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## **QUANTITY THEORY OF MONEY: STYLIZED FACTS, MODELING, AND EMPIRICAL EVIDENCE**

# QUANTITY THEORY OF MONEY: STYLIZED FACTS, MODELING, AND EMPIRICAL EVIDENCE

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## บทคัดย่อ

บทความทางวิชาการบทความนี้ชี้ให้เห็นถึงวัตถุประสงค์พื้นฐานและความสำคัญของทฤษฎีปริมาณเงินโดยการทบทวนและสังเคราะห์บทบาทของทฤษฎีในเศรษฐศาสตร์มหภาคสมัยใหม่ผ่านข้อเท็จจริง แบบจำลอง และผลการศึกษาเชิงประจักษ์ ข้อเท็จจริงที่ค้นพบในบทความฉบับนี้แสดงให้เห็นถึงความสัมพันธ์ที่แข็งแรงระหว่างระดับราคาและปริมาณเงินเกือบทุกประเทศทั่วโลก และแบบจำลองทางเศรษฐศาสตร์มหภาคเกือบทั้งหมดที่มีพื้นฐานจากเศรษฐศาสตร์จุลภาค ยังให้ผลตามทฤษฎีปริมาณเงิน นอกเหนือนั้นผลการศึกษาเชิงประจักษ์โดยทั่วไปยังแสดงให้เห็นถึงความสัมพันธ์ระยะยาวระหว่างเงินและราคาซึ่งมาจากการทฤษฎีปริมาณเงินโดยไม่ขึ้นอยู่กับนโยบายการเงินที่ใช้ในประเทศ การเป็นเหตุเป็นผลยังเริ่มจากปริมาณเงินเป็นเหตุที่ส่งผลต่อราคาซึ่งสนับสนุนทฤษฎีปริมาณเงิน ดังนั้นทฤษฎีปริมาณเงินก็ยังเป็นจริงอยู่ถึงแม้ว่าเศรษฐกิจจะมีการเปลี่ยนแปลงอย่างรวดเร็ว

## ABSTRACT

This paper shows the primary objective and importance of the Quantity Theory of Money (QTM) by reiterating and synthesizing its role in modern macroeconomics through looking at stylized facts, the macroeconomic model, and empirical evidence. The stylized facts found from this paper show that there is a strong relationship between the price level and monetary aggregates for most countries worldwide. Also, most modern macroeconomic models, which are based on a microeconomic foundation of money, still produce QTM results. In addition, empirical evidence, in general, states that the long term relationship between money and price more or less entails the QTM, regardless of the monetary policy adopted by the country in question. The causality also runs from money to price, which supports the QTM. Therefore, the QTM remains valid even through a rapid change in the economy.

**Keywords:** Quantity theory of money, Money-in-the-utility function, Cash-in-advance, Overlapping generation

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## 1. INTRODUCTION

The demand for money is one of the most important topics in macroeconomics, money and banking and the monetary theory. As mentioned in many text books such as Bain and Howells (2003), theories of money demand mainly range from the quantity theory of money (QTM), liquidity preference theory, Tobin's portfolio model of the demand for money to Friedman's modern quantity theory of money. This paper deals mainly with the QTM, which is based on the classical economics theory.

It can be assumed that the QTM is rooted to David Hume's influential 1752 publication, *"Of Money"*. Chacholiades (1972) mentioned that gold flows tended to produce price-level changes, as explained by Hume, who also seemed to involve the QTM. However, as mentioned by many authors such as Snowdon et al. (1994) and Mishkin (1992), the two most influential versions of the QTM are based on well known economists such as Irving Fisher from Yale University, and Alfred Marshall and Arthur C. Pigou from Cambridge University.

The QTM has been subjected to many criticisms, for example, money demand that derived from the QTM and reverse causation. Therefore, this paper shows how important the QTM is. The prime objective is to reiterate and synthesize the QTM's role in modern macroeconomics by looking at stylized facts, the macroeconomic model, and empirical evidence, which is based on various studies over the past thirty years.

This paper starts by describing the QTM concept. Then, some stylized facts related to the QTM have been based on well known studies such as those of McCandless and Weber (1995), which shed some light on the correlation between the variables concerned. Next, the modern macroeconomic model, which is based on the foundation of microeconomics, shows that it can produce the QTM. Lastly, empirical studies from different techniques are reviewed in order to paint a clear picture of how the QTM performs.

## 2. THE QUANTITY THEORY OF MONEY (QTM)

As mentioned earlier, there are two influential version of the QTM, those being from Irving Fisher, and Alfred Marshall and Arthur C. Pigou. The QTM is presented first from the concept of Fisher, as published in his well known book, *"The Purchasing Power of Money"* in 1911. It is acknowledged widely that the QTM mainly deals with the utilization of money for transaction purposes, which is sometimes called the transaction version QTM.

This version of QTM shows the relationship between the total quantity of money  $M$  and value of transaction in the economy  $P_t T$ . Value of the transaction includes the exchange of newly-produced and second-hand goods, and financial assets.  $P_t$  is the average price of transaction and  $T$  the overall number of transactions. The total quantity of money and value of transaction are linked through the so called *"velocity of money"*  $V_t$  or transaction velocity of money, which measures the rate of monetary turnover. On the other hand, maybe the rate shows the average time (perhaps in one year) of the unit of monetary spending for goods and services. From this concept, the transaction velocity of money is

$$V_t = \frac{P_t T}{M} \quad (1)$$

Bain and Howell (2003) mentioned that the transaction velocity of money is subjected to slow change when the financial system changes. Many economists take the concept of slow changing as a constant velocity of money.

The equation of exchange can be obtained by rewriting the above equation, as follows:

$$MV_t = P_t T \quad (2)$$

The right-hand side of the equation of exchange describes the money used to carry out all transactions in the economy and the left-hand side shows the number of monetary units exchanged.

Bain and Howell (2003) also stated that Fisher (1911) distinguished between transactions related to national income  $Y$  and those related to financial transactions  $F$ . The price of newly-produced and second-hand goods is  $P_y$  and the price of financial assets is  $P_f$ . This leads to another type of exchange equation as follows:

$$MV = P_y Y + P_f F \quad (3)$$

On the left hand side of the above equation, Bain and Howell (2003) stated that the total quantity of money can be disaggregated into the money held for purchasing newly-produced and second-hand goods  $M_y$ , purchasing financial assets  $M_f$ , and saving  $M_s$ . Also, velocity can be disaggregated into the velocity of purchasing newly-produced and second-hand goods  $V_y$ , and money for purchasing financial assets  $V_f$ . Equation (3) can be written as

$$M_y V_y + M_f V_f + M_s = P_y Y + P_f F \quad (4)$$

The money used for purchasing newly-produced and second-hand goods  $M_y V_y$  in a certain time (perhaps one year) plus that used for purchasing financial assets  $M_f V_f$  is equal to the money used for purchasing newly-produced and second-hand goods and financial assets  $MV$ , when the money held for saving  $M_s$  is equal to zero. Also, the total nominal income  $PY$  is equal to the total income from newly-produced and second-hand goods  $P_y Y$  plus the total income from financial assets  $P_f F$ . Therefore, the equation of exchange in equation (4) can be transformed and written as

$$MV = PY \quad (5)$$

where  $M$  is the quantity of money,  $Y$  is the amount of output represented by the real  $GDP$ ,  $P$  is the price of output represented by the  $GDP$  deflator, and consequently,  $PY$  is the nominal  $GDP$ .  $V$  is the so called income velocity of money. This income velocity states the number of times money is circulated in the economy in a given period of time, mainly one year. This type of equation is called income version of the QTM.

However, the above two versions of the QTM have shown only the equation of exchange. There is no analysis on how the demand for money is determined. The QTM was looked at differently in the well known articles; “*The value of Money*” in 1917 and “*Money, Credit and Commerce*” in 1923 by A. C. Pigou and A. Marshall, respectively. These two Cambridge economists explained the QTM by using demand and supply analysis. The change in money supply would definitely lead to the change of equilibrium in the money market. The demand for money under the Cambridge approach is a proportion of nominal income and written in the form of equation as follows:

$$M^d = k \bullet PY \quad (6)$$

where  $M^d$  is the demand for money,  $PY$  is the usual nominal income and  $k$  is a fraction of nominal income.

Since money supply is determined exogenously by the monetary authorities and  $Y$  is predetermined at full employment under the classical concept, the money market equilibrium applies as follows:

$$M = M^d = k \bullet PY \quad (7)$$

The Cambridge approach demanding money also leads to the equation of exchange as follows:

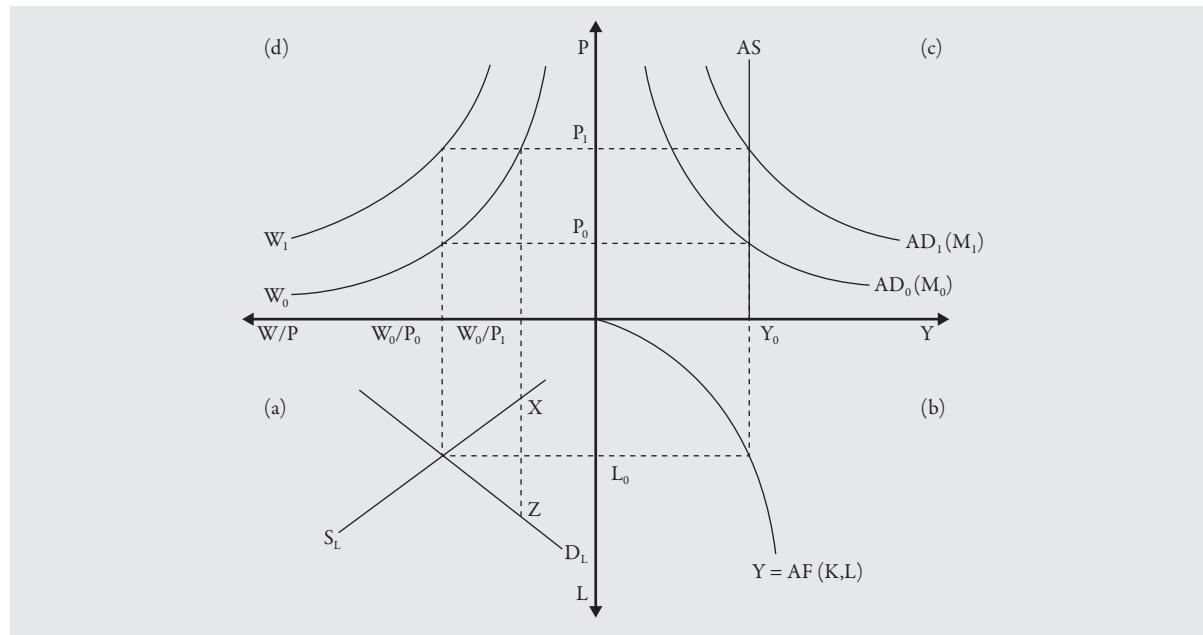
$$M \left( \frac{1}{k} \right) = PY, \quad \text{where } V = \left( \frac{1}{k} \right) \quad (8)$$

The velocity seems to be constant under stylized presentation. Snowdon et al. (1994) stated that the velocity  $V$  is constant because of the institutional factors that determine its changes slowly. However, they also mentioned that under the Cambridge approach,  $k$  could vary in the short term. Apart from the issue of constant monetary velocity, the factor affecting it also is of interest. Bain and Howell (2003) stated that factors affecting  $V$  are the sub-set of those affecting  $k$ . This is because, under the Cambridge approach, the opportunity cost of holding money is included. This implies that people might hold money for purposes other than using it for transactional purposes.

Now, with  $k$  or  $V$  and  $Y$  being constant, the money supply determines the price level. If there is a change in money supply, then the price will be adjusted to restore equilibrium in the money market. To have a clear picture of how the price level is determined in the classical concept, let us look at the diagram below, as introduced by Snowdon et al. (1994).

Figure 1 Determination of the price level in the classical model.

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This diagram has four quadrants; quadrant (a) and (b) show the labor market and goods market, respectively; quadrant (c) shows the **AD** and **AS**; and the last one (d) shows the relationship between real wage and price level. As widely acknowledged, under the classical school of thought, the labor and goods market are competitive. Clearing of the labor market yields the equilibrium of real wage at  $\frac{W_0}{P_0}$  and full employment at  $L_0$  in quadrant (a). Consequently, production is employed at full output  $Y_0$ , as in quadrant (b). Under the classical concept, the aggregate supply curve (AS) is inelastic, and the aggregate demand curve (AD) is based on the QTM. From the equation of exchange [equation (5)], when the money supply and velocity of money is constant, the price level has a negative relationship with the real output. With the inelastic AS curve and negatively sloped  $AD_0$  curve, the equilibrium price level is at  $P_0$  and output fully employed at  $Y_0$ .

Now, if the money supply rises from  $M_0$  to  $M_1$ , it would lead to a shift of the aggregate demand curve to  $AD_1$ . Consequently, the price level rises from  $P_0$  to  $P_1$ , whereas the output stays the same at  $Y_0$ . This increase in the price level has led to change in the real wage from  $\frac{W_0}{P_0}$  to  $\frac{W_0}{P_1}$  and it also creates excess demand for labor of  $XZ$ . To clear the labor market, the nominal wage curve has to shift from  $W_0$  to  $W_1$ . It can be seen from the increase at the money supply level that the price level increases, but there is no change at all in real variables such as real output and real wage. This implies the concept of money neutrality.

### 3. STYLIZED FACTS

After having a clear theoretical concept of the QTM in this section, stylized facts related to money supply and the price level are presented. Although there are numerous empirical studies concerning this issue, they can be divided into cross sectional, long time series, and very old historical data. However, this paper presents one group. Lucas (1980) presented empirical results based on the QTM. The U.S. quarterly time series data, which covered from 1953 to 1977, was used in this study as the money supply  $M_1$ , consumer price level  $P_t$ , and ninety-day Treasury bill rate  $r_t$ . The result shows that there is a strong positive correlation between monetary growth and inflation. In other words, his empirical illustration seems to state that a change in the quantity of money leads to an equal change in price level.

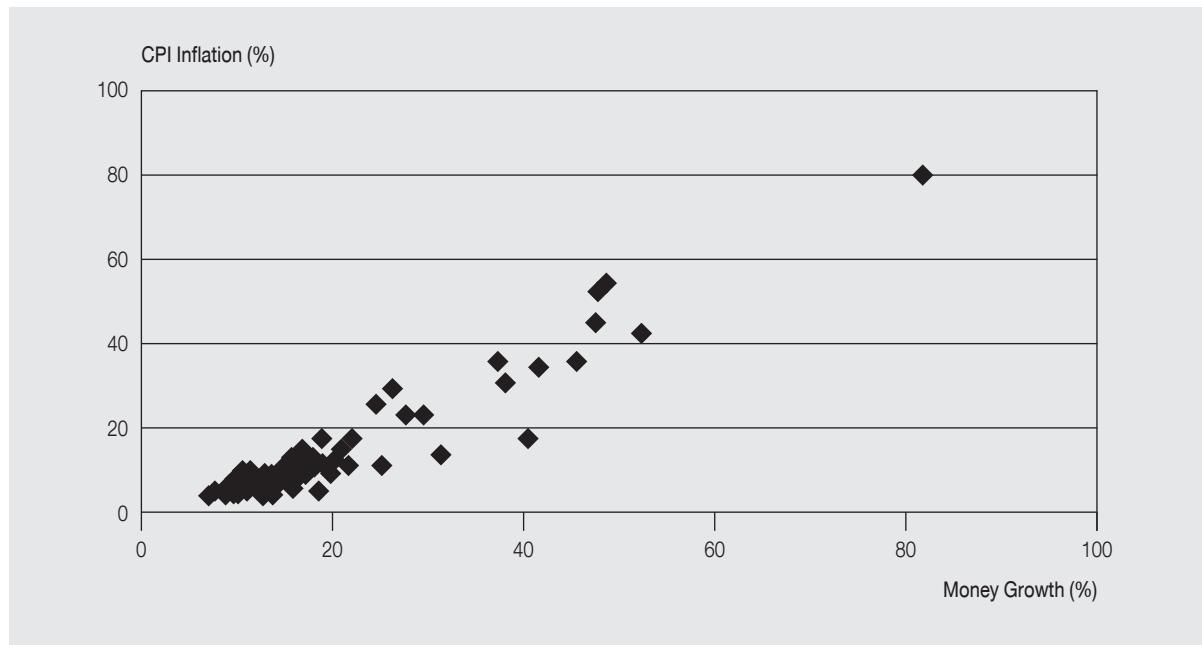
Dwyer and Hafer (1988) studied the relationships between money, income and inflation in both the short and long term. Their studies employed nominal and real income, price level, and the money stock of time series data of 62 countries from 1979 to 1984. The short term evidence from one particular year (1979 or 1984) showed that monetary growth and inflation did not state a one-to-one relationship even though the relationship between the two was positive. However, a one-to-one relationship existed between these two variables, as shown from long term evidence. Furthermore, the estimated regression between monetary growth, as an independent variable, and inflation, as a dependent variable showed that the estimated coefficient for the former is 1.031. In addition, countries with a higher monetary growth seem to have higher inflation on average.

Rolnick and Weber (1995) carried out studies on relationships between inflation, money, and output under alternative monetary standards. This study was different from the previous two because it focused on both commodity and fiat standard. It covered each of 15 countries over at least 80 years. The results showed that correlation between aggregate monetary growth and inflation under fiat standards is higher than that under commodity standards. Correlation under the fiat standard is .99 or close to unity.

McCandless and Weber (1995) studied long term monetary facts among money supply growth, inflation, and real output growth. All definitions of money supply ( $M_0$ ,  $M_1$ , and  $M_2$ ) were applied for money supply growth. The study covered 110 countries from 1960 to 1990. The result showed that the correlation coefficient for all definitions of monetary growth and inflation is approximately .944. This correlation is very high and close to unity. Due to the availability of data, a chart of correlation between monetary growth and inflation can be replicated and is shown as follows:

Figure 2 Correlation between monetary growth (M2) and inflation in 110 countries from 1960 to 1990.

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Grauwe and Polan (2005) utilized the International Financial Statistics of the International Monetary Fund (IMF), which covered most countries between 1969 and 1999, to study the relationship between monetary growth (M1 and M2) and consumer price index (CPI). The correlation between M1 and inflation, and M2 and inflation was reported to be 0.877 and 0.89, respectively. Although the correlations of monetary growth and inflation in this paper were less than in others, they covered more countries, and this paper still confirms that there is almost a one-to-one long term relationship between monetary growth and inflation.

Benati (2009) also carried out a study on the long term evidence of monetary growth and inflation. It employed cross spectral analysis that covered countries in the Euro area, United States, United Kingdom, France, Switzerland, Italy, Japan, Canada, The Netherlands, Australia, West Germany, Norway and Sweden. Quarterly data were used in this study with various ranges. However, the result was not synonymous with coherence between monetary growth and inflation being close to one.

The overall results confirm that the correlation between monetary growth, with different definitions, and inflation under the fiat standards is strongly positive and the magnitude is close to one. Not only the correlation, but also other types of studies such as regression estimation and cross spectral analysis confirm the same results. Therefore, it can be confirmed that a strong long term relationship exists between monetary growth and inflation. In other words, it can be proclaimed, as mentioned in Vogel (1974), that an increase in the growth rate of money supply causes a proportionate increase in the rate of inflation, since money supply is determined exogenously by the central bank. The studies in this part are summarized in Table 1.

**Table 1** Studies of the relationship between money supply growth and inflation.

Author and Year Published	Countries	Time Period	Findings
Lucas (1980)	U.S.	1953-1977	Strong positive correlation and coefficient close to one
Dwyer and Hafer (1988)	62 Countries	1979-1984	Strong positive correlation and estimated coefficient close to one
Rolnick and Weber (1995)	9 Countries	Various	Strong positive correlation and coefficient close to one
McCandless and Weber (1995)	110 Countries	1960-1990	Strong positive correlation and coefficient close to one
De Grauwe and Polan (2005)	About 160 countries	1969-1999	Strong positive correlation and coefficient close to one
Benati (2009)	Euro area and 12 countries	Various	Strong positive correlation and coefficient close to one

## 4. MACROECONOMIC MODELS

Since concluding and confirming the relationship between money supply and price level from the data, modern macroeconomic models, based on microeconomic foundations, have been introduced in order to shed some light on how the QTM still plays an important role in modern modeling.

As widely acknowledged, currently there are two main macroeconomic workhorse models: the infinite life model and overlapping model, which are presented in this paper. According to Croushore (2007), modern macroeconomic models with microeconomic foundations of money include (1) money-in-the-utility function model; (2) transaction cost models; (3) search model; (4) cash-in-advance model; (5) shopping-time model; and (6) overlapping-generation model. In this paper, all models except those for transaction cost, shopping time, and search are presented to show the QTM's role in modern macroeconomic models.

### 4.1 THE MONEY-IN-THE-UTILITY (MIU) FUNCTION MODEL

The MIU model is based on the well-known model of Sidrauski (1967a) and has been used extensively in many theoretical and empirical papers. This model can be used also to show its ability to yield QTM results. It is set in the form of a decentralized economy, and adapted from the model shown in *"Lecture in Macroeconomics"* by Blanchard and Fisher (1989).

#### The decentralized economy

#### The problem of households

The economy is composed of many infinite lives and, for simplicity, the population within it is assumed to be constant or have a growth of zero. Each household solves the following maximization problem as follows:

$$\max U_t = \int_0^\infty u(c_t, m_t) e^{-\theta t} dt, \quad (9)$$

where  $c_t$  is consumption per capita,  $m_t$  is real balance per capita, and  $\theta$  is the discount factor. The utility is assumed to have usual properties that are positive, and diminishing marginal utilities with respect to consumption and real balance, or  $u_c, u_m > 0, u_{cc}, u_{mm} < 0$ .

The budget constraint of the household can be explained simply by using the concept of source and use of funds. The total household source of funds is obtained by income from labor  $w$  (labor is normalized to 1), capital income  $rk$ , and government transfer  $x$ . The income of the household can be used for consumption  $c$ , and held in the form of capital,  $\frac{dk}{dt}$ , and money,  $\frac{dm}{dt} + \pi m$ . The household budget constraint can be written in the form of

$$c + \frac{dk}{dt} + \frac{dm}{dt} + \pi m = w + rk + x \quad (10)$$

It can be seen that the total household wealth  $a$ , is composed of capital and real money balance and, consequently, the constraint can be transformed as

$$\frac{da}{dt} = [ra + w + x] - [c + (\pi + r)m] \quad (11)$$

This equation implies that the change rate of total wealth per capita is the difference between income and consumption. Consumption is the combination of consumption  $c$ , and the foregone interest of holding money  $(\pi + r)m$ .

Lump-sum transfers are equal to the seigniorage from money issued in order that

$$x = \frac{dM/dt}{PN} = \left( \frac{dM}{M} \right) \left( \frac{M}{PN} \right) = \sigma m, \quad (12)$$

where  $\sigma$  is the rate of monetary growth.

In addition, the usual no-Ponzi-game condition is imposed as follows:

$$\lim_{t \rightarrow \infty} a_t \exp -[\int_0^t (r_v - n) dv] = 0 \quad (13)$$

The current-value Hamiltonian of this problem is set to solve this problem in households as follows:

$$\hat{H} = u(c, m) + \lambda [ra + w + x - c - (\pi + r)m]$$

where  $\lambda$  is the current value shadow price or costate variable.

The first-order conditions for maximization are as follows:

$$u_c(c, m) = \lambda, \quad (14)$$

$$u_m(c, m) = \lambda(\pi + r), \quad (15)$$

$$\frac{d\lambda}{dt} - \theta\lambda = -r\lambda, \quad (16) \quad / 17$$

The transversality condition of this problem is as follows:

$$\lim_{t \rightarrow \infty} a_t \lambda_t e^{-\theta t} = 0 \quad (17)$$

The first first-order condition states that the marginal utility of consumption is equal to the current value of shadow price, and combining the first two first-order conditions, equation (14) and (15), implies that the marginal rate of transformation between consumption and real balance is equal to the nominal interest rate or real interest rate plus inflation, as follows:

$$\frac{u_m}{u_c} = (\pi + r) \quad (18)$$

Maximization problem of a firms

The objective of a firm is to maximize profit and, under competitive equilibrium, set its maximization problem in static form as follows:

$$\max \pi = F(K, N) - rK - wN$$

The first-order conditions for maximization are

$$r = f'(k) \quad (19)$$

$$w = f(k) - kf'(k) \quad (20)$$

After obtaining all first order conditions from households and firms, the steady state of the model is presented for analyzing the model's properties by

$$\frac{da}{dt} = \frac{dm}{dt} = \frac{d\lambda}{dt} = 0 \quad (21)$$

As mentioned earlier in equation (12), lump-sum transfers are equal to the seigniorage,

$$x = \frac{X}{N} = \frac{dM/dt}{PN} = \frac{dM}{dt} + m\pi \quad (22)$$

If the above equation is divided by  $m$ , and taking into account that transfer payment per capita  $x$  is equal to  $\sigma m$ , the equation can be written as follows:

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$$\frac{\sigma m}{m} = \frac{dm/dt}{m} + \pi \quad (23)$$

Consequently, in steady state ( $dm/dt = 0$ ), the monetary growth is equal to inflation

$$\sigma = \pi \quad (24)$$

In combining equation (16) and (19), the real rate of return is equal to the discount rate in steady state

$$r = f'(k^*) = \theta \quad (25)$$

The steady state of consumption can be obtained by combining equation (10), (19), and (20) to (25), as follows:

$$c^* = f(k^*) - nk^* \quad (26)$$

From the steady state of consumption and real rate of return, it can be seen that these two variables are not affected by monetary growth. It shows that this model has a money neutrality property, which is confirmed also by this discrete and continuous version of the MIU. As mentioned by Walsh (1998), the rate of inflation is determined by the nominal monetary growth rate. Walsh stated further that the Sidrauski MIU model displays both neutrality and superneutrality

#### 4.2 THE CASH-IN-ADVANCE (CIA) MODEL

The CIA model was pioneered by Lucas (1980, 1982) and has been used widely in modeling. It assumes that money is held in the period prior to financing current consumption. In this model, households comprise a consumer and worker. The worker goes to work and gets paid at the end of the day. Meanwhile, the consumer buys goods and therefore must have enough money to finance consumption.

In addition, the model in this paper is in deterministic form and is set to decentralize an economy. In addition, this model ignores the labor and leisure choice, or its labor is elasticized and normalized as one.

#### Household maximization problem

The representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \quad (27)$$

where  $0 < \beta < 1$ ,  $C_t$  is consumption. The utility is assumed to be strictly increasing, concave and twice differentiable. It is seen clearly that consumers in this model care only for consumption and do not derive utility at all from money.

In the CIA model, the timing of transaction is very important. The consumer enters the period with  $M_t$  amount of currency, and the money is used to finance consumption expenditure  $P_t C_t$ , where  $P_t$  is the price level. This is the cash-in-advance constraint that states the nominal demand for money as follows:

$$M_t = P_t C_t \quad (28)$$

where the money supply  $M_t$  is determined exogenously.

The consumer also faces budget constraint, as their sources of funds come from income and asset holdings. Incomes come from wages  $W_t$  and profit  $\Pi_t$ . Consumers start their life by holding assets that earn them interest on bonds  $(1 + i_t)$  and return on lending capital  $(1 + r_t) P_t K_t$ , where  $i_t$  is the nominal interest rate paid by bonds, and  $r_t$  is the rental rate on capital  $K_t$ . Consumers also hold money  $M_t$  and receive government transfer  $X_t$ , and they use these funds for expense on consumption  $P_t C_t$ , acquiring new capital  $P_t K_{t+1}$ , holding new bonds  $B_{t+1}$ , and money  $M_{t+1}$ . The consumer's budget constraint can be written as follows:

$$P_t C_t + M_{t+1} + B_{t+1} + P_t K_{t+1} = W_t + \Pi_t + (1 + i_t) B_t + (1 + r_t) P_t K_t + X_t \quad (29)$$

Let  $\lambda_t$  be the multiplier associated with the budget constraint and  $\mu_t$  the multiplier associated with the cash-in-advance constraint. The first order conditions of this model are obtained by differentiating the Lagrangian with respect to  $C_t$ ,  $M_{t+1}$ ,  $B_{t+1}$ , and  $K_{t+1}$ , as follows:

$$C_t : U'(C_t) = (\lambda_t + \mu_t) P_t \quad (30)$$

$$M_{t+1} : \lambda_t = \beta (\lambda_{t+1} + \mu_{t+1}) \quad (31)$$

$$B_{t+1} : \lambda_t = \beta (1 + i_{t+1}) \lambda_{t+1} \quad (32)$$

$$K_{t+1} : \lambda_t P_t = \beta (1 + r_{t+1}) \lambda_{t+1} P_{t+1} \quad (33)$$

Combining equation (32) and (33), the following equation is obtained:

$$(1 + i_{t+1}) = (1 + r_{t+1}) \frac{P_{t+1}}{P_t} = (1 + r_{t+1}) (1 + \pi_{t+1}) \quad (35)$$

By forwarding equation (30) by one period and substituting it, equation (31) becomes

$$\lambda_t = \frac{\beta U'(C_{t+1})}{P_{t+1}} \quad (36)$$

This equation can be written in the previous period as follows:

$$\lambda_{t+1} = \frac{\beta U'(C_{t+2})}{P_{t+2}} \quad (37)$$

By substituting equation (36) and (37) in equation (32), the following equation can be written as

$$\frac{U'(C_{t+1})}{P_{t+1}} = \beta(1 + i_{t+1}) \frac{U'(C_{t+2})}{P_{t+2}} \quad (38)$$

In dividing equation (38) by  $(1 + i_{t+1})$  and multiplying the right hand side of it by  $\frac{(1 + i_{t+2})}{(1 + i_{t+2})}$ , as well as using equation (35), equation (38) becomes

$$\frac{U'(C_{t+1})}{(1 + i_{t+1})} = \beta(1 + r_{t+2}) \frac{U'(C_{t+2})}{(1 + i_{t+2})} \quad (39)$$

Providing the nominal interest rate is assumed to be constant –  $(1 + i_{t+1}) = (1 + i_{t+2})$ , then equation (39) becomes the standard Euler equation of

$$U'(C_{t+1}) = \beta(1 + r_{t+2}) U'(C_{t+2}) \quad (40)$$

which shows that the marginal utility of this period is equal to the discounted marginal utility of the next period, multiplied by real interest rate.

Next, the money demand equation can be looked at by combining the first order conditions of money and bonds, and the following equation is obtained as  $\mu_{t+1} = i_{t+1} \lambda_{t+1}$ . Providing the nominal interest rate is positive, the real money demand is

$$\frac{M_t}{P_t} = C_t \quad (41)$$

This real money demand equation clearly shows that consumption elasticity is constant, and this equation is purely the QTM.

Maximization problem of a representative firm

Now, let us turn to the representative firm, which maximizes the following profit:

$$\Pi = P_t F(K_t, N_t) + \delta K_t - W_t N_t - R_t K_t \quad (42)$$

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The first order conditions of the firm are

$$P_t F_N(K_t, N_t) = W_t \quad (43)$$

$$F_K(K_t, N_t) = \frac{R_t}{P_t} = r_t \quad (44)$$

To close the model, the money and bond market are presented in orderly fashion. Money is introduced into the economy through government lump sum transfer  $X_t$ . The money market equilibrium is

$$M_{t+1} = M_t + X_t \quad (45)$$

In this model, bonds are issued by consumers and since all consumers are identical, the equilibrium in the bond market is

$$B_{t+1} = B_t = 0 \quad (46)$$

or outstanding bonds are zero.

When substituting equation (42) - (46) in the consumer's budget constraint, the following equation is obtained:

$$P_t C_t + P_t K_{t+1} = P_t F_N(K_t, 1) + (1 + F_K(K_t, 1) - \delta) P_t K_t \quad (47)$$

Then, when dividing equation (47) by  $P_t$  and using the standard Euler theorem, the national income identity is obtained as the non monetary economy as follows:

$$K_{t+1} = F(K_t, 1) + (1 - \delta) K_t - C_t \quad (48)$$

Steady state and dynamic

Equation (39) shows the steady state of real rate return on capital or  $\frac{1}{\beta} = (1+r)$ . The steady state of capital and consumption are determined by equation (44) and (48), respectively. Furthermore, if the functional form of the utility function is set equal to  $U(C) = \log(C_t)$ , and the real money demand is  $\frac{M_t}{P_t} = C_t$ , then the first order condition of consumption can be written as

$$\frac{P_{t+1}/M_{t+1}}{(1+r)(P_{t+1}/P_t)} = \beta (1+r) \frac{P_{t+2}/M_{t+2}}{(1+r)(P_{t+2}/P_{t+1})} \text{ or } \frac{P_{t+1}}{P_t} = \frac{M_{t+2}}{M_{t+1}} \quad (49)$$

In steady state, the growth rate of money supply is equal to the inflation rate. Therefore, it can be seen clearly that real variables have no effect on inflation, and consequently, the model possessing money neutrality property is applied. Furthermore, this also confirms that the QTM holds in this basic CIA model.

#### 4.3 THE OVERLAPPING GENERATION (OLG) MODEL

The OLG model based on Diamond (1965) is the most well known, but in this case of money, it is based on the model of *“Modeling Monetary Economies”*, written by Scott and Freeman (2001), in which fiat money or money that has no intrinsic value, and cannot be used for consumption or production, is introduced. As mentioned by these authors, fiat money allows the possibility of trading. In other words, having fiat money in the model enables agents to use it in exchange for something they want to consume. Fiat money costs little to produce and store.

##### The environment

OLG assumes that households or individuals live for two periods. The first and second period (1 and 0) of their life is called young and old, respectively. The economy starts in period 1 when  $N_t$  individuals are born, thus  $N_t$  represents young individuals who are born in period 1 and  $N_{t-1}$  old individuals who still exist in period 0. In this economy, there is only one consignment of goods and it cannot be stored from one period into the next. When individuals are young, they are endowed with consumer goods denoted as  $y$ , but when they are old, they receive no endowment. In this model, the population is assumed to be constant throughout the period.

Now, let us look at the preferences of individuals. Each individual derives his or her utility from the consumption of goods throughout his or her lifetime, or in the first and second period. The consumption for a generation in period  $t$  is  $(c_{1t}, c_{2t+1})$ , where  $c_{1t}$  is the consumption of generation  $t$  in the first period and  $c_{2t+1}$  is that of the same generation in the second one. The utility function of this agent is as follows:

$$u(c_{1t}, c_{2t+1}) \quad (50)$$

This utility function has the usual properties; strict monotonicity and strict concavity.

After obtaining the preference of individuals, their budget constraint is explained. As mentioned earlier, the individual is endowed in the first period with  $y$  consumer goods, which can either be consumed or sold for money. Most likely, the individual will consume some of these goods and sell the rest. The amount of money that an individual obtains from selling goods is  $m_t$ , and the unit value of money (or dollars) is  $v_t$  for goods consumed. Therefore, the total value of the individual's holding money is  $v_t m_t$ . Bearing this in mind, the individual's budget constraint in the first period of life is

$$c_{1,t} + v_t m_t \leq y \quad (51)$$

It can be seen in the above equation that its left hand side is for use of funds and the right hand side for sources of funds.

In the second period of life, the individual receives no endowment and must therefore finance his or her consumption by holding money from the first period. In this way, money serves as a medium of exchange in this model. The value of money in the first and second period is not the same. In the second period, the value of money is  $v_{t+1}$  and, therefore, the value of the individual's holding money is  $v_{t+1} m_t$ . The budget constraint in the second period is

$$c_{2,t+1} \leq v_{t+1} m_t \quad (52)$$

However, constraint in the second period can be written as follows:

$$m_t \geq \frac{c_{2,t+1}}{v_{t+1}} \quad (53)$$

By combining constraints in the first and second period, the lifetime budget constraint of the individual is

$$c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq y \quad (54)$$

The term,  $\frac{v_t}{v_{t+1}}$ , is the inverse of return on fiat money. This term also implies inflation.

In this simple model, endowments, population, and preferences of all individuals are the same and, consequently, the value of money is constant over time. When the value of money is constant across generations, individuals choose the same consumption bundle or  $(c_1, c_2)$ . The equilibrium in this case is called stationary equilibrium, since the household behavior is constant. However, it does not mean that  $c_1$  is equal to  $c_2$ , but implies that  $c_1$  and  $c_2$  are identical across individuals. Furthermore, this model expects perfect foresight or that individuals make no forecast errors. This implies that individuals born in period  $t$  can make a perfect prediction of the value of money in period  $t+1$ .

To decide the value of money, demand and supply must be looked at. For example, when looking at demand for fiat money, it is equal to the amount of goods sold by individuals in the first period, so as they can obtain holding money for the second period. The demand for fiat money is equal to

$$N_t (y - c_{1,t}) \quad (55)$$

The supply of money in this model is fixed at the level  $M_t$ . Therefore, the value of money supply is  $v_t M_t$ . By setting the demand for money equal to supply, the following equation is obtained:

$$v_t M_t = N_t (y - c_{1,t}) \quad (56)$$

Consequently, the value of money  $v_t$  is

$$v_t = \frac{N_t (y - c_{1,t})}{M_t} \quad (57)$$

This equation shows that the ratio of real money demand  $N_t (y - c_{1,t})$  and the money available in the economy determines the value of money. The value of money at time  $t+1$ , which can be obtained by using the same logic, is

$$v_{t+1} = \frac{N_{t+1} (y - c_{1,t+1})}{M_{t+1}} \quad (58)$$

By combining equation (57) and (58), the real rate of return for money is

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1} (y - c_{1,t+1})}{M_{t+1}}}{\frac{N_t (y - c_{1,t})}{M_t}} \quad (59)$$

This equation can be simplified by using the stationary equilibrium as follows:

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1} (y - c_1)}{M_{t+1}}}{\frac{N_t (y - c_1)}{M_t}} = \frac{\frac{N_{t+1}}{M_{t+1}}}{\frac{N_t}{M_t}} \quad (60)$$

Also, since the money supply and population in this model are assumed to be constant, the real rate of return for money is

$$\frac{v_{t+1}}{v_t} = 1$$

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Now, let us look at the maximization problem of this model by using the constraint from equation (54), which is

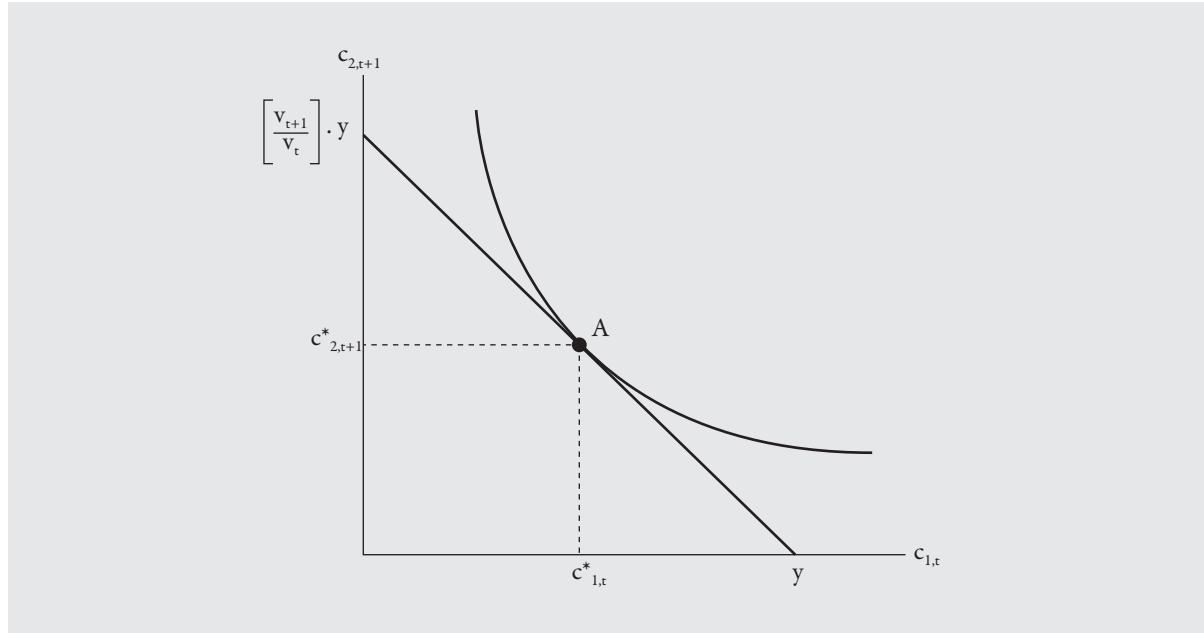
$$\text{Max}_{c_1, c_2}^u (c_1, c_2) \quad (62)$$

subject to

$$c_1 + \frac{v_t}{v_{t+1}} c_2 = y$$

The utility maximization of the individual can be presented by using the following diagram:

Figure 3 The Choice of Consumption with Fiat Money.



The above diagram clearly shows the marginal rate of transformation (MRS) that must equal the real rate of return on money ration or

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$$MRS = \frac{\frac{\partial u}{\partial c_1}}{\frac{\partial u}{\partial c_2}} = \frac{v_{t+1}}{v_t} = 1 \quad (63)$$

It can be recalled that the price level of goods in the unit of money is  $p_t = \frac{1}{v_t}$  and, from the above equation, it is clear that the price is constant over time. It is acknowledged widely that the simple form of the QTM states the price level as proportional to the amount of money. To see whether the QTM is held in this framework or not, let us look again at the value of money equation

$$v_t = \frac{N_t(y - c_{1t})}{M_t} \quad (64)$$

Since consumption is identical across generations and the price level is equal to the inverse of one unit value of money, the equation can be written as

$$p_t = \frac{M_t}{N_t(y - c_1)} \quad (65)$$

This equation clearly states that the price level is proportional to the stock of fiat money. An increase by one unit of money stock will lead to an increase of the price level by  $N_t(y - c_1)$ . Therefore, if the supply of money stock is doubled, then the price level will be doubled as well. Consequently, the QTM holds in the basic overlapping generation model.

Furthermore, the supply of fiat money has no effect on real value of consumption or money demand  $(y - c_1)$ . This can be confirmed by looking at the previous diagram, in which an individual bases his or her choices of consumption and financial balance on the real return of money  $\frac{v_{t+1}}{v_t}$ . The rate of return on money is equal to one, and is not affected by the money supply  $M_t$ . Therefore, it can be proclaimed that the monetary equilibrium of this model possesses money neutrality property.

From the MIU to OLG model, it can be seen clearly that most of the modern macroeconomic models, based on the micro foundation of money, can produce or show the QTM result. This indicates the role of the QTM in the modern macroeconomic theory.

However, as mentioned by Mishkin (2010), macroeconomists have rethought on details of the basic framework model, due to the recent financial crisis. They recognize that the sophisticated financial sector must be included into the New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model. For example, Christiano and Motto (2010) introduced financial markets in a standard monetary DSGE model.

## 5. ESTIMATION EVIDENCE

As stated in section 2, the QTM starts with the equation of exchange or equation (5), which can be transformed into growth form and express inflation as

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$$p = m - y + v \quad (66)$$

where  $p$  is the growth rate of price level,  $m$  is the growth rate of money supply,  $y$  is the growth rate of output and  $v$  the velocity. In general, if one would like to test the QTM, the following equation can be estimated:

$$p_t = \beta_0 + \beta_1 m_t - \beta_2 y_t + \beta_3 v_t + \mu_t \quad (67)$$

where  $\beta_1 = 1$ ,  $\beta_2 < 0$ , and  $\beta_3 = 0$  are predicted. However, many studies that test the QTM implicitly, and estimate the money demand equation, utilize the following equation:

$$m_t = \beta_0 + \beta_1 p_t - \beta_2 y_t + \beta_3 v_t + \mu_t \quad (68)$$

Now, let us look at the empirical estimations tested during the past thirty years of the QTM. To have a clear picture of empirical estimations, it is divided into two groups. The first group is concerned with the estimation of each individual country and the second is based on estimating a number of them. Estimations start in the first group with Argentina, Bangladesh, Brazil, Chile, India, Indonesia, Iran, People's Republic of China, Sudan, Turkey, and United States of America. The estimation of these countries mainly focuses on cointegration and causality between money supply or growth and price level or inflation. The cointegration method is based mainly on the Engle and Granger and Johansen method. Only one paper has used the Autoregressive Distributed Lag (ARDL) model for estimations.

Most empirical estimations are carried out by using US data. Rousseau (2007) employed annual historical data from 1790 to 1850 in stating that long term behavior between money and price existed. Other studies by Mehra (1989), Bagliano and Morana (2004) and Emerson (2006), employed quarterly data that ranged from 1952 to 2006. There are also long term relationships between money supply and price level or monetary growth and inflation. Furthermore, the estimated coefficient is very close to one, therefore, it can be proclaimed that the QTM holds for the US case.

In other countries, studies by Mishra et al. (2010), Hatemij and Irandoust (2008), Heidari (2010), Aslan and Korap (2007), Arintoko (2011), Hansan (1999), Amin (2011), Cerqueira (2009), Ahmed and Suliman (2011), and Gabrielli et al. (2004), show that long term relationships exist between monetary aggregates and price level or the growth rate of monetary aggregates and inflation. Concerning causality, only study on Bangladesh, Brazil, China, Iran, Turkey and Sudan confirms causality from money to price. However, the study of some countries, such as Argentina and India, shows that causality moves from price to monetary aggregates. Furthermore, many studies show that the estimated parameter between price and monetary aggregates is close to one.

After individual countries, our attention turns to group studies. Seven studies under revision in this part are presented individually and in orderly fashion. Duck (1993) studied 33 countries by using annual data from 1962 to 1988, from which the pool time series cross sectioned ordinary least squares was adopted. That study showed a relationship between price level and monetary aggregates and, furthermore, the estimated coefficient was close to one. Gerlach (1995) carried out a study of 79 countries at most, which started from 1950 by using cross-sectional regression. The result showed that a one-to-one relationship existed between money and prices. However, this one-to-one relationship was sensitive by including countries with high inflation.

Herwartz and Reimers (2001) studied the long term links between money, prices and output. The study adopted panel cointegration that covered 110 countries from 1960 to 1990. The result confirmed that monetary aggregates determined the price level in the long term. The study of de Grauwe and Polan (2005) applied the panel ordinary least squares that covered most countries (160 countries) in the world. It confirmed that there is a strong positive relationship between the growth rate of money and inflation, although it is not a proportional one. Furthermore, their study also showed the same result as that from Gerlach (1995), in that a strong link between the two variables is due to including countries with high inflation in the sample. On the other hand, the link between the two variables is quite weak in countries with low inflation. Shirvani and Wilbratte (2006) studied 24 countries individually from 1959 to 1988 by using cointegration. The result also is the same as that from de Grauwe and Polan (2005) and Gerlach (1995). It showed that only money and inflation are related in the case of countries with high inflation. Benamar et al. (2010) studied money and prices in Maghreb countries individually - Algeria, Morocco and Tunisia. This study adopted the cointegration and causality test. The result from Morocco and Tunisia showed that a long term relationship exists between money and price, and money influences price level. However, in the case of Algeria, there was no long term relationship between money and prices.

Sargent and Surico (2011) studied the relationships between inflation and money growth and interest rate and money growth by estimating the Dynamic Stochastic General Equilibrium Model (DSGE) using Bayesian methods. The result showed that the QTM was more or less confirmed in the period from 1955 to 1975 and 1960 to 1983, and was broken between 1900 and 1928, 1929 and 1954, and 1984 and 2005. Alterations in monetary policies were the reason for the breakdown periods.

Teles and Uhlig (2010) investigated whether the QTM was still alive from the IMF for all OECD countries, excluded transition countries and Finland. They used the ordinary least squares from 1970 to 2005, and two subperiods from 1970 to 1990 and 1990 to 2005. The result showed that the QTM has been alive with some brown spots since 1990, when the success of targeting low inflation, dispersion of regulation, and adoption of new technology made establishment of a long-term relationship between monetary growths harder.

From all empirical evidence presented in this paper, it can be seen that both the group and individual study mostly confirm a long term relationship between monetary aggregates and price level, or growth of monetary aggregates and inflation. Furthermore, some studies show that a one-to-one relationship exists between these variables. Obtaining these results may look rather simple, but if one thinks carefully about how they are acquired from various countries running different policies in varied situations, it does not seem so easy.

As mentioned in McCandless and Weber (1996), a major instrument for conducting the monetary policy is (growth rate) monetary aggregates. Furthermore, Kahn and Benolkin (2007) mentioned that two of the most important and powerful central banks — the Federal Reserve (Fed) and European Central Bank (ECB) — pursue monetary policy differently. The Fed looks at the monetary aggregates as one of many economic indicators for an outlook on the economy, whereas, the ECB takes the monetary aggregate as one of the pillars for policy decisions. In addition, if the central bank of one country is monetarist by nature and pursues the constant growth rate of money supply, the result is not supposed to show a long term relationship. Consequently, it could be proclaimed that by looking at a particular country, the artifact of its specific policy can be involved.

For example, the strategy adopted by Canada, the United Kingdom and United States of America during the 1970s and 80s, of using the monetary policy as a target, was not successful. This may be for two reasons, as mentioned by Mishkin (2000). Firstly, monetary targeting was not performed seriously. Secondly, the relationship between monetary aggregates and inflation was not stable, which is explained in the QTM. Therefore, inflation targeting has been adopted since the 1990s in many industrialized countries.

However, the results of empirical evidence presented in this paper, as mentioned before, seem to confirm the relationship between monetary aggregates and price level anonymously, regardless of the policy adopted in each country. Also, the results mainly show that causality runs from money to price. Therefore, having presented broad empirical evidence based on individual and a group of countries, it could be mentioned that a long term relationship between money and price does exist and the validity of the QTM can be more or less confirmed, regardless of the policy pursued by the countries concerned.

## 6. CONCLUDING REMARKS

The equation of exchange under the QTM seems to be very simple and nothing more than an identity equation, which declares that total spending or state of the economy is equal to the total quantity of money spent or amount of money circulating in the economy during an economic period. However, if one looks in great detail, the QTM shows only the inflation determination. Furthermore, this theory was set during the 1930s, but is still valid now in the twenty first century, when many new financial instruments and new transaction technologies, such as electronic banking, are available. The data also confirm the relationship between price level and monetary aggregates for most countries in the world.

In addition, most modern macroeconomic models, which are based on a microeconomic foundation of money, still produce QTM results. The models; either in the form of infinite life or overlapping generations, can confirm validity of the QTM. Also, the way of introducing money into the model still has no effect on generating the QTM. Although Feenstra (1995) pointed out the similarity among different models, the MIU and CIA produce different money demand equations, and still produce or involve the QTM.

Also, empirical evidence in general states the long term relationship between money and price, which more or less involves the QTM, regardless of the monetary policy that the country concerned has adopted. However, the estimated coefficients between price level and monetary aggregates are not equal to one conclusively, due to many reasons such as missing variables in estimated equations, when estimated coefficients are quite high. The causality also runs from money to price, which supports the QTM and implies exogenous money.

In addition, the QTM, which is based on classical economic ideas and was set in the 1930s, is still one of the most important economic theories. The QTM is still alive with some slippage, despite rapidly changing economies.

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