_1) - \xi_G^2(x_1)|$
and $\Rightarrow |\rho_E^2(x_1) - \rho_F^2(x_1)| \leq |\rho_E^2(x_1) - \rho_G^2(x_1)|$, $|\rho_F^2(x_1) - \rho_G^2(x_1)| \leq |\rho_E^2(x_1) - \rho_G^2(x_1)|$
$\therefore \frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)| \leq \frac{\pi}{2}|\xi_E^2(x_1) - \xi_G^2(x_1)| \Rightarrow \cos\left(\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)|\right) \leq \cos\left(\frac{\pi}{2}|\xi_E^2(x_1) - \xi_G^2(x_1)|\right)$ and

$\Rightarrow \cos\left(\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)|\right) \leq \cos\left(\frac{\pi}{2}|\rho_E^2(x_1) - \rho_G^2(x_1)|\right)$

Adding above equations

$\Rightarrow \cos\left(\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)|\right) + \cos\left(\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)|\right) \leq \cos\left(\frac{\pi}{2}|\xi_E^2(x_1) - \xi_G^2(x_1)|\right) + \cos\left(\frac{\pi}{2}|\rho_E^2(x_1) - \rho_G^2(x_1)|\right)$

$\Rightarrow \frac{1}{2n}\sum_{i=1}^{n}\cos\left(\frac{\pi}{2}|\xi_E^2(x_1) - \xi_F^2(x_1)|\right) + \cos\left(\frac{\pi}{2}|\rho_E^2(x_1) - \rho_F^2(x_1)|\right) \leq \frac{1}{2n}\sum_{i=1}^{n}\cos\left(\frac{\pi}{2}|\xi_E^2(x_1) - \xi_G^2(x_1)|\right) + \cos\left(\frac{\pi}{2}|\rho_E^2(x_1) - \rho_G^2(x_1)|\right))$

$\Rightarrow$ $Csim^9$ (E, G) $\leq$ $Csim^9$ (E, F) similarly we can proof for $Csim^9$ (E, G) $\leq$ $Csim^9$ (F, G)

Similarly, we can prove similarity measures proposed in equation (14), (15), and (16).

3.2 Numerical verification

In this subsection we will verify the cosine similarity measures based on properties proposed by [20]. Table 1 shows the values of similarity measure proposed in equation (13), (14), (15) and (16).

Let $E, F, G \in PFS(X)$ for $X = \{x_1, x_2, x_3\}$.

Suppose:

$E = \{\langle x_1, 0.6, 0.2\rangle, \langle x_2, 0.4, 0.6\rangle, \langle x_3, 0.5, 0.3\rangle\}$

$F = \{\langle x_1, 0.8, 0.1\rangle, \langle x_2, 0.7, 0.3\rangle, \langle x_3, 0.6, 0.1\rangle\}$

$G = \{\langle x_1, 0.9, 0.2\rangle, \langle x_2, 0.8, 0.2\rangle, \langle x_3, 0.7, 0.3\rangle\}$

$Csim^9(E,F) = \frac{1}{6}\left(\cos\frac{\pi}{2}|0.6^2 - 0.8^2| + \cos\frac{\pi}{2}|0.2^2 - 0.1^2|\right) + \cos\frac{\pi}{2}|0.4^2 - 0.7^2| + \cos\frac{\pi}{2}|0.6^2 - 0.3^2| + \cos\frac{\pi}{2}|0.5^2 - 0.6^2| + \cos\frac{\pi}{2}|0.3^2 - 0.1^2|)$

$= \frac{1}{6}(\cos\frac{\pi}{2}|0.36\text{-}0.64| + \cos\frac{\pi}{2}|0.04\text{-}0.01| + \cos\frac{\pi}{2}|0.16\text{-}0.49| + \cos\frac{\pi}{2}|0.36\text{-}0.09| + \cos\frac{\pi}{2}|0.25\text{-}0.36| + \cos\frac{\pi}{2}|0.09\text{-}0.01|)$

$= 0.943495958$

$Csim^9(F,G) = \frac{1}{6}\left(\cos\frac{\pi}{2}|0.8^2 - 0.9^2| + \cos\frac{\pi}{2}|0.1^2 - 0.2^2|\right) + \cos\frac{\pi}{2}|0.7^2 - 0.8^2| + \cos\frac{\pi}{2}|0.3^2 - 0.2^2| + \cos\frac{\pi}{2}|0.6^2 - 0.7^2| + \cos\frac{\pi}{2}|0.1^2 - 0.3^2|)$

$= \frac{1}{6}(\cos\frac{\pi}{2}|0.64\text{-}0.81| + \cos\frac{\pi}{2}|0.01\text{-}0.04| + \cos\frac{\pi}{2}|0.49\text{-}0.64| + \cos\frac{\pi}{2}|0.09\text{-}0.04| + \cos\frac{\pi}{2}|0.36\text{-}0.49| + \cos\frac{\pi}{2}|0.01\text{-}0.09|)$

$= 0.9840$

$Csim^9(E,F) = \frac{1}{6}\left(\cos\frac{\pi}{2}|0.6^2 - 0.9^2| + \cos\frac{\pi}{2}|0.2^2 - 0.2^2|\right) + \cos\frac{\pi}{2}|0.4^2 - 0.8^2| + \cos\frac{\pi}{2}|0.6^2 - 0.2^2| + \cos\frac{\pi}{2}|0.5^2 - 0.7^2| + \cos\frac{\pi}{2}|0.3^2 - 0.3^2|)$

$= \frac{1}{6}(\cos\frac{\pi}{2}|0.36\text{-}0.81| + \cos\frac{\pi}{2}|0.04\text{-}0.04| + \cos\frac{\pi}{2}|0.16\text{-}0.64| + \cos\frac{\pi}{2}|0.36\text{-}0.04| + \cos\frac{\pi}{2}|0.25\text{-}0.49| + \cos\frac{\pi}{2}|0.09\text{-}0.09|)$

$= 0.882576293$

From the computations above we can conclude that Property 1: $0 \leq Csim^i(E, F) \leq 1$; $i = 9$ to12, Property 2: $Csim^i(E, F) = 1 \Leftrightarrow E = F$; $i = 9$ to 12, Property 3: $Csim^i(E, F) = Csim^i(F, E)$; $i = 9$ to 12, Property 4: $Csim^i(E, G) \leq Csim^i(E, F)$ and $Csim^i(E, G) \leq Csim^i(F, G)$; $i = 9$ to 12.

Table 1 Numerical computation of proposed similarity measures

Similarity measures	(E, F)	(F, G)	(E, G)
$Csim^9$(E, F)	0.9434	0.9840	0.8825
$Csim^{10}$(E, F)	0.9532	0.9781	0.8869
$Csim^{11}$(E, F)	0.3135	0.3278	0.2912
$Csim^{12}$(E, F)	0.3157	0.3261	0.2909

3.3 Application of similarity measure

Validity of the proposed measures and their applicability in pattern recognition and medical diagnosis are discussed in the following section.

3.3.1 Pattern recognition

Let us suppose there are three patterns A, B, and C. conveyed by PFSs in $X = \{x_1, x_2, x_3\}$ as follows. Table 2 depicts the similarity between three known patterns and one unknown pattern with the help of proposed similarity measure.

A = $\{\langle\frac{1,0}{x_1}\rangle, \langle\frac{0.8,0}{x_2}\rangle, \langle\frac{0.7,0.1}{x_3}\rangle\}$, B = $\{\langle\frac{0.8,0.1}{x_1}\rangle, \langle\frac{1,0}{x_2}\rangle, \langle\frac{0.9,0.1}{x_3}\rangle\}$, C = $\{\langle\frac{0.6,0.2}{x_1}\rangle, \langle\frac{0.8,0}{x_2}\rangle, \langle\frac{1,0}{x_3}\rangle\}$ and let X = $\{\langle\frac{0.5,0.3}{x_1}\rangle, \langle\frac{0.6,0.2}{x_2}\rangle, \langle\frac{0.8,0.1}{x_3}\rangle\}$ be the pattern the similarity of which with A, B and C is to be determined. Let the weight of x_i are 0.5, 0.3, and 0.2, respectively.

Table 2 Similarity measure between A, B, C and X.

Similarity measures	(A, X)	(B, X)	(C, X)
$Csim^9(E, F)$	0.8746	0.8847	0.9548
$Csim^{10}(E, F)$	0.8510	0.8605	0.9452
$Csim^{11}(E, F)$	0.2752	0.2930	0.3217
$Csim^{12}(E, F)$	0.2644	0.2854	0.3197

3.3.2 Medical diagnosis

Let us assume that a patient has been diagnosed by a medical practitioner on the basis five symptoms namely Temperature (x_1), Headache (x_2), Stomach pain (x_3), Cough (x_4) and Chest pain (x_5) and set of prognosis P = $\{Viral\ fever, Malaria, Typhoid, Stomach\ problem\ and\ Chest\ problem\}$. Table 3 depicts the similarity between medical diagnosis and patient with the help of proposed similarity measure.

$P_1 = \{\langle\frac{0.4,0}{x_1}\rangle, \langle\frac{0.3,0.5}{x_2}\rangle, \langle\frac{0.1,0.7}{x_3}\rangle, \langle\frac{0.4,0.3}{x_4}\rangle, \langle\frac{0.1,0.7}{x_5}\rangle\}$, $P_2 = \{\langle\frac{0.7,0}{x_1}\rangle, \langle\frac{0.2,0.6}{x_2}\rangle, \langle\frac{0,0.9}{x_3}\rangle, \langle\frac{0.7,0}{x_4}\rangle, \langle\frac{0.1,0.8}{x_5}\rangle\}$, $P_3 = \{\frac{0.3,0.3}{x_1}\rangle, \langle\frac{0.6,0.1}{x_2}\rangle, \langle\frac{0.2,0.7}{x_3}\rangle, \langle\frac{0.2,0.6}{x_4}\rangle, \langle\frac{0.1,0.9}{x_5}\rangle\}$, $P_4 = \{\frac{0.1,0.7}{x_1}\rangle, \langle\frac{0.2,0.4}{x_2}\rangle, \langle\frac{0.8,0}{x_3}\rangle, \langle\frac{0.2,0.7}{x_4}\rangle, \langle\frac{0.2,0.7}{x_5}\rangle$, $P_5 = \{\frac{0.1,0.8}{x_1}\rangle, \langle\frac{0,0.8}{x_2}\rangle, \langle\frac{0.2,0.8}{x_3}\rangle, \langle\frac{0.2,0.8}{x_4}\rangle, \langle\frac{0.8,0.1}{x_5}\rangle\}$ and let, C = $\{\langle\frac{0.8,0.1}{x_1}\rangle, \langle\frac{0.6,0.1}{x_2}\rangle, \langle\frac{0.2,0.8}{x_3}\rangle, \langle\frac{0.6,0.1}{x_4}\rangle, \langle\frac{0.1,0.6}{x_5}\rangle\}$ be the pattern the similarity of which with A, B and C is to be determined. Let the weight of w_i are 0.15, 0.25, 0.20, 0.15 and 0.25 respectively.

Table 3 Similarity measure for medical diagnosis among patients.

Similarity measures	(P_1, C)	(P_2, C)	(P_3, C)	(P_4, C)	(P_5, C)
$Csim^9(E, F)$	0.9463	0.9547	0.9103	0.7834	0.7318
$Csim^{10}(E, F)$	0.9401	0.9587	0.9049	0.8470	0.8012
$Csim^{11}(E, F)$	0.1870	0.1893	0.1840	0.1613	0.1477
$Csim^{12}(E, F)$	0.1894	0.1905	0.1823	0.1725	0.1609

4. Discussion

4.1 Comparative analysis

A comparison between proposed similarity measures and similarity measures proposed by some of the authors has been done in Table 4 and the results are incorporated in the table for pattern recognition and medical diagnosis respectively. Table 4 and 5 show the comparative analysis done between the proposed similarity measure and measure proposed by [17,20,24,27,41] for pattern recognition and medical diagnosis respectively.

Table 4 Comparative analysis for pattern recognition.

Similarity measures	(A, X)	(B, X)	(C, X)
$Csim^1(E, F)$	0.6066	0.6	0.7116
$Csim^2(E, F)$	0.6573	0.7627	0.9329
$Csim^3(E, F)$	0.8843	0.9228	0.9782
$Csim^4(E, F)$	0.6573	0.7627	0.9329
$Csim^5(E, F)$	0.6573	0.7627	0.9329
$Csim^6(E, F)$	0.5462	0.5291	0.686
$Csim^7(E, F)$	0.8261	0.7995	0.8614
$Csim^8(E, F)$	0.7532	0.7801	0.8545
$Csim^9(E, F)$	0.8746	0.8847	0.9548
$Csim^{10}(E, F)$	0.8514	0.8605	0.9452
$Csim^{11}(E, F)$	0.2929	0.2838	0.3325
$Csim^{12}(E, F)$	0.2799	0.272	0.3165

Table 5 Comparative analysis for medical diagnosis.

Similarity measures	(P_1, C)	(P_2, C)	(P_3, C)
$Csim^1$(E, F)	0.7460	0.7820	0.7170
$Csim^2$(E, F)	0.9191	0.9250	0.8599
$Csim^3$(E, F)	0.9623	0.9554	0.9449
$Csim^4$(E, F)	0.9151	0.9244	0.8599
$Csim^5$(E, F)	0.9151	0.9244	0.8599
$Csim^6$(E, F)	0.6927	0.7248	0.6769
$Csim^7$(E, F)	0.8767	0.8915	0.9627
$Csim^8$(E, F)	0.8883	0.9314	0.8744
$Csim^9$(E, F)	0.9463	0.9547	0.9103
$Csim^{10}$(E, F)	0.9357	0.9788	0.9408
$Csim^{11}$(E, F)	0.1870	0.1893	0.1840
$Csim^{12}$(E, F)	0.1919	0.2933	0.1823

Figures 1 and 2 depict the similarity between the proposed entropy for pattern recognition whereas Figure 3 and 4 shows the comparative analysis of these similarity measures with the existing one in pattern recognition and medical diagnosis environment.

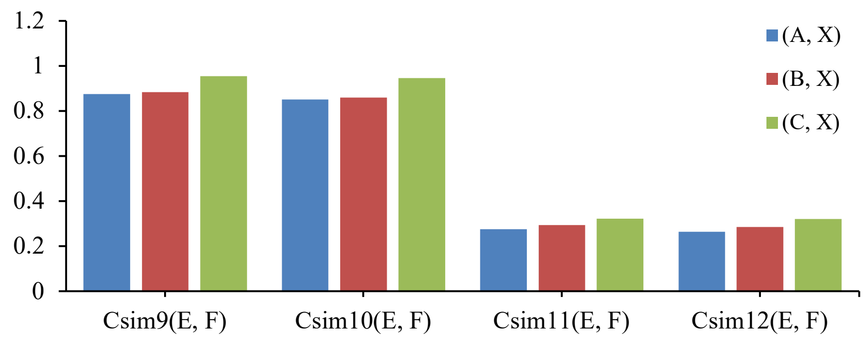

Figure 1 Similarity measures of A, B, C and X for pattern recognition.

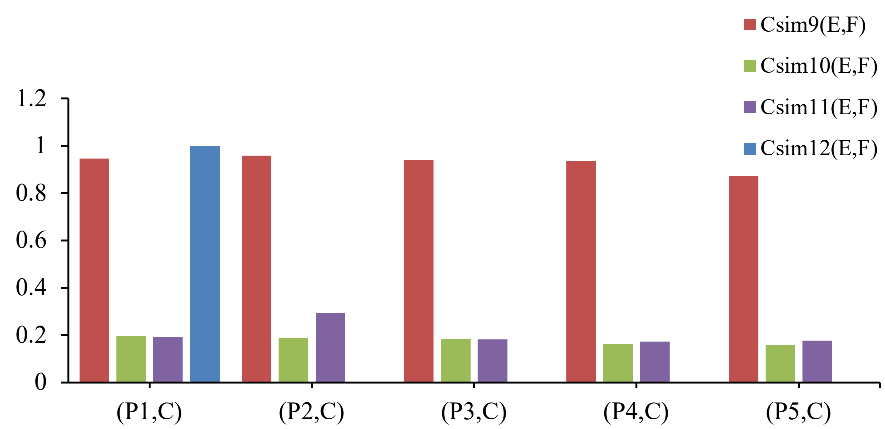

Figure 2 Similarity measures of P_1, P_2, P_3, P_4, P_5 and C for medical diagnosis.

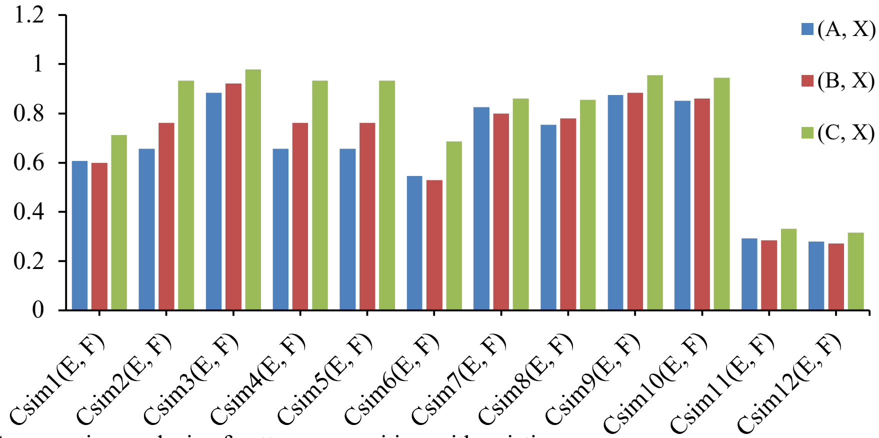

Figure 3 Comparative analysis of pattern recognition with existing measures.

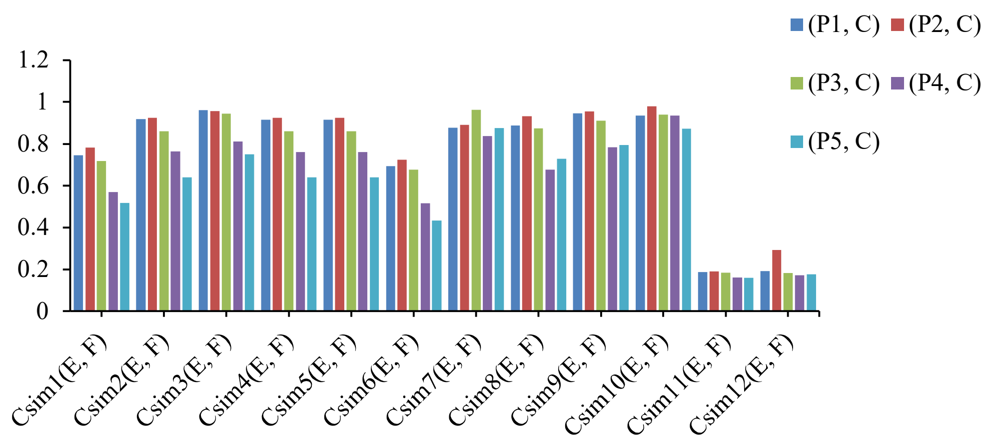

Figure 4 Comparative analysis of medical diagnosis with existing measures.

5. Conclusion

This paper offers some new trigonometric similarity measures based on cosine function considering membership and non-membership function along with hesitancy. Numerical computations are also being performed to see the reliability of these measures. We have implemented trigonometric measures and weighted similarity measures in pattern recognition and medical diagnosis. Comparative analysis of proposed measures has also been done to ensure the virtue of these measures. In coming years these criteria can be used to make complicated decisions. The measures are malleable and easily applicable with other decision making techniques which make them suitable to be used in diverse situations.

This study can be extended to q-rung orthopair fuzzy sets that are extension to PFS, spherical, T-spherical fuzzy sets and complex PFS. Since the measure proposed is supple and can be combined with MCDM techniques like TOPSIS, VIKOR, AHP, SAW and in clustering analysis, image processing etc. to make better decisions. This measure is not only easily comprehensible but also flexible. As not much work has been done with cosine measure this will help future researchers to work more on this topic, find and enhance more about the richness of cosine measures but will also help to solve real life problems more appropriately. The limitation of this study is lack of machine which can do the fuzzification that is needed to convert raw data into PFS data.

6. Acknowledgments

Authors thank the anonymous reviewers and editors for their careful reading of our manuscript and their insightful comments and suggestions.

7. References

[1] Zadeh LA. Fuzzy sets. Inform Control. 1965;8(3):338-356.
[2] Atanassov K. Intuitionistic fuzzy sets. Fuzzy Sets Syst. 1986;20(1):87-96.
[3] Atanassov K. (1989). More on intuitionistic fuzzy sets. Fuzzy Sets Syst. 1989;33(1):37-45.
[4] De SK, Biswas R, Roy AR. An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Sets and Syst. 2001;117(2):209-213.
[5] Li D, Cheng C. New similarity measures of intuitionistic fuzzy sets and application to pattern recognition. Pattern Recognition Lett. 2002;23(1-3):221-225.
[6] Li YH, Olson DL, Zheng Q. Similarity measures between intuitionistic fuzzy (vague) sets: a comparative analysis. Pattern Recognition Lett. 2007;28(2):278-285.
[7] Ye J. Cosine similarity measures for intuitionistic fuzzy sets and their applications. Math Comput Modell. 2011;53(1-2):91-97.
[8] Rajarajeswari P, Uma N. Intuitionistic Fuzzy Multi Similarity Measure Based on Cotangent Function, Int J Eng Res Technol. 2013;2(11):1323-1329.
[9] Zhou W, Xu Z. Extended intuitionistic fuzzy sets based on the hesitant fuzzy membership and their application in decision making with risk preference. Int J Intell Syst. 2007;33(2):417-443.
[10] Dutta P, Goala S. Fuzzy decision making in medical diagnosis using an advanced distance measure on intuitionistic fuzzy sets. Open Cybernet Syst J. 2018;12:136-149.
[11] Immaculate H, Evanzalin E, Sebastian T. A new similarity measure based on cotangent function for multi period medical diagnosis. Int J Mech Eng Technol. 2018;9:1285-1293.
[12] Garg H, Singh S. Algorithm for solving group decision-making problems based on the similarity measures under type 2 intuitionistic fuzzy sets environment. Soft Comput. 2020;24:7361-7381.
[13] Yager R, editor. Pythagorean fuzzy subsets. Proceeding of the Joint IFSA World Congress and NAFIPS Annual Meeting. 2013 June 24-28; Edmonton, Canada. New York: IEEE; 2013.
[14] Yager RR. Pythagorean membership grades in multicriteria decision making. IEEE Trans Fuzzy Syst. 2014;22(4):958-965.
[15] Yager RR, Abbasov AM. Pythagorean membership grades, complex numbers, and decision making. Int J Intell Syst. 2013;28(5):436-452.
[16] Yager R. Properties and applications of Pythagorean fuzzy sets. In: Angelov P, Sotirov S, editors. Imprecision and uncertainty in information representation and processing. Berlin: Springer; 2016. p. 119-136.
[17] Peng X, Yuan H, Yang Y. Pythagorean fuzzy information measures and their applications. Int J Intell Syst. 2017;32(10):991-1029.
[18] Verma R, Merigó J, Sahni M. Pythagorean fuzzy graphs: some results. 2018;ArXiv:1806.06721.doi.org /10.48550/arXiv.1806.06721.
[19] Augustine PA. Distance and similarity measures for Pythagorean fuzzy sets. Granular Computing. 2018;2:1-17.
[20] Wei G, Wei Y. Similarity measures of Pythagorean fuzzy sets based on the cosine function and their applications. Int J Intell. Syst. 2018;33:634-652.
[21] Nguyen X, Nguyen VD, Nguyen VH, Garg H. (2019). Exponential similarity measures for Pythagorean fuzzy sets and their applications to pattern recognition and decision-making process. Complex Intell Syst. 2019;5:217-228.
[22] Augustine PA. Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition. Complex Intell Syst. 2019;5:165-175.
[23] Zhang Q, Hu J, Feng J, Liu A, Li Y. New Similarity Measures of Pythagorean Fuzzy Sets and Their Applications. IEEE Access. 2019;7:138192-138202.
[24] Augustine PA. New similarity measures for Pythagorean fuzzy sets with applications. Int J Fuzzy Comp Modelling. 2020;3(1):1-17.
[25] Hussain Z, Abbas S, Hussain S, Ali Z, Jabeen G. Similarity measures of Pythagorean fuzzy sets with applications to pattern recognition and multi criteria decision making with Pythagorean TOPSIS. J Mech Continua Math Sci. 2021;16(16):64-86.
[26] Agheli B, Firozja MA, Garg H. Similarity measure for Pythagorean fuzzy sets and application on multiple criteria decisions making. J Stat Manag Syst. 2022;25(4):749-769.
[27] Zhang Q, Yao H, Zhang ZH. Some similarity measures of interval-valued intuitionistic fuzzy sets and application to pattern recognition. Appl Mech Mater. 2010;(44-47):3888-3892.
[28] Sharma DK, Tripathi R. 4 Intuitionistic fuzzy trigonometric distance and similarity measure and their properties. In: Nola AD, Cerulli R, editors. Soft Computing, Berlin: De Gruyter; 2020. p. 53-66.
[29] Tian M. A new fuzzy similarity measure based on cotangent function for medical diagnosis. Adv Model Optim. 2013;15(3):151-156.

[30] He Y, Xiao F. A new distance measure of Pythagorean fuzzy sets based on matrix and and its application in medical diagnosis. 2018;ArXiv:2102.01538.doi.org/10.48550/arXiv.2102.01538.
[31] Molodtsov D. Soft set theory - first result. Comput Math Appl. 1999;37:19-31.
[32] Zhang WR. Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multi agent decision analysis. The 1st International Joint Conference of the North American Fuzzy Information Processing Society Biannual Conference. 1994 Dec 18-21; Texas, United States. New York: IEEE; 1994. p. 305 - 309.
[33] Mahmood T. A Novel Approach towards Bipolar Soft Sets and Their Applications. J Mathematics. 2020;5:1-11.
[34] Riaz M, Hashmi MR. Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems. J Intell Fuzzy Syst. 2019;37:5417-5439.
[35] Peng X, Yuan H. Pythagorean fuzzy multi-criteria decision making method based on multiparametric similarity measure. Cognit Comput. 2021;13(2):466-484.
[36] Yager RR. Generalized Orthopair Fuzzy Sets. IEEE Trans Fuzzy Syst. 2017;25(5):1222-1230.
[37] Mahmood T, Ullah K, Khan Q, Jan N. An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. Neural Comput Applic 2019;31:7041-7053.
[38] Ashraf S, Abdullah S, Mahmood T. GRA method based on spherical linguistic fuzzy Choquet integral environment and its application in multi-attribute decision-making problems. Math Sci 2018;12:263-275.
[39] Khan A, Ashraf S, Abdullah S, Qiyas M, Luo J, Khan S. Pythagorean fuzzy dombi aggregation operators and their application in decision support system. Symmetry. 2019;11(3):383.
[40] Kesavan J, Veerakumari KP, Vasanth KR. Complex pythagorean fuzzy einstein aggregation operators in selecting the best breed of Horsegram. Expert Syst Appl. 2022;187:115990.
[41] Verma R, Merigó JM. On generalized similarity measures for Pythagorean fuzzy sets and their applications to multiple attribute decision-making. Int J Intell Syst. 2019;34:2556-2583.
[42] Peng X. New similarity measure and distance measure for Pythagorean fuzzy set. Complex Intell Syst. 2019;5:101-11.
[43] Barukab O, Abdullah S, Ashraf S, Arif M, Khan SA. A new approach to fuzzy TOPSIS method based on entropy measure under spherical fuzzy information. Entropy. 2019;21(12):1231.
[44] Ashraf S, Abdullah S, Abdullah L. Child development influence environmental factors determined using spherical fuzzy distance measures. Mathematics. 2019;7(8):661.
[45] Ashraf S, Abdullah S, Mahmood T. Spherical fuzzy sets and their applications in multi-attribute decision making problems. J Intell Fuzzy Syst. 2019;36:2829-2844.
[46] Jin Y, Ashraf S, Abdullah S. Spherical fuzzy logarithmic aggregation operators based on entropy and their application in decision support systems. Entropy. 2019;21(7):628.
[47] Ashraf S, Abdullah S, Smarandache F, Amin N. Logarithmic Hybrid Aggregation Operators Based on Single Valued Neutrosophic Sets and Their Applications in Decision Support Systems. Symmetry. 2019;11(3):364.
[48] Rafiq M, Ashraf S, Abdullah S, Mahmood T, Muhammad S. The cosine similarity measures of spherical fuzzy sets and their applications in decision making. J Intell Fuzzy Syst. 2019;36(6):6059-6073.
[49] Ashraf S, Abdullah S, Aslam M. Symmetric sum based aggregation operators for spherical fuzzy information: Application in multi-attribute group decision making problem. J Intell Fuzzy Syst. 2020;38(4):5241-5255.
[50] Ashraf S, Abdullah S, Mahmood T. Spherical fuzzy Dombi aggregation operators and their application in group decision making problems. J Ambient Intell Human Comput. 2020;11:2731-2749.
[51] Ashraf S, Abdullah S. Spherical aggregation operators and their application in multi attribute group decision-making. Int J Intell Syst. 2019;34:493-523.
[52] Batool B, Ahmad M, Abdullah S, Ashraf S. Chinram R. Entropy based pythagorean probabilistic hesitant fuzzy decision making technique and its application for fog-haze factor assessment problem. Entropy. 2020:22(3):318.
[53] Ashraf S, Abdullah S. Decision support modeling for agriculture land selection based on sine trigonometric single valued neutrosophic information. IJNS. 2020;9:60-73.
[54] Ashraf S, Abdullah S, Zeng S, Jin H, Ghani F. Fuzzy decision support modeling for hydrogen power plant selection based on single valued neutrosophic sine trigonometric aggregation operators. Symmetry. 2020;12(2):298.