Choice of Methodologies to Estimation of Total Factor Productivity Growth[1]

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ABSTRACT

This study presents two methodologies to estimation of total factor productivity (TFP) growth. One is referred to a Törnqvist index number technique and the other is a parametric technique using a dual approach of a stochastic cost frontier function. This study provides a discussion of their relative merits and illustrates the use of these two techniques in an empirical analysis using panel data (1986-1998) on 61 U.S. electricity generation businesses. The main purpose is to examine the sensitivity of the estimates obtained to the choice of TFP measurement methodology. The results indicate that the TFP growth indices between these two techniques are quite different. Therefore, researches who are interested in measuring TFP growth must choose the choice of alternative methodologies to estimation of TFP growth with care.

1. INTRODUCTION

Productivity is used to measure the performance of firms which convert inputs into outputs. Theoretically, it is defined as the ratio of the outputs produced to the inputs used by a firm in the production process. When the production process involves only a single output and a single input, the productivity can be easily calculated to compare the performance of

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an industry over time or across geographical regions. However, most of the productions involve more than one output and one input in the production process. Then, a method for aggregating the outputs into a single output index and aggregating the inputs into a single input index must be used to obtain a ratio measure of productivity. The productivity measured within the multi-output and multi-input production technology is referred as *total factor productivity* (TFP) which is productivity measure involving all factors of production.

In early studies of productivity, index number techniques were used to construct a TFP index. The TFP index is defined as the ratio of an aggregate output quantity index to an aggregate input quantity index. TFP growth occurs when an index of outputs changes at a different rate than an index of inputs. The first and foremost of productivity measurement was the use of the Fisher (1922) and Törnqvist (1936) indices. It was subsequently developed and based upon the idea of Malmquist (1953) and Shephard (1953) who independently introduced the notion of a distance function. The input and output quantity index numbers, and productivity indices are all based on the ideas of Malmquist and the distance function. Although the index number techniques are easy to compute but they carry some shortcomings. They require quantity and price information, as well as assumptions concerning the structure of technology and the behavior of producers. Moreover, they cannot provide the sources that contribute to productivity growth which are of broad interest for researchers. These problems lead to the development of new empirical techniques known as nonparametric and parametric approaches to measure the TFP growth decomposition. These two techniques do not require price information or technological and behavioral assumptions.

The nonparametric approach applied in the literature to measure TFP growth decomposition is based upon the previous work of Färe et al. (1994). They extended the Malmquist TFP index defined in Caves et al. (1982) and illustrated how the Malmquist TFP index can be computed using the nonparametric, linear programming techniques of data envelopment analysis (DEA) to fit distance functions. Färe et al. (1994) computed the TFP growth as the geometric mean of two Malmquist TFP index without using price data and defining a specific functional form. They showed how the resulting TFP index could be decomposed into two sources which consist of technical change (or frontier shifts) and efficiency change (or catching up effects). Their approach requires a constant returns to scale (CRS) restriction on the frontier technology. The DEA based Malmquist TFP index to measure the TFP growth decomposition is extensively applied in the empirical literature. However, the issue concerning statistical noise in the analysis became one of the main criticisms in the DEA. Since the DEA is nonstochastic, all departures from the frontier technology are attributed to inefficiency so that noise is excluded in the measure of efficiency change.

The parametric approach to measure TFP growth decomposition has been extensively applied using both primal and dual representations. The primal approach relates the conventional TFP measure to the characteristics of the production technology based on the aggregate production, while the dual approach uses the inverse relationship between the production and cost functions to establish the link between the conventionally measured TFP growth to the shift of aggregate cost function. These two approaches differ only in that the primal approach is developed to disentangle the contribution of factors other than technological progress from shifts in the production function, while the dual approach relates the observed growth to shift of the cost function.

The primal approach to the econometric estimation of productivity growth originated with Solow (1957), who assumed constant returns to scale and technical efficiency, and associated productivity growth with technical change. The conventionally measured productivity growth can be decomposed through the explicit specification of the production structure originated with Griliches (1963, 1964). The primal approach allows decomposition of TFP growth into a number of components by explicitly using the production function framework. TFP growth is decomposed into components associated with technical change and non-constant scale effects.

The dual approach to the econometric estimation of productivity growth originated with Ohta (1974), who derived the relationships between primal and dual cost measures of scale economies and technical change. Caves, Christensen, and Swanson (1980), Denny, Fuss, and Waverman (1981), and Nadiri and Schankerman (1981) used a flexible cost function and applied the duality theory to improve and refine the measurement of sources of TFP growth.

Nishimizu and Page (1982) originally presented a measurement of TFP growth decomposition in the presence of inefficiency. The efficiency change is presented as a source of TFP growth. They used a translog production frontier to decompose TFP growth into technical change and technical efficiency change. Extending the study of Nishimizu and Page (1982), Bauer (1990a) derives detailed primal and dual decompositions of TFP growth in the presence of inefficiency.

The purpose of this study is to apply two methods– a Törnqvist index number technique and a parametric technique using a dual approach of stochastic cost frontier function to measure TFP growth– as well as providing a discussion of their relative merits. This study illustrates the use of these methods in an empirical analysis that uses panel data on 61 US electricity generation businesses, observed over a 13-year period from 1986 to 1998.

The outline of this paper is as follows. In the next section the two performance measurement methods to measure TFP growth are presented. This is followed in Section 3 with a discussion of the data set used in this study and key assumptions underlying the construction. The next section provides the estimation results of the two performance measurement methods, and then conclusions follow in the final section.

2. METHODOLOGY

The methods that are used to measure the TFP growth can be roughly classified into two groups according to the types of prices employed, i.e. market price and shadow prices. Market prices are the actual prices that people must pay for the goods and services while shadow prices (internal prices to the firms) are derived from the shape of the underlying production technology. Three TFP measurement approaches that are widely applied in the literature are: the Törnqvist price-based index number, a parametric technique known as stochastic frontier analysis (SFA) and a nonparametric technique known as data envelopment analysis (DEA). The Törnqvist price-based index number approach uses market prices, while the SFA and the DEA approaches involve the estimation of a production technology, and hence the use of shadow prices derived from the shape of the estimated frontier.

The Törnqvist price-based index number approach has the advantage that it can be used when limited data are available (e.g. aggregate industry-level data). The SFA and DEA frontier approaches require more data (i.e. firm-level panel data), however they have the advantage that they allow one to identify various components of the TFP growth (such as technical change, efficiency change and scale effects), which are often of particular interest to regulators. The SFA approach has an advantage over the DEA approach when analyzing data in a stochastic environment. This is because DEA typically does not attempt to take statistical noise into account (and consequently may provide inaccurate efficiency measures), while the parametric approach does attempt to accommodate statistical noise.

This study applies the SFA approach of cost frontier function, in a TFP analysis of panel data on 61 U.S. electric utilities observed over the time period of 1986-1998, and compares the results with those obtained using the traditional Törnqvist price-based index number approach. The main objective is to examine the sensitivity of the estimates obtained to the choice of TFP measurement methodology.

2.1 The Törnqvist Price-Based Index Number (TPIN) Approach

Following Caves, Christensen and Diewert (1982), a Törnqvist TFP index can be constructed as the ratio of a Törnqvist output index to a Törnqvist input index. The logarithmic form of the Törnqvist TFP growth index between periods t and $t+1$ is defined as

$$\ln\left(TFP_{i,t+1}/TFP_{i,t}\right)^{T} = \frac{1}{2}\sum_{m=1}^{M}\left[\left(r_{mi,t+1}+r_{mi,t}\right)\left(y_{mi,t+1}-y_{mi,t}\right)\right] - \frac{1}{2}\sum_{k=1}^{K}\left[\left(s_{ki,t+1}+s_{ki,t}\right)\left(x_{ki,t+1}-x_{ki,t}\right)\right], \quad (1)$$

where the T superscript refers to *Törnqvist*; $i = 1,...,I$ indexes firms; $t = 1,...,T$ indexes time periods; $k = 1,...,K$ indexes input variables; $m = 1,...,M$ indexes output variables; $x_{ki,t}$ is the log of the *k*-th input quantity, $X_{ki,t}$; $y_{mi,t}$ is the log of the *m*-th output quantity, $Y_{mi,t}$; $r_{mi,t}$ is the observed revenue share of the *m*-th output; and $s_{ki,t}$ is the observed cost share of the *k*-th input.

For the single-output case, which is considered in the empirical part of this study, equation (1) is rewritten as

$$\ln TFPC^{T} = \ln\left(TFP_{i,t+1}/TFP_{i,t}\right)^{T} = \left(y_{i,t+1}-y_{i,t}\right) - \frac{1}{2}\sum_{k=1}^{K}\left[\left(s_{ki,t+1}+s_{ki,t}\right)\left(x_{ki,t+1}-x_{ki,t}\right)\right]. \quad (2)$$

As noted earlier, the Törnqvist TFP index approach has the advantage that it can be used to measure the TFP growth when limited data is available. However, it requires information on both quantities and prices of outputs and inputs. In addition, it cannot provide the sources that contribute to productivity growth and which are of broad interest for researchers. This problem can be addressed by gaining access to panel data and using a frontier technique such as a stochastic cost frontier (SFA) to decompose the measured TFP growth into its components.

2.2 The Stochastic Cost Frontier Approach

2.2.1 Derivations of Total Factor Productivity Decomposition

TFP growth is defined as the residual growth in outputs growth ($\hat{Y}$) not explained by input growth ($\hat{X}$). For the multiple-output and multiple-input case, TFP growth ($T\hat{F}P$) can be defined as

$$T\hat{F}P = \hat{Y} - \hat{X} = \sum_m r_m \hat{Y}_m - \sum_k s_k \hat{X}_k \,, \tag{3}$$

where "^" denotes the percentage rate of growth over time, r_m is the observed revenue share of *m*-th output and s_k is the observed cost share of *k*-th input.

Following Kumbhakar and Lovell (2000), a stochastic cost frontier function incorporating a time trend can be written as

$$E_{it} = C_{it}\left(Y_{mit}, W_{kit}, t; \beta\right)\exp\left\{v_{it} + u_{it}\right\}, \tag{4}$$

where $i = 1,...,I$ index of firms; $t = 1,...,T$ index of time periods; Y_m is the *m*-th output quantity; W_k is the *k*-th input price; *t* is a time trend index serving as a proxy for a technical change; β s are unknown parameters to be estimated; v_{it} s are the two-side random statistical noise accounting for measurement error or other random factors such as weather, luck, strike, etc. and the u_{it} s are non-negative random errors associated with the cost inefficiency effects.

The stochastic cost frontier function in logarithm form (omitting the firm index *i* and the time index *t*) is written as

$$\ln E = \ln C\left(Y_m, W_k, t; \beta\right) + v + u \,. \tag{5}$$

The measurement of TFP growth associated with the cost frontier function is derived by totally differentiating the stochastic cost frontier function in equation (5) with respect to time. This yields

$$\frac{\partial \ln E}{\partial t} = \frac{\partial \ln C(\cdot)}{\partial t} + \sum_m \frac{\partial \ln C(\cdot)}{\partial \ln Y_m}\frac{\partial Y_m}{\partial t}\left(\frac{1}{Y_m}\right) + \sum_k \frac{\partial \ln C(\cdot)}{\partial \ln X_k}\frac{\partial W_k}{\partial t}\left(\frac{1}{W_k}\right) + \frac{\partial u}{\partial t}. \tag{6}$$

Substituting equation (6) into equation (3) yields

$$T\hat{F}P = \frac{\partial \ln E}{\partial t} - \frac{\partial \ln C(\cdot)}{\partial t} + \left[\sum_m r_m - \sum_m \frac{\partial \ln C(\cdot)}{\partial \ln Y_m}\right]\hat{Y}_m - \sum_k s_k \hat{X}_k$$
$$- \sum_k \frac{\partial \ln C(\cdot)}{\partial \ln W_k}\hat{W}_k - \frac{\partial u}{\partial t}. \tag{7}$$

Defining c as the log of total cost, C; w_k as the log of k-th input price, W_k; y_m as the log of output quantity, Y_m, equation (7) is rewritten as

$$T\hat{F}P = -\frac{\partial c(\cdot)}{\partial t} + \left[\sum_m r_m - \sum_m \frac{\partial c(\cdot)}{\partial y_m}\right]\frac{\partial y_m}{\partial t} + \left[\frac{\partial \ln E}{\partial t} - \sum_k s_k \frac{\partial x_k}{\partial t}\right]$$
$$- \sum_k \frac{\partial c(\cdot)}{\partial w_k}\frac{\partial w_k}{\partial t} - \frac{\partial u}{\partial t}. \tag{8}$$

Rearranging equation (8) yields

$$T\hat{F}P = -\frac{\partial c(\cdot)}{\partial t} + \left[\sum_m r_m - 1\right]\frac{\partial y_m}{\partial t} + \left[1 - \sum_m \frac{\partial c(\cdot)}{\partial y_m}\right]\frac{\partial y_m}{\partial t} + \left[s_k - \frac{\partial c(\cdot)}{\partial w_k}\right]\frac{\partial w_k}{\partial t} - \frac{\partial u}{\partial t}. \tag{9}$$

(Note that $\frac{\partial \ln E}{\partial t} - \sum_k s_k \frac{\partial x_k}{\partial t} = \hat{E} - \sum_k s_k \frac{\partial x_k}{\partial t} = \sum_k s_k \frac{\partial w_k}{\partial t}$).

Defining ε_m as the first partial derivation of the logarithmic form of cost frontier function with respect to the m-th output and κ_k as the first partial derivation of the logarithmic form of cost frontier function with respect to the k-th input price, equation (9) can be rewritten as

$$T\hat{F}P = -\frac{\partial c(\cdot)}{\partial t} + \sum_m \left[r_m - \frac{\varepsilon_m}{\varepsilon}\right]\frac{\partial y_m}{\partial t} + \sum_m \left[\frac{\varepsilon_m}{\varepsilon} - \varepsilon_m\right]\frac{\partial y_m}{\partial t} + \sum_k (s_k - \kappa_k)\frac{\partial w_k}{\partial t} - \frac{\partial u}{\partial t}. \tag{10}$$

where $\varepsilon_m = \left(\partial c(\cdot)/\partial y_m\right)$ represents the production elasticities; $\varepsilon = \sum_m \varepsilon_m$ represents the inverse of the standard returns to scale elasticity; $\kappa_k = \left(\partial c(\cdot)/\partial w_k\right)$ represents the implicit cost shares for the k-th input; and $\kappa = \sum_k \kappa_k$ represents the sum of implicit cost shares for all inputs.

Rearranging equation (10) yields

$$\hat{TFP} = -\frac{\partial c(\cdot)}{\partial t} + \sum_m \left[SF \cdot \varepsilon_m\right]\frac{\partial y_m}{\partial t} + \left[\sum_m \left(r_m - \pi_m\right)\frac{\partial y_m}{\partial t} + \sum_k \left(s_k - \kappa_k\right)\frac{\partial w_k}{\partial t}\right] - \frac{\partial u}{\partial t}, \quad (11)$$

where $SF = (1-\varepsilon)/\varepsilon$ is scale factors at each data point and $\pi_m = \left(\varepsilon_m/\varepsilon\right)$ is implicit revenue share.

TFP growth in equation (11) comprises of four components. The first term measures the technical change representing a shift in the cost frontier function. The second component measures the change in scale efficiency, which requires the calculation of the production elasticities. For the case of constant returns to scale, the term ε will be equal to 1, and hence the second component in equation (11) will be equal to 0. The third term measures the change in allocative efficiency, which consists of two components. The first component measures the change in output mix allocative efficiency effects. This component will be zero if the market (observed) revenue shares, r_m, equal to the implicit revenue shares, π_m. The second component of the change in allocative efficiency measures the change in input mix allocative efficiency effects. This component will be zero if the market (observed) cost shares, s_k, equal to the implicit cost shares, κ_k. Finally, the last term in equation (11) measures the cost efficiency change.

Equation (11) for the single-output case can be rewritten as

$$\hat{TFP} = -\frac{\partial c(\cdot)}{\partial t} + (1-\varepsilon)\frac{\partial y}{\partial t} + \sum_k \left(s_k - \kappa_k\right)\frac{\partial w_k}{\partial t} - \frac{\partial u}{\partial t} \quad (12)$$

2.2.2 Estimation Approach

In order to measure the components of TFP growth discussed in Section 2.2.1, a flexible functional form of the cost frontier function must be specified. This study adopts a translog functional form. A log-quadratic translog functional form of stochastic cost frontier function for the single-output and three-input case can be defined as follows.

$$c_{it} = \beta_0 + \sum_{k=1}^{3} \beta_k w_{kit} + \frac{1}{2} \sum_{k=1}^{3} \sum_{l=1}^{3} \beta_{kl} w_{kit} w_{lit} + \beta_y y_{it} + \frac{1}{2} \beta_{yy} y_{it}^2 + \sum_{k=1}^{3} \beta_{ky} w_{kit} y_{it}$$
$$+ \beta_t t + \frac{1}{2} \beta_{tt} t^2 + \sum_{k=1}^{3} \beta_{kt} x_{kit} t + \beta_{yt} y_{it} t + v_{it} + u_{it}, \qquad (13)$$

where the β s are unknown parameters to be estimated, and all other notation is as previously defined. This study follows the standard practice of assuming a normal distribution for v and a half-normal distribution for u. That is, we set $v \sim N(0, \sigma_v^2)$ and $u \sim |N(0, \sigma_u^2)|$. Given these distributional assumptions, the parameters of this model can then be estimated using the method of maximum likelihood. Following the suggestion of Battese and Corra (1977), and replace the two variance parameters with the two new parameters $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\gamma = \sigma_u^2 / \sigma^2$. By doing this the parameter space of γ is searched between 0 and 1, to provide good starting values for the iterative maximization routine which is used to calculate the maximum likelihood parameter estimates.

Young's theorem requires that the symmetry restriction is imposed so that $\beta_{kl} = \beta_{lk}$ for all $k, l = 1,2,3$ and homogeneity of degree +1 in input prices. From Euler's Theorem, this implies

$$\sum_{k=1}^{3} \beta_k + \sum_{k=1}^{3} \beta_{kl} + \sum_{k=1}^{3} \beta_{ky} + \sum_{k=1}^{3} \beta_{kt} = 1, \qquad (14)$$

which will be satisfied if

$$\sum_{k=1}^{3} \beta_k = 1; \; \sum_{k=1}^{3} \beta_{kl} = 0 \text{ for } l = 1,2,3; \; \sum_{k=1}^{3} \beta_{ky} = 0; \; \sum_{k=1}^{3} \beta_{kt} = 0. \qquad (15)$$

The restrictions of homogeneity constraints upon equation (13) can be imposed by estimating a model where all input prices are normalized by one of the input prices. By normalizing the K-th input price, the translog stochastic cost frontier function in equation (13) is rewritten as

$$c_{kit}^* = \beta_0 + \sum_{k=1}^{2} \beta_k w_{kit}^* + \frac{1}{2}\sum_{k=1}^{2}\sum_{l=1}^{2} \beta_{kl} w_{kit}^* w_{lit}^* + \beta_y y_{it} + \frac{1}{2}\beta_{yy} y_{it}^2 + \sum_{k=1}^{2} \beta_{ky} w_{kit}^* y_{it} + \beta_z t$$
$$+ \frac{1}{2}\beta_{zz} t^2 + \sum_{k=1}^{2} \beta_{kz} w_{kit}^* t + \beta_{yz} y_{it} t + v_{it} + u_{it}, \tag{14}$$

where $c_{kit}^* = \left(c_{kit}/w_{Kit}\right)$, $w_{kit}^* = \left(w_{kit}/w_{Kit}\right)$ and $w_{lit}^* = \left(w_{lit}/w_{Kit}\right)$.

Once the equation (14) is estimated using the maximum likelihood estimation, the parameter estimates and the point estimates of the cost efficiencies are used to calculate the components of $T\hat{F}P$ in equation (12).

Following Orea (2002), a measure of TFP growth, for each firm between any two time periods, can be calculated by using the estimates of the coefficients of the cost frontier in equation (14) and the firm-level sample data. The logarithmic form of the TFP growth between period t and $t+1$ for the i-th firm is defined as[1]

$$\ln TFPC = \ln\left(TFP_{i,t+1}/TFP_{i,t}\right) = \ln\left(CE_{i,t}/CE_{i,t+1}\right) - \frac{1}{2}\left[\partial c_{i,t+1}/\partial t + \partial c_{i,t}/\partial t\right]$$
$$+ \frac{1}{2}\left[\left(1 - \partial c_{i,t+1}/\partial y\right) + \left(1 - \partial c_{i,t}/\partial y\right)\right]\left(y_{i,t+1} - y_{i,t}\right) \tag{15}$$
$$+ \frac{1}{2}\sum_{k=1}^{3}\left[\left(s_{ki,t} - \left(\partial c_{i,t}/\partial w_{ki,t}\right)\right) + \left(s_{ki,t+1} - \left(\partial c_{i,t+1}/\partial w_{ki.t+1}\right)\right)\right]\left(w_{ki,t+1} - w_{ki,t}\right),$$

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[1]This formula is quite similar to that provided in Bauer (1990), which was alternatively derived using a differential approach expressed in Section 2.2.1. The main differences between the two sets of TFP decomposition formula is that the TFP growth components in equation (12) are evaluated at the t and $t+1$ data points, while the Bauer (1990) formula is only evaluated at the t data point. This difference will have minimal effect on the empirical measures obtained in most instances.

where the three terms on the right-hand-side of equation (15) represents the cost efficiency change $\left(CEC_{it,t+1}\right)$, technical change $\left(TC_{it,t+1}\right)$, scale efficiency change $\left(SEC_{it,t+1}\right)$, and input allocative efficiency change $\left(AEC_{it,t+1}\right)$, respectively. The cost efficiency measure, $\left(CE_{it}\right)$, in equation (15) is the cost efficiency prediction of the *i*-th firm in the *t*-th time period, and is calculated from the cost frontier in equation (14). The technical change measure, $\left(TC_{it,t+1}\right)$, is the mean of the technical change measures evaluated at the period t and period $t+1$ data points. The scale efficiency change measure, $\left(SEC_{it,t+1}\right)$, relates to the change in scale efficiency, which requires calculation of the output elasticity in period t, $\partial c_{i,t}/\partial y$, and period $t+1$, $\partial c_{i,t+1}/\partial y$. The input allocative efficiency change $\left(AEC_{it,t+1}\right)$ component is equal to the difference between the $\left(TFPC_{it,t+1}\right)$ measure obtained from the cost frontier in equation (15) and the Törnqvist *TFPC* index in equation (2).

3. DATA DISCUSSIONS

This study uses data on fossil-fuel fired steam electric power generation for major investor-owned utilities in the United States. The primary sources of data are obtained from the Energy Information Administration, the Federal Energy Regulatory Commission and the Bureau of Labor Statistics. Panel data on 61 electric utilities over the time period of 1986-1998 are used in the empirical analysis.

The data set used to measure and decompose TFP growth contains the measurements of firm outputs and input quantities. The variable inputs are fuel (*F*), labor and maintenance (*L*), and capital (*K*). The definitions of these variables are summarized as follows.

a. Output Variable

Output variable, Y_{it}, is represented by net steam electric power generation in megawatt-hours which is defined as the amount of power produced using fossil-fuel fired boilers to produce steam for turbine generators during a given period of time.

b. Price and Quantity of Fuels Input Variables

The price of fuel aggregate, W_{1it}, is a Törnqvist price index of fuels (i.e. coal, oil, gas) which is calculated by a weighted geometric average of the price relatives with weights given by the simple average of the value shares in period t and $t+1$.

$$\left(\frac{W_{1it+1}}{W_{1it}}\right) = \prod_{f=1}^{3}\left(\frac{P_{fit+1}}{P_{fit}}\right)^{\left(\frac{a_{fit}+a_{fit+1}}{2}\right)}, \qquad (16)$$

where $a_{fit} = \frac{P_{fit}Q_{fit}}{\sum_{f=1}^{3} P_{fit}Q_{fit}}$, P_{fit} is the price of the *f*-th fuel (i.e. coal, oil, gas), and Q_{fit} is the consumption of the same fuel. The Törnqvist price index of fuels is converted to a multilateral Törnqvist price index for fuels using the formula discussed in Coelli, Rao, and Battese (1998).

The quantities of fuel, X_{1it}, equal the steam power production fuel costs divided by the multilateral Törnqvist price index for fuels.

c. Price and Quantity of Aggregate Labor and Maintenance Input Variables

The price of labor and maintenance aggregate, W_{2it}, is a cost share-weighted multilateral Törnqvist price index for labor and maintenance. The price of labor is a firm-level average wage rate. The price of maintenance and other supplies is an industry-level price index of electrical supplies.

The quantities of labor and maintenance, X_{2it}, are measured as the aggregate costs of labor and maintenance divided by the multilateral Törnqvist price index for labor and maintenance. Data on labor and maintenance costs are calculated by subtracting fuel expenses from total steam power production expenses.

d. Price and Quantity of Capital Input Variables

The price of capital, W_{3it}, is the yield of the firm's latest issue of long term debt adjusted for appreciation and depreciation of the capital good using the Christensen and Jorgenson (1970) cost of capital formula.

$$W_{3it} = p_{kit}\left[i_{dit} + s_{it}\left(r_{eit} - i_{dit}\right) + d_{it} - f_{it}\right] \quad (17)$$

where p_{kit} is a price index for electrical generating plant and equipment; i_{dit} is the adjusted corporate bond rate by firm based upon its bond ratings by Moody's Investor Service; s_{it} is the equity share of total capital defined as total proprietary capital (TPC) divided by the sum of total proprietary capital and total long-term debt (TOTB); r_{eit} is the equity rate of return defined as the ratio of net income to total proprietary capital; d_{it} is a depreciation rate assuming 30 years straight line depreciation; and f_{it} the inflation rate.

The values of capital stocks are calculated by the valuation of base and peak load capacity at replacement cost to estimate capital stocks in a base year and then updating it in the subsequent years based upon the value of additions and retirements to steam power plant as discussed in Considine (2000). The base year capacity is calculated by multiplying the price of new generation capacity in dollars per megawatt and the base year nameplate capacity in megawatts.

$$X_{3it} = P_{cit}C_{it}, \quad t = 1986 \quad (18)$$

where P_{cit} is the price of new generation capacity in dollars per megawatt, and C_{it} is the nameplate capacity in megawatts. For the subsequent years, the values of capital stocks are calculated by

$$X_{3it} = \frac{(1-\upsilon)\left(X_{3it-1}\right)p_{kit}}{p_{kit-1}} + A_{it} - R_{it}, t = 1987,...,1998 \quad (18)$$

where υ denotes the depreciation rate assuming 30 years straight line depreciation; X_{3it} is equal to the nominal stock divided by the price index for electrical generating plant and equipment, p_{kit} ; A_{it} and R_{it} denote additions and retirements to steam power plant.

The final data set is a balanced panel of 61 electric utilities for the years 1986 to 1998 with a total of 793 observations. The availability of panel data generally implies that there are degrees of freedoms in the estimation of parameters and such a data set permits the simultaneous investigation of both the technical change and the technical efficiency change over time. Table 2 represents a summary of the data used in this study. All price indices used in this study are obtained and calculated relative to the base period 1993. The mean steam electric power generation across electric utilities is 13.71 million megawatt hours with a standard deviation of 12.56 million megawatt hours. The mean of fuel quantity is 300.57 million dollars with a standard deviation of 351.84, and of labor and maintenance is 61.78 million dollars with a standard deviation of 53.37. The mean capital value is 955.22 million dollars with a standard deviation of 877.40. The average expenses of aggregate fuels, aggregate labor and maintenance, and capital are calculated to be 254.77, 66.71, and 113.90 million dollars, respectively. The mean cost shares of fuel, labor and maintenance, and capital account for 58.6, 17.9, and 23.5 percent, respectively.

Table 1 : Data summary for 61 electric utilities over the periods of 1986-98

Variable	Units	Mean	S. D.	Minimum	Maximum
Output	($\times 10^6$ *MWhr*)	13.709	12.561	0.499	79.723
Fuel	($\times 10^6$ *dollars*)	300.568	351.842	12.823	2,522.324
Labor and Maintenance	($\times 10^6$ *dollars*)	61.776	53.366	1.810	444.453
Capital	($\times 10^6$ *dollars*)	955.225	877.403	9.070	3,878.295
Price Index of Fuel		0.861	0.208	0.306	1.338
Price Index of Labor and Maintenance		1.079	0.255	0.443	1.928
User Costs of Capital		0.102	0.019	0.009	0.203

4. EMPIRICAL RESULTS

4.1 Discussion of parameter estimates

The data described in Section 3 were used in the calculation of Törnqvist TFP indices and also in the estimation of the stochastic cost frontier function described in Section 2. The data variables used in the model estimation were each transformed by division by their respective geometric means. This transformation does not alter the performance measures obtained, but does allow one to interpret the estimated first-order parameters as elasticities, evaluated at the sample means. A number of hypothesis tests regarding the structure of the production technology such as the functional form (i.e. Cobb-Douglas versus Translog; $\beta_{kl} = \beta_{yy} = \beta_{ky} = \beta_{tt} = \beta_{kt} = \beta_{yt} = 0, k, l = 1,2$ the presence of technical change $(\beta_t = \beta_{tt} = \beta_{tt} = \beta_{yt} = 0, k = 1,2)$ and the presence of technical inefficiency $(\gamma = 0)$ were conducted using likelihood ratio tests. The results of these likelihood ratio (LR) tests are presented in Table 2. All null hypotheses were rejected. The LR test results indicate the translog functional form is a preferable one and there exist technical change and technical inefficiency in the model.

The maximum likelihood parameter estimates for the stochastic cost frontier function obtained using the computer program FRONTIER 4.1 (Coelli, 1996a) are listed in Table 3. The estimated results indicate that the input elasticities are 0.449, 0.361 and 0.190 for *F*, *L* and *K*, respectively. These elasticities can also be interpreted as shadow input shares. These shadow input shares differ from the average *observed* shares in this data set which are 0.586, 0.179 and 0.235 for *F*, *L* and *K*, respectively. As a result, the TFP growth estimates obtained using the Törnqvist index (which uses observed shares) likely differ from the cost frontier TFP measures.

The estimated parameters in Table 3 also provide information on scale economies and technical change. Using the first order coefficient of the output variable, the elasticities of scale relative to the cost functions can be calculated as $RTS = (\partial c / \partial y)^{-1}$, where a value of *RTS* greater than one imply increasing returns to scale, while values less than one imply decreasing returns to scale, and values equal to one indicate constant returns to scale. The estimated parameters in Table 3 suggest that the average estimate of *RTS* is 1.035. This means that electric utilities in the sample data were operating at modestly increasing returns to scale in the production of electricity. The first order coefficient of the time trend variable in Table 3 provides an estimate of the average annual rate in technical change. The estimate suggests that the technology is improving at a modest rate of 2.2 percent per annum.

Table 2 : Tests of hypothesis for parameters

Null hypothesis	ln[L(H_1)] (unrestricted model)	ln[L(H_0)] (restricted model)	Test-statistic	Critical Value	Decision
(1) H_0: Cobb-Douglas is preferred					
Cost frontier model	200.65	173.22	54.86	18.31	**Reject H_0**
(2) H_0: no technical change					
Cost frontier model	200.65	136.32	128.66	11.07	**Reject H_0**
(3) H_0: no technical inefficiency					
Cost frontier model	200.65	96.70	207.90	3.84	**Reject H_0**

Table 3 : Parameter estimates for the cost frontier function

Parameters	Estimates	S.D.	t-ratio
β_0	-0.270	0.012	-22.055
β_1	0.449	0.022	19.998
β_2	0.361	0.030	11.983
β_3	0.190		
β_{11}	0.267	0.128	2.095
β_{12}	-0.360	0.110	-3.287
β_{13}	0.093		
β_{22}	0.525	0.169	3.111
β_{23}	-0.164		
β_{33}	0.071		
β_y	0.966	0.006	155.078
β_{yy}	0.028	0.012	2.400

β_{1y}	-0.059	0.025	-2.337
β_{2y}	-0.011	0.031	-0.343
β_{3y}	0.069		
B_t	-0.022	0.002	-10.455
B_{tt}	0.002	0.001	1.458
β_{1t}	0.038	0.008	4.756
β_{2t}	-0.048	0.011	-4.453
β_{3t}	0.010		
β_{yt}	-0.005	0.002	-2.297
σ^2	0.113	0.007	16.177
γ	0.974	0.008	122.552
Log likelihood function			200.651
LR test of the one-sided error			207.896

4.2 Discussion of performance measures

Some summary measures of the TFP growth measures (and components) described in Section 2 are listed in Table 5. The mean value reported for the Törnqvist index (TPIN) is 1.518, indicating that the average annual change in this TFP measure over this period is 1.518 percent per year. This is quite different from the value of 3.545 percent per year reported for the cost frontier function case. The difference of the TFP growth measures obtained from the Törnqvist index and the cost frontier function is expected as the estimated shadow input shares differ from the average *observed* shares mentioned in Section 4.1.

In looking at which components contribute most to TFP growth in the cost frontier function section of Table 5, the estimated results indicate that the major contribution is from TC (2.213%),[4] followed by AEC (1.042%), CEC (0.243%) and lastly SEC (0.047). The large contribution from TC conforms with most past studies of this industry (e.g. Atkinson and Primont, 2002). The small contribution of SEC is not surprising, given that the estimated technology exhibits modestly increasing returns to scale (at the sample mean). Furthermore, the modest contribution of AEC is as expected, given the differences between observed and shadow shares.

Table 6 reports weighted annual average TFP growth decomposition, where the firm-level results have been weighted by the output of each firm. These weighted average results are likely to give a more accurate picture of the industry-level changes over time. It is interesting to note that the weighted average TFP growth measures are larger than the unweighted average, in the case of the Törnqvist and cost function results. This suggests that the larger firms are achieving higher productivity growth relative to the smaller firms. This is perhaps due to them having greater resources devoted to research and development, or maybe due to these larger firms having higher growth rates and hence more opportunities to benefit from embodied technical change in new investments. However, further research is required confirm these hypotheses.

Tables 5 and 6 also contain year-by-year averages. These annual measures indicate the degree of volatility in the TFP growth measures. For example, in the final column of Table 6, the estimated results show that TFP growth varies from a high of 9.866% in 1986/87 to a low of negative 1.220% in 1989/90, and from the following CEC column, the estimates indicate that most of this TFP volatility is due to CEC. These measures illustrate the degree to which exogenous factors, such as the business cycle and climatic conditions, can affect efficiency measures. Given this, it would clearly be prudent for a regulator to not base TFP growth measures upon only a handful of years of data, where the danger that an unusual event could significantly affect the measures obtained.

It is reassuring to note that these TC measures, formed by averaging firm-level measures, are similar to those obtained earlier, which were derived by evaluating the time derivative at the sample means.

Annual averages of the firm-level measures are reported in Table 7. There is a wealth of information in this table. Of particular note is the degree to which the TFP performance of some firms varies, varying from an annual average decrease of 2.107% for firm 14 to an increase of 11.221% for firm number 2. The distribution of TFP growth scores is further illustrated using the frequency distributions reported in Table 8. The TFP growth scores obtained from the Törnqvist index indicate that most of firms had productivity progress between 0.1 and 4.0 percent, while those obtained from the cost frontier function had productivity progress between 2.0 and 6.0 percent over the period of study from 1986 to 1998. This variability in performance is one prickly issue that regulators and regulatory consultants must deal with when they design a policy which involves the measures of TFP growth and efficiency levels to the regulated firm. In theory, one should not base upon a firm's individual TFP growth performance, because this will greatly reduce the firm's incentives to seek out productivity improvements. However, in practice, the regulator has to deal with a situation where some firms are earning very high super-normal profits while others are making substantial losses (and facing bankruptcy). Neither case is likely to make the local politicians very happy.

Table 5 : Annual average TFP change measures (unweighted)*

Year	TPIN	Cost Frontier Model				
		CEC	TC	SEC	AEC	TFPC
86-87	6.631	6.455	1.652	0.146	2.001	10.255
87-88	1.575	1.962	1.888	0.042	2.615	6.507
88-89	4.003	2.748	2.073	0.250	1.477	6.548
89-90	-2.570	-4.401	2.062	-0.165	-0.371	-2.874
90-91	-2.287	-2.052	2.109	-0.062	1.971	1.966
91-92	0.999	0.096	2.208	-0.069	1.222	3.457
92-93	0.109	-0.922	2.273	-0.282	1.156	2.225
93-94	1.767	0.012	2.361	0.114	0.421	2.907
94-95	0.428	-0.817	2.467	-0.032	1.008	2.626
95-96	2.377	0.089	2.514	-0.120	0.362	2.845
96-97	3.488	0.702	2.451	0.498	0.516	4.167
97-98	1.696	-0.961	2.500	0.244	0.128	1.912
Mean	**1.518**	**0.243**	**2.213**	**0.047**	**1.042**	**3.545**

(* All measures are in percentage terms)

Table 6 : Weighted annual average TFP change measures

Year	TPIN	Cost Frontier Model				
		CEC	TC	SEC	AEC	TFPC
86-87	5.867	5.641	2.099	-0.049	2.169	9.861
87-88	1.465	1.378	2.367	0.023	2.567	6.335
88-89	2.108	0.452	2.532	0.055	1.104	4.142
89-90	-1.416	-3.662	2.541	-0.040	-0.059	-1.220
90-91	-1.133	-1.497	2.605	-0.002	2.028	3.133
91-92	1.227	1.431	2.702	0.007	2.886	7.025
92-93	1.557	-1.723	2.752	-0.020	-0.344	0.665
93-94	1.040	-0.027	2.835	0.019	1.538	4.365
94-95	0.864	-0.805	2.982	0.029	1.273	3.479
95-96	3.371	0.475	3.090	0.012	0.613	4.189
96-97	2.156	-0.699	3.105	0.121	0.812	3.339
97-98	1.624	-0.852	3.116	0.125	0.373	2.762
Mean	**1.561**	**0.009**	**2.727**	**0.023**	**1.246**	**4.006**

Table 7 : Average TFPC decomposition by firm (in percentage)

Firm	TPIN TFPC	CEC	TC	SEC	AEC	TFPC	Firm	TPIN TFPC	CEC	TC	SEC	AEC	TFPC
1	0.683	-0.328	2.247	0.050	1.227	3.196	32	0.784	0.294	1.082	0.060	0.697	2.133
2	11.175	8.650	0.756	0.663	1.153	11.221	33	2.415	1.959	0.953	0.006	0.537	3.456
3	-0.047	-2.192	2.892	-0.045	0.578	1.232	34	-0.392	-0.556	1.169	0.047	1.041	1.701
4	2.877	1.652	1.319	-0.023	0.117	3.064	35	0.861	-1.402	3.106	-0.080	0.724	2.348
5	0.285	0.530	1.281	-0.243	1.307	2.875	36	1.627	0.278	1.685	0.106	0.972	3.041
6	1.758	1.533	2.649	-0.089	2.413	6.505	37	0.147	0.710	2.359	0.329	3.167	6.565
7	0.098	-0.333	1.065	0.086	0.624	1.443	38	-0.826	-1.146	1.911	0.005	1.468	2.238
8	0.827	-0.203	1.996	0.029	0.984	2.807	39	-0.532	-2.680	3.683	0.051	1.541	2.595
9	0.337	-0.615	1.882	0.020	0.736	2.024	40	0.918	0.588	0.500	0.115	0.374	1.578
10	-0.152	-1.079	0.710	0.058	-0.447	-0.759	41	2.023	-0.719	4.718	-0.007	1.930	5.921
11	1.971	1.238	0.979	-0.078	0.253	2.393	42	1.727	-0.806	4.174	0.106	1.740	5.215
12	1.934	-0.232	3.493	0.018	1.338	4.617	43	-3.943	-3.891	2.891	-0.833	2.041	0.207
13	1.615	0.758	1.424	0.028	0.615	2.825	44	2.556	1.526	1.142	0.336	1.078	4.082
14	-0.960	-2.175	0.847	-0.175	-0.605	-2.107	45	1.674	0.559	2.234	0.015	1.184	3.991
15	0.569	-0.578	3.209	-0.023	2.022	4.630	46	2.915	-1.245	4.742	-0.072	0.479	3.904
16	1.854	0.084	3.172	0.015	1.458	4.729	47	-0.580	-1.569	1.568	0.084	0.530	0.614
17	2.529	1.252	1.659	0.056	1.001	3.968	48	1.433	0.580	2.788	-0.012	1.935	5.291
18	1.907	0.325	2.064	0.073	0.550	3.012	49	2.191	1.608	0.297	0.199	-0.046	2.057
19	2.601	1.033	3.026	0.203	1.690	5.952	50	3.498	0.582	4.079	0.171	1.657	6.489
20	9.341	5.622	3.032	0.288	-0.341	8.600	51	1.679	0.446	1.246	0.010	0.031	1.733
21	1.121	0.212	2.905	0.014	2.123	5.253	52	1.521	-0.787	3.509	0.018	0.871	3.610
22	3.318	1.704	2.491	-0.098	0.811	4.909	53	2.079	0.135	2.626	-0.032	0.875	3.604
23	2.017	1.806	0.636	0.373	0.862	3.677	54	4.422	3.449	1.881	0.082	1.162	6.574
24	0.593	1.026	-0.562	0.182	0.094	0.740	55	0.785	-0.067	-0.152	0.103	-0.904	-1.019
25	2.447	0.549	3.042	-0.254	0.907	4.244	56	-1.966	-2.953	2.268	-0.121	1.077	0.272
26	-0.337	-0.866	1.023	-0.012	0.461	0.606	57	0.102	-0.049	1.641	0.333	1.776	3.701
27	1.475	-1.366	4.921	0.018	2.056	5.630	58	2.115	0.200	4.284	-0.037	2.340	6.787
28	2.848	0.909	2.568	0.045	0.702	4.225	59	0.418	-0.755	3.620	-0.012	2.289	5.141
29	3.222	1.211	3.376	-0.037	1.371	5.920	60	0.582	-0.072	1.488	0.478	1.307	3.202
30	0.906	-0.146	2.443	0.066	1.419	3.782	61	1.539	0.596	2.149	0.102	1.351	4.199
31	2.017	0.001	2.813	0.109	0.875	3.798	**Mean**	**1.518**	**0.243**	**2.213**	**0.047**	**1.042**	**3.545**

Table 8 : Distribution of Average Total Factor Productivity Change

TFPC (%)	Number of Firms	
	TPIN	Cost Frontier Approach
less than -2.01	1	1
-2.01-0.00	9	2
0.01-2.00	31	10
2.01-4.00	19	25
4.01-6.00	1	16
6.01-8.00	0	5
8.01-10.00	1	1
greater than 10.00	1	1

5. CONCLUSIONS

Measuring TFP growth in infrastructure industries such as electricity, gas, airline and telecommunication has been the subject matter for extensive research over the past three decades. This study attempts to draw attentions of researches in selecting the choice of alternative methodologies to estimation of TFP growth. Two TFP measurement approaches: the Törnqvist price-based index number and a parametric technique known as stochastic frontier analysis (SFA) are presented in this study. The Törnqvist price-based index number approach uses market prices, while the SFA approach involves the estimation of a production technology, and hence the use of shadow prices derived from the shape of the estimated frontier. The Törnqvist price-based index number is easy to compute and can be used when limited data are available. However, it cannot provide the sources that contribute to TFP growth which are of broad interest for researchers. The SFA frontier approach requires more data (i.e. panel data) and allows one to identify various components of the TFP growth (such as technical change, efficiency change and scale effects). This study illustrates the use of these two approaches in an empirical analysis using panel data on 61 U.S. electric utilities observed over the time period of 1986-1998. The main purpose is to examine the sensitivity of the estimates obtained to the choice of TFP measurement methodology. Since the two techniques mentioned in this study are extensively applied in the literature, this study demonstrates the choice of methodologies can lead to the different results. Therefore, regulators and regulatory consultants who design a policy involving the uses of TFP growth and efficiency levels to the regulated firm must select the choice of alternative methodologies to estimation of TFP growth with care.

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