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Two-level capacitated facility location problem under disruption and fortification

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Abstract

A two-level capacitated facility location problem (TCFLP) is a facility location problem that limits capacity and mainly considers the products shipped between two consecutive levels. Nowadays, in some situations, main facilities may be at risk of disruptions that affect their failure, which may lead to a higher cost of backup products or use of backup facilities. This situation can be prevented by fortifying the main facilities with a fixed budget. In this paper, we have proposed an integer nonlinear programming model for TCFLP that the main facilities were addressed to fortify under the risk of disruption. Furthermore, the linearization technique was used to reduce the algorithm's complexity for solving it. Moreover, a numerical example was illustrated, the results of the simulated small problems were tested with the Gurobi optimizer, and the sensitivity analysis was also provided.

Keywords: Disruption, Fortification, Integer nonlinear programming, Linearization technique, Two-level capacitated facility location problem

1. Introduction

A facility location problem (FLP) is one of the optimization problems in supply chain network design related to determining the best location for facilities such as warehouses, plants, and machines. The achievement of this objective is constrained by the requirement that the established facilities must service demands at several points. The objective of the FLP is to minimize total costs, specifically by choosing the facility that can reduce established costs and total distance from the customer to the facility. Certain facilities may or may not have limited capacities for services, and this classifies the problems into uncapacitated and capacitated variants.

An uncapacitated facility location problem (UFLP) focuses on producing and distributing a single commodity over a single time period. It deals with finding facility sites where facilities have no capacity limit and is also known as the "simple" facility location problem. The objective of UFLP is to minimize the sum of the fixed setup costs and variable costs of serving to meet the customers' demands. However, there are more realistic problems with incorporating capacity limitations on the facilities to be established. Each facility limits the number of customers that it can serve. This version of UFLP is called the capacitated facility location problem (CFLP). It deals with finding facility sites under capacity constraints to meet customers' demands. In some situations, each client can be served by more than one facility, and this is called a multi-source problem [1-3]

Although core facility location models such as the uncapacitated and capacitated facility location problems are a long way from approaching realistic problems in strategic supply chain planning, they have been beneficial for building comprehensive models that include supply chain management decisions. In addition, many realistic location problems may take into account the existence of different facilities that play a specific role (e.g., production, warehousing) and a natural material flow (a hierarchy) between them. Each facility with the same type and role is usually denoted by a layer or echelon and is defined as a level in the hierarchy of facilities. The facility location problem has two types of facilities and is called a "two-level facility location problem (TFLP)." TFLP can be classified by the limitation of the facility's capacity: uncapacitated and capacitated variants. If facilities at each level do not have limited capacities for service, it is called a two-level uncapacitated facility location problem (TUFLP). Otherwise, it is called a two-level capacitated facility location problem (TCFLP).

In 1996, Aardal et al. [4] studied TCFLPs that considered an additional facility level between the customers and main facilities. The main facilities were called "depots" and the lower-level facilities were called "satellites." The objective of TCFLPs is to minimize the total cost; thus, we considered depots and satellites that were opened and the assignment of customers to depots and satellites that were established.

Subsequently, in 1997, Tragantalerngsak et al. [5] proposed a mathematical model for TCFLP and considered six heuristics based on Lagrangian relaxation for a solution. They used a subgradient optimization procedure to solve the dual problem and presented numerical results for many test problems. Later, in 2000, Tragantalerngsak et al. [6] further studied TCFLP and proposed a Lagrangian relaxation-based branch and bound algorithm to solve the problem. Furthermore, numerical results were presented for a large suite of test problems of realistic and practical size.

In the classical FLP and TFLP, it is assumed that the facilities are set up and are always available. However, in reality, the facilities may become unavailable due to various reasons such as natural disasters (earthquakes, floods, etc.) and human actions (terrorism, cyber-attacks, change of ownership [7], etc.). These situations are called disruption, and there are two main types that are widely interesting for studying: facility disruption and route disruption. A facility disruption may be caused by natural hazards and accidents that occur during the production in facilities and delivery of products to customers. This results in the establishment of the inability to provide services that the occurrence of time delays and uncertainties to transport produces. When a facility failure emerges, customers may need to be reallocated from their primary facilities to others which necessitates higher transportation costs [8-10].

As illustrated in Figure 1, primary and backup depots are assigned to each customer. Primary depots are the main service providers of products to customers and when they fail, backup depots are used to serve the customers; thus, this assumes that depots lose some of their available capacity when they fail.

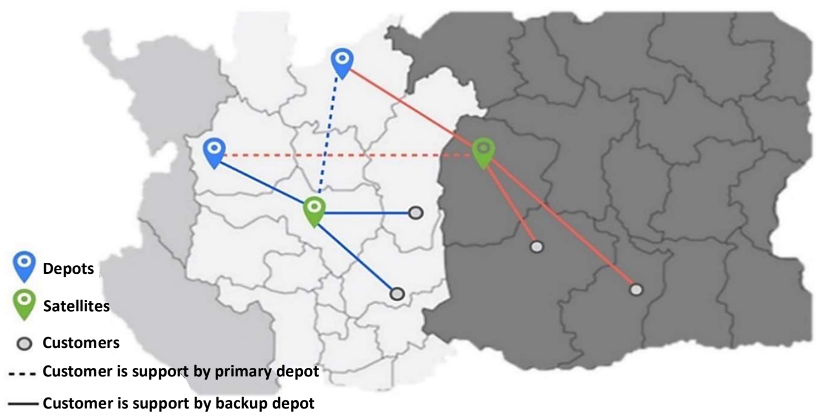

Figure 1 The primary and backup depots are assigned to each customer in disruption scenarios of TCFLP.

The disruption of facilities has been a topic of much literature, and the probability of disruption at a facility in the model formulation of FLPs increases the reliability of the distribution chain. The study of a heuristic algorithm of the supply chain in disruption was initially considered and presented in a model by Drenzner [11] in 1987. The p-median and p-center problems were presented to consider the possibility that one or more of the facilities may become inactive. An unreliable p-median problem was defined by introducing the probability that a facility becomes inactive, while a p-q center problem was defined as when p facilities need to be located but up to q of them may become unavailable simultaneously and computational results were provided.

In 2019, Ramshani et al. [7] studied a single assignment with route disruption uncertainty in TUFLP. Single assignment constraints mentioned by Gendron et al. [12] are considered on TCFLP to ensure that each level of facility is served only by one facility at its previous level. Each customer received the product from only one satellite and depot. They also examined a two-level distribution chain. The flow started from the production unit, went through distribution centers, and ended at the customer level. Two formulations were developed for the problem: a tabu search algorithm and a problem-specific heuristic (route subset selector - RSS) were proposed.

Later, in 2021, Afify et al. [13] proposed an integer nonlinear programming formulation for reliable capacitated facility location (RCFL) problems. They focused on a facility fortification within a finite budget. Moreover, they used the CPLEX solver to present the separation approach to solving the model proposed.

As mentioned above, many studies on RFLP or FLP with the disruption have been popular in succeeding years. Table 1 summarizes the disruption studies in FLP and TFLP.

Table 1 A summary of interruption studies in FLP and TFLP

Year	Authors	Category	Disruption	Model	Approach
1987	Zvi Drenzner [11]	TFLP	Facility	p-median problem	Heuristic
1996	Aardal et al. [4]	TUFLP	-	INLP	LP-relaxation
1997	Tragantalerngsak et al. [5]	TCFLP	-	IP	Lagrangian relaxations, FORTRAN 77
2000	Tragantalerngsak et al. [6]	TCFLP	-	IP	FORTRAN, DEC2000 300AXP
2009	Farahani et al. [9]	FLP	-	BIP	LINGO, CPLEX
2010	Cui et al. [8]	UFLP, disruption	Facility	MIP	C++
2013	Li et al. [14]	FLP, disruption, Fortification	Facility	NLP, INLP	CPLEX, C++
2015	Rohaninejad et al. [3]	CFLP, disruption	Facility	MINLP	Relax & Fix a Heuristics
2018	Santiváñez and Carlo [15]	CFLP, disruption	Facility	MILP	LINGO 17.0
2019	Ramshani et al. [7]	TUFLP, disruption	Route	INLP	Heuristics & Gurobi
2021	Afify et al. [13]	CFLP, disruption, fortification	Facility	INLP	Heuristics & CPLEX
2023	This paper	TCFLP, disruption, fortification	Facility	INLP	Linearization & Gurobi

Although the literature on CFLP with disruption is presently abundant, the disruption in FLP with more than one level has been studied. Some studies consider disruption in a TCFLP due to independent failures of their consisting levels, i.e., satellites (a lower-level facility) and depots. However, the prevention of a facility disruption, namely *fortification*, has not yet been studied in the two-level facility location problem. No previous study has considered fortification of depot in TCFLP when the facility had disruptions due to independent failures of the depot. Therefore, in this paper, we were interested in studying the fortification to protect the facility in a disruption scenario for the two-level capacitated facility location problems. We focused on the higher-level (depot) failure which can be fortified to protect against disruption. Consequently, we developed two reliable models of TCFLP through integer nonlinear programming models to minimize the facility setup and transportation costs while incorporating the failure probability in different levels of facilities.

The remainder of this paper is organized as follows: The definition of the TCFLP under disruption and fortification and a mathematical formulation including the linearization technique for TCFLP are presented in Section 2. The benchmark results are presented in section 3. Finally, the conclusion and future research directions are summarized in Section 4.

2. Materials and method

In this paper, we devised the TCFLP model under disruption by accounting for the failure of a higher-level facility (depot) following which a specified budget would fortify the facility when it fails. Fortification can be achieved by increasing the cost of procurement, installation of protective measure units, purchase and storage of spare inventory, and hiring staff. We intended to determine the optimal locations in which to establish facilities and pair the depot and satellite in this problem assigned to each customer to serve their demands, as well as which depots should be fortified so that the total cost can be minimized.

A model of TCFLP was formulated through an integer nonlinear programming problem (INLP). However, a nonlinear model, in the process of solving a problem, is complicated and cumbersome, and it takes a long time to solve the problem. Therefore, to simplify the problem and reduce the computational time, we were interested in converting a nonlinear model to a linear model and solving the problem with a Gurobi optimizer.

2.1 Problem definitions

For a facility location problem under disruption, a backup facility will be considered to produce the product to serve each customer. Researchers have tried to propose a model for finding a primary depot and a backup depot to minimize total cost; however, fortification is another way to protect the facility in a disruption scenario. If the facility is fortified, the customer does not need support by a backup facility, thereby reducing the setup and transportation costs. However, since each supplier's financial limitations for each depot allows for the fortification of a subset of the established depots within a limited budget, the question of which depots should be chosen for fortification arises.

In this problem, we minimized an objective function to find the optimal solution for establishing a depot and satellite to required demands. The decision determined the appropriate number of facilities needed to minimize the number of establishments and allocation costs to meet all customers' demands. The assumptions to define TCFLP were as follows:

- Single assignment constraints must be considered.
- Consider only the disruption that occurs in the depot.
- Depots and satellites have limited capacity.
- Depot failures are independent.
- Each customer is supported by a primary and backup depot unless the primary depot is fortified, in which case the customer is not supported by a backup depot.
- If a depot fails, it will lose a proportion of its capacity in the range [0,1].
- The fortification budget is fixed.

2.2 Model formulation

In this subsection, we first introduce the notations of the mathematical model for the TCFLP under disruption and fortification shown in Table 2. Next, we presented the model of TCFLP with an objective function to minimize cost, which considered the probability of depot disruption.

Table 2 Notation

Index	Definition
i	Index of depot
j	Index of satellite
k	Index of customer
Decision variable	
x_i	$x_i = 1$, if the depot i is established; otherwise, $x_i = 0$
y_j	$y_j = 1$, if the satellite j is established; otherwise, $y_j = 0$
z_i	$z_i = 1$, if the depot i is fortified; otherwise, $z_i = 0$
w_{ijk}	$w_{ijk} = 1$, if the customer k is supported by primary depot i and satellite j; otherwise, $w_{ijk} = 0$
w'_{ijk}	$w'_{ijk} = 1$, if the customer k is supported by a backup depot i and satellite j; otherwise, $w'_{ijk} = 0$
Parameter	
c_{ijk}	Allocation cost from depot i and satellite j to customer k
F_i	Setup cost of the depot i
M_j	Setup cost of satellite j
p_i	Failure proportion of depot i
cap_i	The capacity of the depot i
cap'_j	The capacity of the satellite j
d_k	The demand of customers k
fc_i	Fortification cost of the depot i
B	Fortification budget

The model of TCFLP with disruption and fortification can be written as follows:

$$\min \quad \sum_{i\in I} F_i x_i + \sum_{j\in J} M_j y_j + \sum_{i\in I} fc_i z_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[c_{ijk} w_{ijk}\left(1 - p_i\left(1 - z_i\right)\right) + c_{ijk} w'_{ijk} \sum_{\substack{r\in I\\ r\neq i}} \sum_{o\in J} w_{rok} p_r \left(1 - z_r\right)\right] \tag{1}$$

s.t.

$$w_{ijk} + w'_{ijk} \leq x_i \quad \forall i,j,k \tag{2}$$

$$w_{ijk} + w'_{ijk} \leq y_i \quad \forall i,j,k \tag{3}$$

$$\sum_{j\in J}\left(w_{ijk} + w'_{ijk}\right) \leq 1 \quad \forall i,k \tag{4}$$

$$\sum_{i\in I}\sum_{j\in J} w_{ijk} = 1 \quad \forall k \tag{5}$$

$$\sum_{i\in I}\sum_{j\in J} w'_{ijk} = 1 \quad \forall k \tag{6}$$

$$\sum_{j\in J}\sum_{k\in K}\left[d_k\left(w_{ijk}\left(1-p_i\left(1-z_i\right)\right)+\sum_{\substack{r\in I\\ r\neq i}}\sum_{o\in J}w'_{ijk}w_{rok}p_r\left(1-z_r\right)\right)\right] \leq \left(1-p_i\left(1-z_i\right)\right)cap_i \quad \forall i \qquad (7)$$

$$\sum_{i\in I}\sum_{k\in K}\left[d_k\left(w_{ijk}\left(1-p_i\left(1-z_i\right)\right)+\sum_{\substack{r\in I\\ r\neq i}}\sum_{o\in J}w'_{ijk}w_{rok}p_r\left(1-z_r\right)\right)\right] \leq cap'_j \quad \forall j \qquad (8)$$

$$z_i \leq x_i \quad \forall i \qquad (9)$$

$$\sum_{i\in I} fc_i z_i \leq B \qquad (10)$$

$$w_{ijk}, w'_{ijk} \in \{0,1\} \quad \forall i,j,k \qquad (11)$$

$$x_i, z_i \in \{0,1\} \quad \forall i \qquad (12)$$

$$y_j \in \{0,1\} \quad \forall j. \qquad (13)$$

The objective function (1) aims to minimize the total cost, i.e., transportation cost, depot establishment cost, and satellite establishment cost, while considering the failure probability and fortification of the depots. In the fourth term, it is the expected transportation cost of the primary depot i, satellite j to customer k where $(1 - p_i)$ is the probability that the primary depot i will be available. Constraints (2) and (3) guarantee that the depots and satellites must be established before sending the product to any customer. Constraint (4) ensures that the primary and backup depot must not be the same facility. In constraints (5) and (6), each customer k must be supplied by only one primary and one backup depot. Constraints (7) and (8) are capacity constraints that ensure that the total demand assigned to the depot will be no more than the available capacity. Constraint (9) ensures the depot must be fortified after being established. Constraint (10) ensures that the total fortification cost will be no more than the budget. Constraints (11), (12), and (13) guarantee the integrity of the binary variables of the model.

Since the proposed model was nonlinear and had many constraints and decision variables, it took a long time to solve the problem. Therefore, we were interested in decreasing the time to solve the complex problem by converting a nonlinear model to a linear model.

2.2.1 Model linearization

Since the objective function of the proposed model was nonlinear, the optimal solution to such a problem required combinatorial optimization that often renders the solution search procedure intractable for large-size supply networks. One of the possible approaches was to linearize the nonlinear term in the objective function consisting of the products of $w_{ijk}z_i$, $w'_{ijk}z_i$, $w_{ijk}w'_{ijk}$ and $w_{ijk}w'_{ijk}z_r$ by introducing a new decision variable in the model as follows:

Let $t_{ij} = w_{ijk}z_i$, $t'_{ij} = w'_{ijk}z_i$, $u_{ijrok} = w_{rok}w'_{ijk}$ and $v_{ijrok} = w_{rok}w'_{ijk}z_r$; the terms are defined as follows Table 3.

Table 3 Decision variables and definition

Decision variable	Definition
t_{ij}	t_{ij} =1, if customer k is supported by primary depot i, satellite j and depot i is fortified $\forall i \in I, j \in J$; otherwise, $t_{ij} = 0$.
t'_{ij}	t'_{ij} =1, if customer k is supported by backup depot i, satellite j and depot i is fortified $\forall i \in I, j \in J$; otherwise, $t'_{ij} = 0$.
u_{ijrok}	u_{ijrok} =1, if customer k is supported by primary depot r, satellite o and backup depot i, satellite j $\forall i, r \in I, j, o \in J, k \in K, r \neq i$; otherwise, $u_{ijrok} = 0$.
v_{ijrok}	v_{ijrok} =1, if customer k is supported by primary depot r, satellite o and backup depot i, satellite j and depot r is fortified $\forall i, r \in I, j, o \in J, k \in K, r \neq i$; otherwise, $v_{ijrok} = 0$.

Since $t_{ijk} = w_{ijk} z_i$, the value of t_{ijk} depends on w_{ijk} and z_i. That is, t_{ijk} is equal to 1 when w_{ijk} and z_i are equal to 1, otherwise $t_{ijk} = 0$. Thus, $t_{ijk} \in \{0,1\}$, and to ensure that the results obtained from the linearized model were consistent with the previous model, the following constraints were added for consideration, as follows:

$$2t_{ijk} \leq w_{ijk}+z_i \qquad \forall i,j,k \tag{14}$$
$$w_{ijk}+z_i \leq t_{ijk}+1 \qquad \forall i,j,k \tag{15}$$

From constraints (14)-(15), if $z_i = 0$ or $w_{ijk} = 0$, then $2t_{ijk} \leq 1$. Since $t_{ijk} \in \{0,1\}$, $t_{ijk} = 0$. If $z_i = 1$ and $w_{ijk} = 1$, then $2t_{ijk} \leq 2$ and $2 \leq t_{ijk} + 1$. Since $t_{ijk} \in \{0,1\}$, $t_{ijk} = 1$, which corresponds to the value of $w_{ijk}\, z_i$.

Similarly, for variables t'_{ijk} , u_{ijrok} and v_{ijrok}, we can add constraints (26)-(33) to the model. Thus, the value of the given variable is equivalent to the previous model. The linearized model of TCFLP under disruption and fortification can be written as follows:

$$\min \quad \sum_{i\in I} F_i x_i + \sum_{j\in J} M_j y_j + \sum_{i\in I} fc_i z_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left[\left(c_{ijk} w_{ijk} - c_{ijk} w_{ijk} p_i + c_{ijk} t_{ijk} p_i\right) + c_{ijk} \sum_{\substack{i\in I\\ r\neq j}}\sum_{o\in J}\left(u_{ijrok} - v_{ijrok}\right) p_r\right] \tag{16}$$

s.t.

$$w_{ijk} + w'_{ijk} \leq x_i \qquad \forall i,j,k \tag{17}$$
$$w_{ijk} + w'_{ijk} \leq y_i \qquad \forall i,j,k \tag{18}$$
$$\sum_{j\in J}\left(w_{ijk} + w'_{ijk}\right) \leq 1 \qquad \forall i,k \tag{19}$$
$$\sum_{i\in I}\sum_{j\in J} w_{ijk} = 1 \qquad \forall k \tag{20}$$
$$\sum_{i\in I}\sum_{j\in J} w'_{ijk} = 1 \qquad \forall k \tag{21}$$
$$\sum_{j\in J}\sum_{k\in K}\left[d_k\left(\left(w_{ijk} - w_{ijk} p_i + t_{ijk} p_i\right) + \sum_{\substack{r\in I\\ r\neq i}}\sum_{o\in J}\left(u_{ijrok} - v_{ijrok}\right) p_r\right)\right] \leq \left(1 - p_i\left(1 - z_i\right)\right) cap_i \qquad \forall i \tag{22}$$
$$\sum_{i\in I}\sum_{k\in K}\left[d_k\left(\left(w_{ijk} - w_{ijk} p_i + t_{ijk} p_i\right) + \sum_{\substack{r\in I\\ r\neq i}}\sum_{o\in J}\left(u_{ijrok} - v_{ijrok}\right) p_r\right)\right] \leq cap'_j \qquad \forall j \tag{23}$$
$$z_i \leq x_i \qquad \forall i \tag{24}$$
$$\sum_{i\in I} fc_i z_i \leq B \tag{25}$$
$$2t_{ijk} \leq w_{ijk} + z_i \qquad \forall i,j,k \tag{26}$$
$$w_{ijk} + z_i \leq t_{ijk} + 1 \qquad \forall i,j,k \tag{27}$$
$$2t'_{ijk} \leq w'_{ijk} + z_i \qquad \forall i,j,k \tag{28}$$
$$w'_{ijk} + z_i \leq t'_{ijk} + 1 \qquad \forall i,j,k \tag{29}$$
$$2u_{ijrok} \leq w_{ijk} + w'_{ijk} \qquad \forall i,j,r,o,k,r\neq i \tag{30}$$
$$w_{ijk} + w'_{ijk} \leq u_{ijrok} + 1 \qquad \forall i,j,r,o,k,r\neq i \tag{31}$$
$$3v_{ijrok} \leq w_{rok} + w'_{ijk} + z_r \qquad \forall i,j,r,o,k,r\neq i \tag{32}$$
$$w_{rok} + w'_{ijk} + z_r \leq v_{ijrok} + 2 \qquad \forall i,j,r,o,k,r\neq i \tag{33}$$
$$u_{ijrok}, v_{ijrok} \in \{0,1\} \qquad \forall i,j,r,o,k,r\neq i \tag{34}$$
$$w_{ijk}, w'_{ijk}, t_{ijk}, t'_{ijk} \in \{0,1\} \qquad \forall i,j,k \tag{35}$$
$$x_i, z_i \in \{0,1\} \qquad \forall i \tag{36}$$
$$y_j \in \{0,1\} \qquad \forall j\,. \tag{37}$$

From the linearized model, if the number of depots, satellites, and customers are m, n, and l, respectively, then, the number of decision variables is $6\,(m \times n \times l) + 2m + n$.

3. Results

In this section, we propose a numerical example and experimental results. The experiments for TCFLP with disruption and fortification were conducted using the locations of depots and satellites from a company in Bangkok. The data were randomly generated in the ranges shown in Table 4. The problems of TCFLP with disruption and fortification were solved with the Gurobi optimizer (python) package in jupyter notebook. The results were obtained using a 64-bit core i5-1137G7 machine with 8GB RAM, running the Windows operating system.

Table 4 Parameters used in experiments.

Parameter	value	Parameter	value
Allocation cost (c_{ijk})	[10, 300]	Failure proportion (p_i)	[0.1, 0.2]
Depot setup cost (F_i)	[2,000, 4,000]	Depot capacity (cap_i)	[70, 90]
Satellite setup cost (M_j)	[2,000, 4,000]	Satellite capacity (cap'_i)	[100, 140]
Fortification cost (fc_i)	[1,000, 3,000]	Customer demand (d_k)	[10, 100]

3.1 Numerical example

To illustrate TCFLP with disruption and fortification, a 20-node instance was solved where the number of depots, satellites, and customers were 5, 5, and 10 respectively. To compare the results between the different fortification budgets, budget $0, $2,000 and $4,000 were tested. Table 5 presents the solution details, i.e., sets of established depots and satellites, sets of fortified depots, budget cost, cost without the fortification budget, objective value (cost), and CPU time.

Table 5 Results of 20 nodes of TCFLP.

Facilities established		Budget	Fortified depot	Cost ($) (Not include fc_i)*	Total cost ($)	CPU time (s)
Depot	Satellite					
1,4,5	2,5	0	-	14,538	14,538	33.02
1,4,5	2,5	2,000	1	14,538	15,752	412.87
1,4	2,5	4,000	1,4	12,326	16,730	684.12

*The total cost does not include a fortified cost (fc_i)

Table 5 shows the optimal solution to the 20-node instance in the disruption scenario (where the depots are disrupted) and the difference in fortification budgets. The established depots in the disruption scenario with a budget of $0 were 1, 4, and 5, and the optimal cost was $14,538. After we increased the budget by $2,000, the established depots were 1, 4, and 5, while depot 1 was selected for fortification, and the optimal cost was $15,752. In this case, we see that the cost that excluded the fortification cost was still not different from the budget of $0. It means that the transportation cost was still similar to the case of a budget of $0; therefore, the fortification depot was not useful. However, when the budget was increased to $4,000, the established depots were only two. The cost excluding the fortification cost decreased to $12,326. Although the total cost was higher than the $0 budget, from a supply chain activity and sustainability point of view, it may have represented daily, weekly, or monthly savings as opposed to one-time savings. Therefore, spending a fixed amount of fortification may be beneficial once to have repeated savings over the long term. Figure 2 shows the established facilities and pairs of depots and satellites assigned to each customer in the fortification budget of $4,000.

3.2 Experimental results

To validate the model, the dataset in the small problems comprised of 15–30 nodes where the number of depots was 5–10, the number of satellites ranged from 5–10, and the number of customers ranged from 10–20. The data were randomly generated in the ranges as shown in Table 3. Budgets were varied by $0, $3,000, and $6,000.

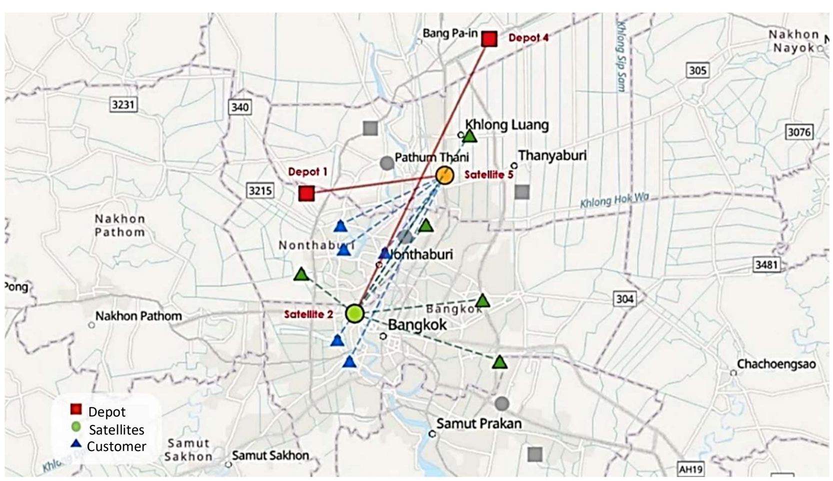

Figure 2 Illustrative solutions in the disruption scenarios (Budget = $4,000).

Table 6 shows the details of the solutions. Each solution includes established depots and satellites, sets of fortified depots, budget cost, cost without the fortification budget, objective value (cost), and CPU time.

Table 6 Solution details of TCFLP

Number of facilities			Customer	Facilities established		Budget	Fortified depot	Cost ($) (Not include ($fc_i$)*	Cost ($)	CPU time (s)
Node	Depot	Satellite		Depot	Satellite					
20	5	5	10	1,2,4,5	1,2,5	0	-	19,652	19,652	28.32
				1,2,4	1,2,5	3,000	4	18,299	20,244	432.49
				1,2,4	1,2,5	6,000	4	18,299	20,244	731.43
25	5	10	10	1,2,3,4	1,3,10	0	-	20,958	20,958	154.29
				1,2,4	1,3,10	3,000	4	19,227	22,911	502.80
				1,2,4	1,3,10	6,000	4	19,227	22,911	782.91
30	10	10	10	1,3,4,8	3,9,10	0	-	21,052	21,052	802.08
				1,3,4,8	3,9,10	3,000	4	21,052	23,145	6,914.78
				1,4,8	3,9,10	6,000	4,8	19,138	24,389	24,467.33

*The total cost does not include a fortified cost (fc_i).

As shown in Table 6, the cost, excluding the fortification budget, decreases when the budget increases. Moreover, the results obtained with the linearization model could be solved in 28.32 to 24,467.33 seconds in small problems.

3.3 Sensitivity analysis

To compare the effect of a failure proportion, we were interested in studying the effects of different ranges of failure proportions p_i between [0.05, 0.1] and [0.2, 0.3]. The solution details for small-scale problems of these situations are shown in Tables 7 and 8 respectively.

Table 7 Solution details of TCFLP when considering range p_i between [0.05, 0.1]

Number of facilities			Customer	Facilities established		Budget	Fortified depot	Cost ($)	CPU time (s)
Node	Depot	Satellite		Depot	Satellite				
20	5	5	10	1,2,4,5	1,2,5	0	-	19,652	26.78
				1,2,4	1,2,5	3,000	4	20,244	413.54
				1,2,4	1,2,5	6,000	4	20,244	651.23
25	5	10	10	1,2,3,4	1,3,10	0	-	20,958	143.12
				1,2,4	1,3,10	3,000	4	22,911	501.28
				1,2,4	1,3,10	6,000	4	22,911	781.25

Table 8 Solution details of TCFLP when considering range p_i between [0.2, 0.3].

Number of facilities			Customer	Facilities established		Budget	Fortified depot	Cost ($)	CPU time (s)
Node	Depot	Satellite		Depot	Satellite				
20	5	5	10	1,2,4,5	1,2,5	0	-	19,652	31.51
				1,2,4	1,2,5	3,000	4	20,244	479.32
				1,2,4	1,2,5	6,000	4	20,244	787.81
25	5	10	10	1,2,3,4	1,3,10	0	-	20,958	159.26
				1,2,4	1,3,10	3,000	4	22,911	522.92
				1,2,4	1,3,10	6,000	4	22,911	892.33

The established depots and the total cost in the range of failure between [0.05, 0.1], [0.1, 0.2], and [0.2, 0.3] have the same result in small-scale problems, as shown in Tables 6, 7, and 8, respectively. Figure 3 compares the linearized model's solution time over the different failure settings range between [0.05, 0.3]. The solution time is slightly increased for some problems, but it is similar on average.

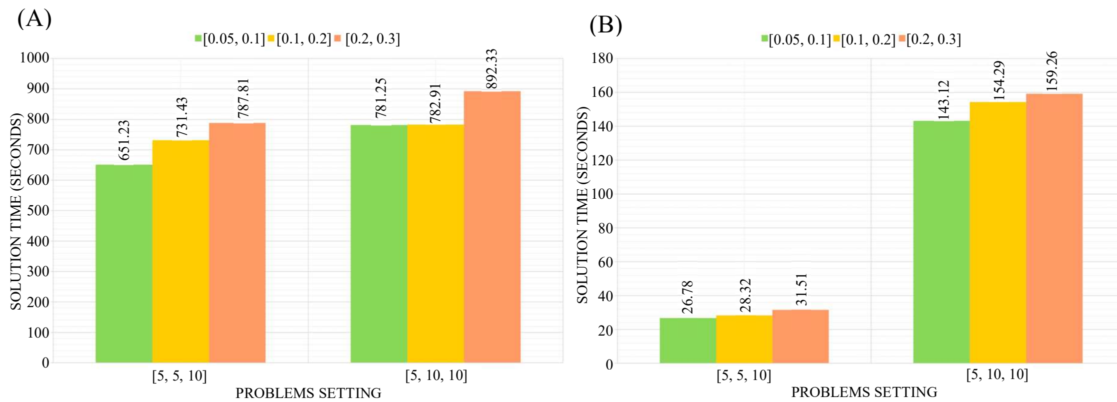

Figure 3 Comparison of the solution time for a linearized model across a range of failure settings [0.05, 0.3], with (A) representing a budget of $6000 and (B) also representing a budget of $0.

4. Conclusions

In this paper, we studied the two-level capacitated facility location problem (TCFLP) by considering the depot's limited capacity and failure probabilities. The fortification budget was considered to ensure the depot's limited capacity was maintained. In this problem, we presented an integer nonlinear programming model of TCFLP with disruption and fortification to minimize the total cost. However, the proposed model was nonlinear. It takes a long time to solve the problem; therefore, the model was linearized to solve the complex problem.

The experiments for TCFLP with disruption and fortification were conducted using the locations of depots and satellites from a company in Bangkok and solved with the Gurobi optimizer (python) to validate the model. Although the total cost, including the fortification cost, was higher than the $0 budget from a supply chain activity and sustainability perspective, it may represent daily, weekly, or monthly savings rather than one-time savings. Therefore, spending a fixed amount of fortification once may be beneficial to have repeated savings over the long term. Our approach could help decision-makers prevent disruption scenarios by choosing fortifications according to the existing budget. For future work, we are interested in studying the technique to solving TCFLP in large-scale problems. Moreover, the consideration of the satisfaction of customers when the facility is disrupted is interesting.

5. Acknowledgments

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