

# Liquidity Effects and Habit Persistence.

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## Abstract

We introduce various types of habit persistence in consumption and leisure into a limited participation model to investigate whether these features can account for persistence in nominal interest rate. We model this by assuming that the past levels of consumption and leisure affect the current utility of agents. We consider five different versions of a limited participation model, the benchmark model, the model with the habit persistence in only consumption, the model with the habit persistence in only leisure, the model with the habit persistence in both consumption and leisure and the model with the habit persistence in only consumption extended for more than one period. Our results show that depending on the nature and the duration of habit persistence, it was possible to generate varying degrees of serial correlation in nominal interest rate movements.

## 1. Introduction

Economists are generally agreed that an expansionary monetary policy shock generates a persistent decrease in nominal interest rates and a persistent increase in the levels of employment and output, at least in the short run. These effects are thought to be the result of two opposing forces. The first is a liquidity effect, whereby an increase in the money supply leads to a fall in the nominal interest rate as agents try to dispose of their excess cash balances. The second is an anticipated inflation effect, whereby an increase in monetary growth pushes nominal interest rates up because of higher expected inflation. For Christiano (1991), the widespread view among economists is that the liquidity effect is stronger than the anticipated inflation effect, at least in the short run. However, existing dynamic general equilibrium business cycle models are inconsistent with this view. Most of these models predict exactly the opposite result that the anticipated inflation effect dominates the liquidity effect.

An exception to the above is the limited participation model which is capable of generating a negative response of nominal interest rates to positive monetary growth shocks. This model derives its name from the limited ability of household's to participate in financial markets. Only firms and financial intermediaries interact directly in financial markets after monetary injections. When the monetary authority injects cash into the economy, this cash is distributed to financial intermediaries which then lend it to firms. To persuade firms to accommodate the new cash injections, the nominal interest rate has to fall.

Although limited participation models can explain the fall in nominal interest rates in response to positive monetary shocks, they find it difficult to explain the persistence of this effect. To do so, it is necessary to add other features to the model. For example, Christiano and Eichenbaum (1992) introduce small costs of adjusting sectoral flows of funds, while Hendry and Zhang (1998) consider frictions in the adjustment of prices, wages and portfolios. In the analysis that follows, we consider another alternative suggested, but not pursued, by Christiano (1991). This is the introduction of habit

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persistence in the consumption and labour supply decisions of agents. We model this by assuming that past levels of consumption and leisure affect the current utility of agents. This implies that the marginal utilities of current consumption and leisure are increasing functions of lagged values of these variables. We consider five different versions of a limited participation model which differ according to the nature and extent of habit persistence. The first model is the benchmark case in which there is no habit persistence. The second and third models focus on habit persistence in only consumption and only leisure, respectively. The fourth model allows for habit persistence in both consumption and leisure. And the fifth model allows habit persistence in consumption to extend for more than one period.

These models are able to generate a persistent fall in nominal interest rates in response to a positive monetary shock. The degree of persistence depends on the nature of habit formation in household preferences.

## 2. The Model Economies

The basic structure of each of our model economies is given by a simplified version of the limited participation model of Christiano et al (1998). The difference between each model lies in the specification of the utility function of agents who may display habit persistence towards their consumption and leisure choices.

### 2.1 Economic Environment

The economy is populated by three types of economic agents: households, goods-producing firms and financial intermediaries. There is also a monetary authority which is responsible for setting the rate of growth of the money supply. Agents of each type are identical and all markets are perfectly competitive.

At the beginning of each period  $t$ , households are in possession of the economy's entire stock of money. During each period, households lend part of their money holding to financial intermediaries (which relend to firms) and spend the rest on consumption. In addition, the monetary authority injects new money into the economy through the financial intermediaries. The flow of money from households, financial intermediaries, and the monetary authority to firms is displayed in Figure 1.

The money from firms and financial intermediaries flows back to households at the end of the period. Households receive their wage payments from firms and their dividends from both firms and financial intermediaries (both of which are owned by households). The dividend payments of financial intermediaries are generated from the profits from lending the cash injection of the monetary authority to firms. Households also receive repayments of loans from firms through financial intermediaries in the form of interest and principal payments. This is because financial intermediaries borrow from households and lend to firms. The circulation of money back to households is displayed in Figure 2.

### 2.2 Households

At the start of period  $t$ , the representative household possesses the economy's entire (beginning-of-period) money stock,  $M_t$ . The household allocates  $Q_t$  dollars to purchase the consumption good  $C_t$  and lends the rest  $M_t - Q_t$  to financial intermediaries. Consumption and investment expenditures,  $P_t(C_t + I_t)$  must be fully financed with cash from two sources:  $Q_t$  and current period wage earnings,  $W_t N_t$ , where  $W_t$  is the nominal wage rate and  $N_t$  is time devoted to work. Investment,  $I_t$ , produces capital,  $K_{t+1}$ , according to

$$K_{t+1} = I_t + (1-\delta)K_t \quad (1)$$

for  $0 < \delta < 1$ . In addition, the household faces the following cash-in-advance constraint

$$P_t(C_t + K_{t+1} - (1-\delta)K_t) \leq Q_t + W_t N_t \quad (2)$$

The household has four sources of money at the beginning of each period. The first is labour income,  $W_t N_t$ . The second is the interest earnings on cash loans,  $R_t(M_t - Q_t)$ , and the lump sum profits from financial intermediaries,  $R_t X_t$ , where  $R_t$  is the gross interest rate and  $X_t$  is the lump sum injections from the monetary authority. The third source is the profits received from firms,  $D_t$ . The fourth source is the capital rental income,  $r_t K_t$ . The fact that capital rental income appears only in the budget constraint and not in the cash-in-advance constraint indicates that this income is received at the end of the period. Each household faces the following budget constraint:

$$M_{t+1} \leq R_t(M_t - Q_t) + R_t X_t + D_t + r_t K_t + [Q_t + W_t N_t - P_t(C_t + K_{t+1} - (1-\delta)K_t)] \quad (3)$$

The household's choices of  $C_t$ ,  $N_t$ ,  $M_{t+1}$  and  $K_{t+1}$  are constrained to be functions of variables dated at time  $t$  and earlier. The choice of  $Q_t$  is constrained to be a function of variables dated at  $t-1$  and earlier which is consistent with the limited participation assumption. Because households are limited in their ability to participate in financial markets, they are unable to adjust the money set aside to purchase consumption goods,  $Q_t$ , following a monetary shock.

The information sets of the household are given as  $\Omega_{t-1}$ ,  $\Omega_t^Q$  and  $\Omega_t$ , and are defined as follows:  $\Omega_{t-1}$  includes all variables dated  $t-1$  and earlier.  $\Omega_t^Q$  includes  $\Omega_{t-1}$  and  $z_t$ .  $\Omega_t$  includes  $\Omega_t^Q$  and  $x_t$ .  $z_t$  and  $x_t$  are the state of technology at time  $t$  and the growth rate of the money supply at time  $t$  respectively.

The representative household's expected lifetime utility for the benchmark model (i.e. the model without any habit persistence) is

$$E \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \quad (4)$$

$$U = \ln C_t + \eta(T - N_t), \quad T = L_t + N_t \quad (5)$$

where  $T$  denotes total time available and  $L_t$  denotes the quantity of leisure time. Each of our other models is based on a modified version of (4) and (5) to allow for habit persistence.

In model 1 - the pure consumption habit persistence model - the household's preferences are characterised as

$$E \sum_{t=0}^{\infty} \beta^t U(C_t, C_{t-1}, L_t), \quad (6)$$

$$U = \ln(C_t - b_1 C_{t-1}) + \eta(T - N_t), \quad T = L_t + N_t \quad (7)$$

Habit persistence is captured by  $b_1 > 0$ , which implies that current period utility depends (negatively) on previous period consumption. Equivalently, the marginal utility of current consumption depends (positively) on last period's consumption. Thus higher consumption in the past motivates higher consumption today, in accordance with the idea of habit formation.

In model 2 - the pure leisure habit persistence model - we have

$$E \sum_{t=0}^{\alpha} \beta^t U(C_t, L_t, L_{t-1}), \quad (8)$$

$$U = \ln C_t + \eta(T - N_t - c_1(T - N_{t-1})), \quad T = L_t + N_t \quad (9)$$

In this case, habit persistence is captured by  $c_1 > 0$ , implying that current period utility depends on previous period leisure. Analogously to the above, higher leisure in the past motivates higher leisure today, reflecting habitual behaviour.

Model 3 is the joint consumption-leisure habit persistence model, being a combination of model 1 and 2. Expected lifetime utility in this case is given by

$$E \sum_{t=0}^{\alpha} \beta^t U(C_t, C_{t-1}, L_t, L_{t-1}), \quad (10)$$

$$U = \ln(C_t - b_1 C_{t-1}) + \eta(T - N_t - c_1(T - N_{t-1})), \quad T = L_t + N_t \quad (11)$$

Finally, we have model 4 which allows for two period habit persistence in consumption. In this case,

$$E \sum_{t=0}^{\alpha} \beta^t U(C_t, C_{t-1}, C_{t-2}, L_t), \quad (12)$$

$$U = \ln(C_t - b_1 C_{t-1} - b_2 C_{t-2}) + \eta(T - N_t), \quad T = L_t + N_t \quad (13)$$

where  $b_2 > 0$  captures the dependence of current period utility on the level of consumption two periods ago.

### 2.3 Firms

The representative firm hires labour,  $N_t$ , at the nominal wage rate,  $W_t$ , and rents capital,  $K_t$ , at the rental price  $r_t$ . To finance the nominal wage bill,  $W_t N_t$ , the firm must borrow cash from financial intermediaries at the interest rate  $R_t$ . Unlike labour, capital is assumed to be a credit good so that the firm does not need to borrow funds from the financial intermediary to finance investment.

The firm uses capital and labour to produce output,  $Y_t$ , according to the following technology

$$Y_t = f(z_t, K_t, N_t) = z_t K_t^\alpha N_t^{1-\alpha} \quad (14)$$

where  $z_t$  is the state of technology at time  $t$  and evolves according to the following law of motion

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t} \quad (15)$$

where  $\varepsilon_{z,t}$  is a serially uncorrelated i.i.d. random variable with standard deviation  $\sigma_{\varepsilon,z}$ .

The firm sells its output at the competitive market price  $P_t$  and then pays dividends  $D_t$  to its shareholders. The dividends paid out equal its cash receipts  $P_t Y_t$  minus its cash outlays:

$$D_t = P_t Y_t - R_t W_t N_t - r_t K_t. \quad (16)$$

## 2.4 Financial Intermediaries

The representative financial intermediary has two sources of funds: deposits from households,  $M_t - Q_t$ , and lump sum cash injections from the monetary authority,  $X_t$ . These funds are lent on the loan market at the gross rate of interest  $R_t$ . The demand for loans comes from firms who need to finance the nominal wage bill,  $W_t N_t$ . The loan market clearing condition is given by:

$$W_t N_t = M_t - Q_t + X_t. \quad (17)$$

At the end of the period, the financial intermediary distributes  $R_t X_t$  in the form of profits and pays  $R_t(M_t - Q_t)$  in return for the deposits from households.

## 2.5 Monetary Growth

The money supply evolves according to  $M_{t+1} = M_t + X_t$ . The growth rate of the money supply is therefore  $(M_{t+1} - M_t) / M_t = X_t / M_t = x_t$ . Following Christiano and Eichenbaum (1991), we assume that this growth rate is governed by the following stochastic process

$$x_t = \rho_x x_{t-1} + \varepsilon_{x,t} \quad (18)$$

where  $\varepsilon_{x,t}$  is a serially uncorrelated i.i.d. random variable with standard deviation  $\sigma_{\varepsilon,x}$ .

## 3. Solving the Models

The method employed for solving each of our models is the undetermined coefficients method which is used for obtaining linear approximations to the solution of dynamic, rational expectations models. According to Christiano (1998), a distinguishing characteristic of this method is that it can accommodate a wide class of models in which different decisions are taken on the basis of different information sets. In our model, the money set aside by households to purchase consumption goods is constrained to be a function of variables dated  $t-1$  and earlier but other variables are constrained to be a function of date  $t$  and earlier.

In what follows, we describe the first order conditions associated with the household's optimization problem as follows.

For  $K_{t+1}$  :

$$E \left[ \frac{U_{C,t}(C_t, T - N_t)}{P_t} P_t - \beta \left[ \frac{U_{C,t+1}(C_{t+1}, T - N_{t+1})}{P_{t+1}} (1-\delta) P_{t+1} + \beta \tilde{\Lambda}_{t+2} r_{t+1} \right] \middle| \Omega_t \right] = 0. \quad (19)$$

For  $Q_t$  :

$$E \left[ \tilde{\Lambda}_t - \beta R_t \tilde{\Lambda}_{t+1} \middle| \Omega_t^0 \right] = 0. \quad (20)$$

For  $N_t$  :

$$U_{N,t}(C_t, T - N_t) + U_{C,t}(C_t, T - N_t) \frac{W_t}{P_t} = 0. \quad (21)$$

We now turn to the maximisation problem of firms. A firm maximises dividends, subject to the production function, with respect to  $N_t$  and  $K_t$ . The first order conditions are

For  $N_t$  :

$$\frac{W_t R_t}{P_t} = f_{N,t} \quad (22)$$

For  $K_t$  :

$$\frac{r_t}{P_t} = f_{K,t} \quad (23)$$

where  $f_{N,t} = \partial f(z_t, K_t, N_t) / \partial N_t$  and  $f_{K,t} = \partial f(z_t, K_t, N_t) / \partial K_t$ .

Having established the optimality conditions for both households and firms, it is now necessary to re-scale some variables in order to make them stationary. For this purpose, we define the following:

$$\lambda_t = \tilde{\Lambda}_t M_t, q_t = Q_t / M_t, p_t = P_t / M_t, w_t = W_t / M_t, 1+x_t = M_{t+1} / M_t. \quad (24)$$

Given the above, we may rewrite the households' and firms' optimality conditions as

$$H_K = E \left[ \begin{array}{l} U_{C,t}(C_t, T - N_t) - \beta U_{C,t+1}(C_{t+1}, T - N_{t+1})(1-\delta) \\ - \beta^2 U_{C,t+2}(C_{t+2}, T - N_{t+2}) p_{t+2}^{-1} f_{K,t+2} p_{t+1} \frac{1}{(1+x_{t+1})} \end{array} \middle| \Omega_t \right] = 0, \quad (25)$$

$$H_\varrho = E \left[ U_{C,t} (C_t, T - N_t) p_t^{-1} - \beta U_{C,t+1} (C_{t+1}, T - N_{t+1}) p_{t+1}^{-1} \frac{R_t}{(1+x_t)} \middle| \Omega_t^\varrho \right] = 0, \quad (26)$$

$$H_N = U_{N,t} (C_t, T - N_t) + U_{C,t} (C_t, T - N_t) \frac{w_t}{p_t} = 0, \quad (27)$$

$$\frac{w_t R_t}{p_t} = f_{N,t} \quad (28)$$

$$\frac{r_t}{p_t} = f_{K,t}. \quad (29)$$

In addition to the above, we have the resource constraint, the cash-in-advance constraint, and the loan market equilibrium condition. After appropriate re-scaling these relationships may be rewritten respectively, as

$$z_t K_t^\alpha N_t^{1-\alpha} = C_t + K_{t+1} - (1-\delta) K_t, \quad (30)$$

$$p_t (C_t + K_{t+1} - (1-\delta) K_t) = 1 + x_t, \quad (31)$$

$$w_t N_t = 1 - q_t + x_t. \quad (32)$$

The expressions (25)- (32) define 8 equations in 8 variables ( $C_t, N_t, P_t, R_t, w_t, K_t, q_t, r_t$ ). Before analysing this system, we describe briefly the main differences in our other models with habit persistence.

In the habit persistence model 1, where utility is given by (7), the only difference from the benchmark case relates to the household's first order condition for consumption which is now given by

$$U_{C,t} (C_t - b_1 C_{t-1}, T - N_t) + \beta U_{C,t} (C_{t+1} - b_1 C_t, T - N_{t+1}) = (\nu_t + \mu_t) P_t. \quad (33)$$

The H functions in (25)-(27) are then replaced by

$$H_K = E \left[ \begin{array}{l} U_{C,t} (C_t - b_1 C_{t-1}, T - N_t) + \beta U_{C,t} (C_{t+1} - b_1 C_t, T - N_{t+1}) \\ - \beta U_{C,t+1} (C_{t+1} - b_1 C_t, T - N_{t+1}) (1-\delta) - \beta^2 U_{C,t+1} (C_{t+2} - b_1 C_{t+1}, T - N_{t+2}) (1-\delta) \\ - \beta^2 U_{C,t+2} (C_{t+2} - b_1 C_{t+1}, T - N_{t+2}) p_{t+2}^{-1} \frac{1}{(1+x_{t+1})} f_{K,t+1} p_{t+1} \\ - \beta^3 U_{C,t+2} (C_{t+3} - b_1 C_{t+2}, T - N_{t+3}) p_{t+2}^{-1} \frac{1}{(1+x_{t+1})} f_{K,t+1} p_{t+1} \end{array} \middle| \Omega_t^\varrho \right] = 0, \quad (34)$$

$$H_Q = E \left[ \begin{array}{c} U_{C,t} (C_t - b_1 C_{t-1}, T - N_t) p_t^{-1} + \beta U_{C,t} (C_{t+1} - b_1 C_t, T - N_{t+1}) p_t^{-1} \\ - \beta U_{C,t+1} (C_{t+1} - b_1 C_t, T - N_{t+1}) \frac{R_t}{(1+x_t) p_{t+1}} \\ - \beta^2 U_{C,t+1} (C_{t+2} - b_1 C_{t+1}, T - N_{t+2}) \frac{R_t}{(1+x_t) p_{t+1}} \end{array} \middle| \Omega_t^Q \right] = 0, \quad (35)$$

$$H_N = E \left[ \begin{array}{c} U_{N,t} (C_t - b_1 C_{t-1}, T - N_t) + \beta U_{C,t} (C_{t+1} - b_1 C_t, T - N_{t+1}) \frac{w_t}{p_t} \\ - \beta U_{C,t} (C_{t+1} - b_1 C_t, T - N_{t+1}) \frac{w_t}{p_t} \end{array} \middle| \Omega_t^Q \right] = 0. \quad (36)$$

In the habit persistence model 2, where utility is given by, (9), the only difference from the benchmark case is the household's first order condition with respect to leisure. This is now given by

$$U_{N,t} (C_t, T - N_t - c_1 (T - N_{t-1})) + \beta U_{N,t} (C_{t+1}, T - N_{t+1} - c_1 (T - N_t)) + (\nu_t + \mu_t) W_t = 0. \quad (37)$$

The H functions become

$$H_K = E \left[ \begin{array}{c} U_{C,t} (C_t, T - N_t - c_1 (T - N_{t-1})) - \beta U_{C,t+1} (C_{t+1}, T - N_{t+1} - c_1 (T - N_t)) (1-\delta) \\ - \beta^2 (U_{C,t+1} (C_{t+2}, T - N_{t+2} - c_1 (T - N_{t+1}))) p_{t+2}^{-1} \frac{1}{(1+x_{t+1})} f_{K,t+1} p_{t+1} \end{array} \middle| \Omega_t^Q \right] = 0, \quad (38)$$

$$H_Q = E \left[ \begin{array}{c} U_{C,t} (C_t, T - N_t - c_1 (T - N_{t-1})) p_t^{-1} \\ - \beta \frac{R_t}{(1+x_t)} U_{C,t+1} (C_{t+1}, T - N_{t+1} - c_1 (T - N_t)) p_t^{-1} \end{array} \middle| \Omega_t^Q \right] = 0, \quad (39)$$

$$H_N = \left[ \begin{array}{c} U_{N,t} (C_t, T - N_t - c_1 (T - N_{t-1})) + \beta U_{N,t} (C_{t+1}, T - N_{t+1} - c_1 (T - N_t)) \\ + U_{C,t} (C_t, T - N_t - c_1 (T - N_{t-1})) \frac{w_t}{p_t} \end{array} \right] = 0. \quad (40)$$

Naturally, in the habit persistence model 3, where utility is given by (11), the first-order conditions for both consumption and leisure are changed, being given by (33) and (37). The H functions in this case are

$$H_K = E \left[ \begin{array}{l} U_{C,t} (C_t - b_1 C_{t-1}, T - N_t - c_1 (T - N_{t-1})) \\ + \beta U_{C,t} (C_{t+1} - b_1 C_t, T - N_{t+1} - c_1 (T - N_t)) \\ - \beta U_{C,t+1} (C_{t+1} - b_1 C_t, T - N_{t+1} - c_1 (T - N_t)) (1 - \delta) \\ - \beta^2 U_{C,t+2} (C_{t+2} - b_1 C_{t+1}, T - N_{t+2} - c_1 (T - N_{t+1})) (1 - \delta) \\ - \beta^2 U_{C,t+2} (C_{t+2} - b_1 C_{t+1}, T - N_{t+2} - c_1 (T - N_{t+1})) p_{t+2}^{-1} \frac{1}{(1+x_{t+1})} f_{K,t+1} p_{t+1} \\ - \beta^3 U_{C,t+2} (C_{t+3} - b_1 C_{t+2}, T - N_{t+3} - c_1 (T - N_{t+2})) p_{t+2}^{-1} \frac{1}{(1+x_{t+1})} f_{K,t+1} p_{t+1} \end{array} \right] = 0, \quad (41)$$

$$H_Q = E \left[ \begin{array}{l} U_{C,t} (C_t - b_1 C_{t-1}, T - N_t - c_1 (T - N_{t-1})) \\ + \beta U_{C,t} (C_{t+1} - b_1 C_t, T - N_{t+1} - c_1 (T - N_t)) p_t^{-1} \\ - \beta U_{C,t+1} (C_{t+1} - b_1 C_t, T - N_{t+1} - c_1 (T - N_t)) \frac{R_t}{(1+x_t) p_{t+1}} \\ - \beta^2 U_{C,t+1} (C_{t+2} - b_1 C_{t+1}, T - N_{t+2} - c_1 (T - N_{t+1})) \frac{R_t}{(1+x_t) p_{t+1}} \end{array} \right] = 0, \quad (42)$$

$$H_N = E \left[ \begin{array}{l} U_{N,t} (C_t - b_1 C_{t-1}, T - N_t - c_1 (T - N_{t-1})) + \beta U_{N,t} (C_{t+1} - b_1 C_t, T - N_{t+1} - c_1 (T - N_t)) \\ + U_{C,t} (C_t - b_1 C_{t-1}, T - N_t - c_1 (T - N_{t-1})) \frac{w_t}{p_t} + \beta U_{C,t} (C_{t+1} - b_1 C_t, T - N_{t+1} - c_1 (T - N_t)) \frac{w_t}{p_t} \end{array} \right] = 0. \quad (43)$$

Finally, in the habit persistence model 4, with utility defined by (13), the first-order condition reads

$$U_{C,t} (C_t - b_1 C_{t-1} - b_2 C_{t-2}, T - N_t) + \beta U_{C,t} (C_{t+1} - b_1 C_t - b_2 C_{t-1}, T - N_{t+1}) + \beta^2 U_{C,t} (C_{t+2} - b_1 C_{t+1} - b_2 C_t, T - N_{t+2}) = (v_t + \mu_t) P_t \quad (44)$$

and the H functions become

$$H_K = E \left[ \begin{array}{l} U_{C,t} (C_t - b_1 C_{t-1} - b_2 C_{t-2}, T - N_t) + \beta U_{C,t} (C_{t+1} - b_1 C_t - b_2 C_{t-1}, T - N_{t+1}) \\ + \beta^2 U_{C,t} (C_{t+2} - b_1 C_{t+1} - b_2 C_t, T - N_{t+2}) - \beta U_{C,t+1} (C_{t+1} - b_1 C_t - b_2 C_{t-1}, T - N_{t+1}) (1 - \delta) \\ - \beta^2 U_{C,t+1} (C_{t+2} - b_1 C_{t+1} - b_2 C_t, T - N_{t+2}) (1 - \delta) - \beta^3 U_{C,t+2} (C_{t+3} - b_1 C_{t+2} - b_2 C_{t+1}, T - N_{t+3}) (1 - \delta) \\ - \beta^2 U_{C,t+2} (C_{t+2} - b_1 C_{t+1} - b_2 C_t, T - N_{t+2}) \frac{1}{(1+x_{t+1}) p_{t+2}} f_{K,t+1} p_{t+1} \\ - \beta^3 U_{C,t+2} (C_{t+3} - b_1 C_{t+2} - b_2 C_{t+1}, T - N_{t+3}) \frac{1}{(1+x_{t+1}) p_{t+2}} f_{K,t+1} p_{t+1} \\ - \beta^4 U_{C,t+2} (C_{t+4} - b_1 C_{t+3} - b_2 C_{t+2}, T - N_{t+4}) \frac{1}{(1+x_{t+1}) p_{t+2}} f_{K,t+1} p_{t+1} \end{array} \right] = 0, \quad (45)$$

$$H_Q = E \left[ \begin{array}{l} U_{C,t} (C_t - b_1 C_{t-1} - b_2 C_{t-2}, T - N_t) p_t^{-1} \\ + \beta U_{C,t} (C_{t+1} - b_1 C_t - b_2 C_{t-1}, T - N_{t+1}) p_t^{-1} \\ + \beta^2 U_{C,t} (C_{t+2} - b_1 C_{t+1} - b_2 C_t, T - N_{t+2}) p_t^{-1} \\ - \beta U_{C,t+1} (C_{t+1} - b_1 C_t - b_2 C_{t-1}, T - N_{t+1}) \frac{R_t}{(1+x_{t+1}) p_{t+1}} \\ - \beta^2 U_{C,t+1} (C_{t+2} - b_1 C_{t+1} - b_2 C_t, T - N_{t+2}) \frac{R_t}{(1+x_{t+1}) p_{t+1}} \\ - \beta^3 U_{C,t+1} (C_{t+3} - b_1 C_{t+2} - b_2 C_{t+1}, T - N_{t+3}) \frac{R_t}{(1+x_{t+1}) p_{t+1}} \end{array} \right] \Omega_t^Q = 0, \quad (46)$$

$$H_N = E \left[ \begin{array}{l} U_{N,t} (C_t - b_1 C_{t-1} - b_2 C_{t-2}, T - N_t) + U_{C,t} (C_t - b_1 C_{t-1} - b_2 C_{t-2}, T - N_t) \frac{w_t}{p_t} \\ + \beta U_{C,t} (C_{t+1} - b_1 C_t - b_2 C_{t-1}, T - N_{t+1}) \frac{w_t}{p_t} + \beta^2 U_{C,t} (C_{t+2} - b_1 C_{t+1} - b_2 C_t, T - N_{t+2}) \frac{w_t}{p_t} \end{array} \right] = 0. \quad (47)$$

#### 4. The Qualitative Properties of the Models

Although a complete analysis of the models requires numerical simulations, it is possible to identify some key properties and implications from an analytical investigation. This is based on the specific functional forms for the utility and production functions as given in equations (7), (9), (11), (13), and (14).

##### 4.1 The Benchmark Model

The main ingredient for generating a liquidity effect in this model is the assumed rigidity in  $Q_t$  (the amount of cash used for purchasing consumption goods). This assumption is made to prevent the proportional distribution of an increase in the money supply among all agents. The surprise lump sum cash injections by the monetary authority is distributed directly to the financial intermediaries which lend this money to firms. The financial intermediaries will lend all this cash to the firms as long as the interest rate is greater than one. For firms to absorb the cash voluntarily, the interest rate must fall which is the liquidity effect in the model. At the same time, the new cash injections will trigger an expected increase in money growth, which leads to the anticipated inflation effect in the model. However, it has already been proved in many works that a limited participation model can generate the dominant liquidity effect and it is, also, shown in the quantitative section in this paper.

We now turn to the responses of employment and output. In figure 3 we plot the labour demand curve (equation (22)) and the labour supply curve (equation (21)). Like the standard Real Business Cycle (RBC) model, an increase in the capital stock or a positive technology shock shifts the labour demand curve to the right. In contrast to the standard RBC model, a fall in the nominal interest rate shifts the labour demand to the

right. This is because firms equate the marginal product of labour to the real cost of hiring labour, taking into account the cost of working capital. A fall in the nominal interest rate decreases the cost of working capital and raises the demand for labour. Due to the domination of the liquidity effect over the anticipated inflation effect, a positive money shock leads to a fall in the nominal interest rate. As a consequence, the labour demand curve shifts to the right ( $Ld'$ ). A positive money shock also leads to an increase in consumption due to the wealth effect. This has the effect of shifting the labour supply curve to the left ( $Ls'$ ). The net result of these movements is that employment increases from  $N$  to  $N1$ , leading to an increase in output as well.

#### 4.2 The Habit Persistence Model 1

The main consequence of introducing habit persistence in consumption is to increase the persistence of nominal interest rate responses to monetary shocks. As mentioned earlier, this persistence is a feature of the data but difficult to generate in standard RBC models, as well as the benchmark model of limited participation.

To see how habit persistence produces persistence in nominal interest fluctuations, we draw the labour demand and labour supply schedules as before. The former is given by (22) while the latter is given by

$$\eta \left[ \frac{(C_t - b_1 C_{t-1})(C_{t+1} - b_1 C_t)}{(C_{t+1} - b_1(1+\beta)C_t + b_1^2 \beta C_{t-1})} \right] = \frac{w_t}{p_t}. \quad (48)$$

These curves are depicted in figure 4.

Equation (48) is different from the benchmark case because the marginal utility of consumption at date  $t$  depends not only on consumption at date  $t$  but also on consumption at date  $t-1$ . As noted by Dynan (2000), this means that the net marginal cost of foregoing one unit of consumption in period  $t$  is a function of the level of consumption in periods  $t-1$ ,  $t$ , and  $t+1$ . This introduces a degree of sluggishness into the dynamic of consumption.

As before, with the assumption that  $Q_t$  is constrained to be a function of variables dated at  $t-1$  and earlier, the liquidity effect dominates the anticipated inflation effect so that the nominal interest rate drops and employment, output and investment increase in response to a positive monetary shock. In the current model, however, the sluggishness in consumption means that the labour supply schedule shifts only gradually back to  $Ls''$  and  $Ls'''$ . At the same time labour demand shifts gradually back from  $Ld'$  to  $Ld''$  to  $Ld'''$  as capital starts to decrease. After its initial increase from  $N$  to  $N1$  employment gradually moves back to  $N2$  and  $N3$ . These movements in employment imply that the fall in the nominal interest rate will display persistence.

#### 4.3 The Habit Persistence Model 2

When there is habit persistence in only leisure, the labour supply function becomes

$$\eta(1 - \beta c_1) C_t = \frac{w_t}{p_t}. \quad (49)$$

This is exactly the same as in the benchmark model (equation(21)) in the sense that it depends only current (not lagged nor future) consumption. As such, it may be expected that this model, like the benchmark model, can not generate a persistent drop in nominal interest rate.

#### 4.4 The Habit Persistence Model 3

With habit persistence in both consumption and leisure, the labour supply function reads

$$\eta(1-\beta c_t) \left[ \frac{(C_t - b_1 C_{t-1})(C_{t+1} - b_1 C_t)}{(C_{t+1} - b_1(1+\beta)C_t + b_1^2 \beta C_{t-1})} \right] = \frac{w_t}{p_t}. \quad (50)$$

Naturally, this equation is essentially a combination of equation (48) and (49) derived from the pure consumption and pure leisure habit persistence models. As in the former model, therefore, it is to be expected that the nominal interest rate fluctuation will display persistence due to the sluggishness in consumption.

#### 4.5 The Habit Persistence Model 4

Finally, we turn to the model in which habit persistence in consumption extends for two, rather than just one period. In this case, the labour supply function is

$$\frac{\eta \left[ (C_t - b_1 C_{t-1} - b_2 C_{t-2})(C_{t+1} - b_1 C_t - b_2 C_{t-1})(C_{t+2} - b_1 C_{t+1} - b_2 C_t) \right]}{\left[ (C_{t+1} - b_1 C_t - b_2 C_{t-1})(C_{t+2} - b_1 C_{t+1} - b_2 C_t) \right.} = \frac{w_t}{p_t} \quad (51)$$

$$\left. - \beta b_1 (C_t - b_1 C_{t-1} - b_2 C_{t-2})(C_{t+1} - b_1 C_t - b_2 C_{t-1}) \right]$$

$$\left. - \beta^2 b_1 (C_{t+1} - b_1 C_t - b_2 C_{t-1})(C_{t+2} - b_1 C_{t+1} - b_2 C_t) \right]$$

Labour supply is now seem to be a function of consumption at dates  $t-2$ ,  $t-1$ ,  $t$ ,  $t+1$  and  $t+2$ . Accordingly we would expect the model to display a greater degree of persistence in the nominal interest rate in response to monetary shocks.

### 5. Quantitative Analysis

#### 5.1 Parameter Values

To analyse the quantitative properties of the models, we need to assign values to the models' parameters. In doing this we assume that the time period in each model corresponds to one quarter.

Nearly all the parameter values are based on Christiano et al (1998). In all five models, the main structural parameters consist of  $\beta$ ,  $\delta$ ,  $\alpha$ ,  $\rho_z$ ,  $\sigma_{z,z}$ ,  $\rho_x$ ,  $\sigma_{x,x}$ ,  $\eta$ , together with the parameters describing habit persistence, where appropriate. The latter are given by  $b_1$ ,  $b_2$  and  $c_1$ .

The capital share of aggregate output,  $\alpha$ , is set equal to 0.4 based on Cooley and Prescott (1995). The autoregressive coefficient in monetary growth,  $\rho_x$ , is set equal to 0.65 which implies that in the benchmark model, the effect of a positive monetary shock is to cause the nominal interest to fall and to return to its steady state value within one period. The rate of capital depreciation,  $\delta$ , the discount factor,  $\beta$ , and the standard deviation of the shocks,  $\sigma_z$  and  $\sigma_x$  are taken directly from Christiano et al (1998).

In the habit persistence models 1 and 3, the habit persistence parameter on consumption,  $b_1$  is set equal to 0.63, as in Christiano et al (2001). In models 2 and 3, the habit persistence parameter on leisure is set equal to 0.5 as in Yum (1996). In model 4, the parameters determining one period and two period habit persistence in consumption are both set equal to 0.3 (which satisfies the restriction  $b_1 + b_2 < 1$ ).

A summary of parameter values is given in the table below

$\alpha$	$\delta$	$\beta$	$\rho_z$	$\sigma_{e,z}$	$\rho_{e,x}$	$\sigma_x$
0.40	0.20	$1.03^{-.25}$	0.9857	0.1369	0.65	0.0041

**Table 1** Common Parameter Values in All Models.

$b_1$	$c_1$
0.63	0.5

**Table 2** Parameter Values for Habit Persistence Models 1, 2, and 3.

$b_1$	$b_2$
0.3	0.3

**Table 3** Parameter Values for Habit Persistence Model 4.

Given the above parameter values, the utility weight on leisure  $\eta$ , is then calibrated for each model to yield a steady state level of employment equal to 0.31.

## 5.2 The Quantitative Results

Having set the parameter values, the models can be solved numerically by using Christiano's method in the Matlab programme to linearise the H functions and obtain the decision rules. The models are then simulated and analysed through a discussion of the impulse responses of the nominal interest rate, output, labour, and consumption to a one standard deviation money growth shock,  $x_t$ , in period 2.

Figure 5 displays the impulse response of the nominal interest rate to a positive monetary shock  $x_t$  in period 2 in all five models. In each case, the interest rate falls as a result of the dominance of the liquidity effect over the anticipated inflation effect. The main difference between the models is the persistence of the response.

By construction, the benchmark model does not generate any persistence at all. With the exception of model 2, all the other models with habit persistence in consumption are capable of generating some degree of persistence in interest rate fluctuations. The exceptional case of model 2 arises from the fact that the labour supply equation, (49), is almost identical to the labour supply equation of the benchmark, (21). In particular, this equation depends only on current period variables, not lagged or future variables (as in the other models). Consequently, there is none of the sluggishness in labour and consumption which can generate a persistent drop in nominal interest. As a result, the implications of model 2 are essentially the same as the implications of the benchmark model. Similarly the implications of model 3 (where there is habit persistence in both

consumption and leisure) are essentially the same as the implications of model 1 (where there is habit persistence in only consumption). It should be noted however that this is largely the result of the fact that utility is linear in leisure. If this were not the case, then current labour supply decisions would be affected by past labour supply decisions such that persistence would be generated.

In model 1, it takes 5 quarters for the nominal interest rate to return to its steady state value following a monetary shock. As expected, this persistence is increased in model 4 where habit persistence in consumption extends for two (rather than just one) periods. In the period after the shock, the interest rate moves back by two thirds towards its steady state value and eventually reaches that value after 7 quarters. With two period habit persistence, labour supply and consumption are more sluggish as compared with one period habit persistence.

Figure 6 shows the impulse response of consumption to the positive monetary shock in each of the models. For the reason given above, the pattern and the magnitude of the response are the same in the benchmark model and the pure leisure habit persistence model. In response to the shock, consumption increases to about 0.06 percent above its steady state and then decreases to 0.02 percent in the subsequent period. Thereafter, it falls below its steady state value. These models generate the largest response of consumption because consumption is not smoothed by the effect of habit persistence.

Again, for reasons given earlier, the pattern and the magnitude of the response of consumption in models 1 and 3 are also the same. In comparison to the previous two models, however, the magnitude of the response is much less. Consumption initially increases to 0.004 percent above its steady state and then gradually declines to approximately 0.01 below its steady state. This illustrates the consumption smoothing effect of habit persistence. As expected, the response of consumption in model 4 is smoother than in any of the previous models. The magnitude of the response is not very different from models 1 and 3, peaking at 0.002 percent above the steady state and then gradually declining to approximately -0.006.

The impulse responses of employment to a positive monetary shock  $x_t$  are shown on figure 7. As a consequence of the limit participation feature, a positive monetary shock leads to an increase in employment  $N_t$ . However, the magnitude of the response is different in each model.

In the case of the benchmark model and model 2, the response of employment are the same as a result of the reasons mentioned before. A positive monetary shock leads to an increase in employment in period 2. It then falls below its steady state value in period 3 and gradually moves back to its steady state by period 10. The magnitude of the response of employment is the strongest in these two models. This is because the labour supply equation in these models depends only on consumption in period  $t$ . Since consumption is relatively unsmoothed and remains high, the employment response is higher than other models.

The response of employment in models 1 and 3 are also the same. This is due to the similarity of labour supply in these models. As noted above, the magnitude of the employment response is lower than the benchmark model and model 2 because of greater consumption smoothing due to habit persistence in consumption.

As expected, the pattern of the response of employment in model 4 is the same as the other models but its magnitude is the lowest among the five models. This comes from the same reason as before.

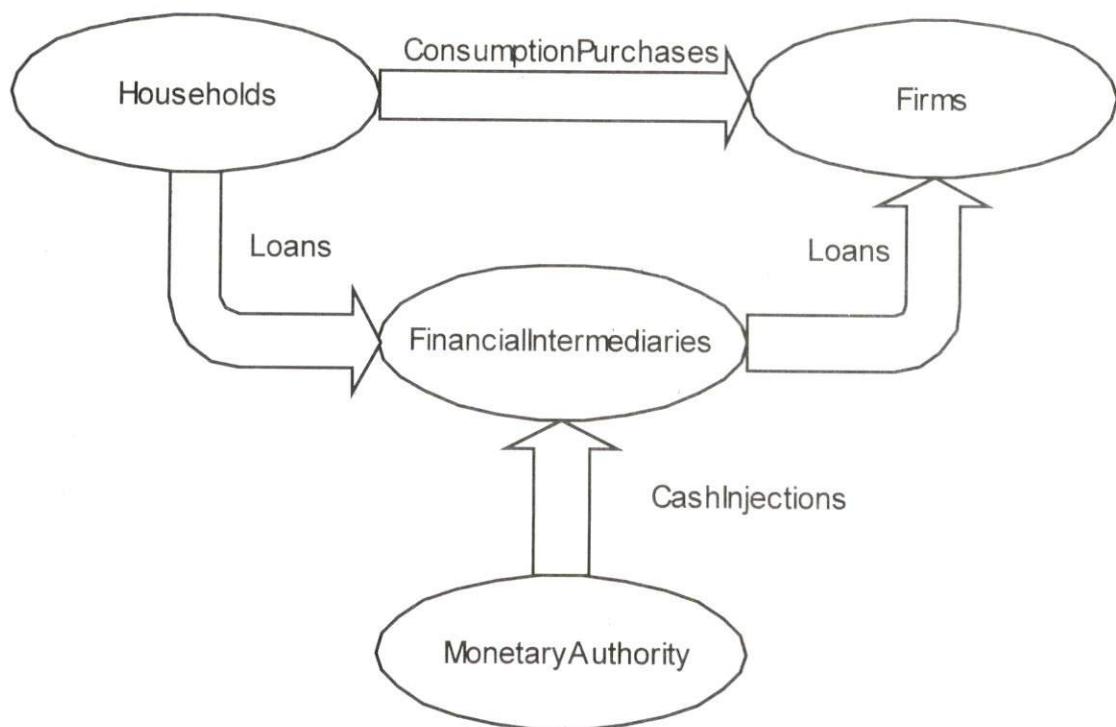
Figure 8 shows the impulse response of output. With the drop in the nominal interest rate, output increases in response to a positive monetary shock.

In the case of the benchmark model and model 2, the responses are the same and are the highest among the models as a consequence of the highest increase in employment. Output increases to nearly 0.2 percent above the steady state and then drops to approximately -0.07 below the steady state. It then gradually declines to -0.0049 below steady state by period 10.

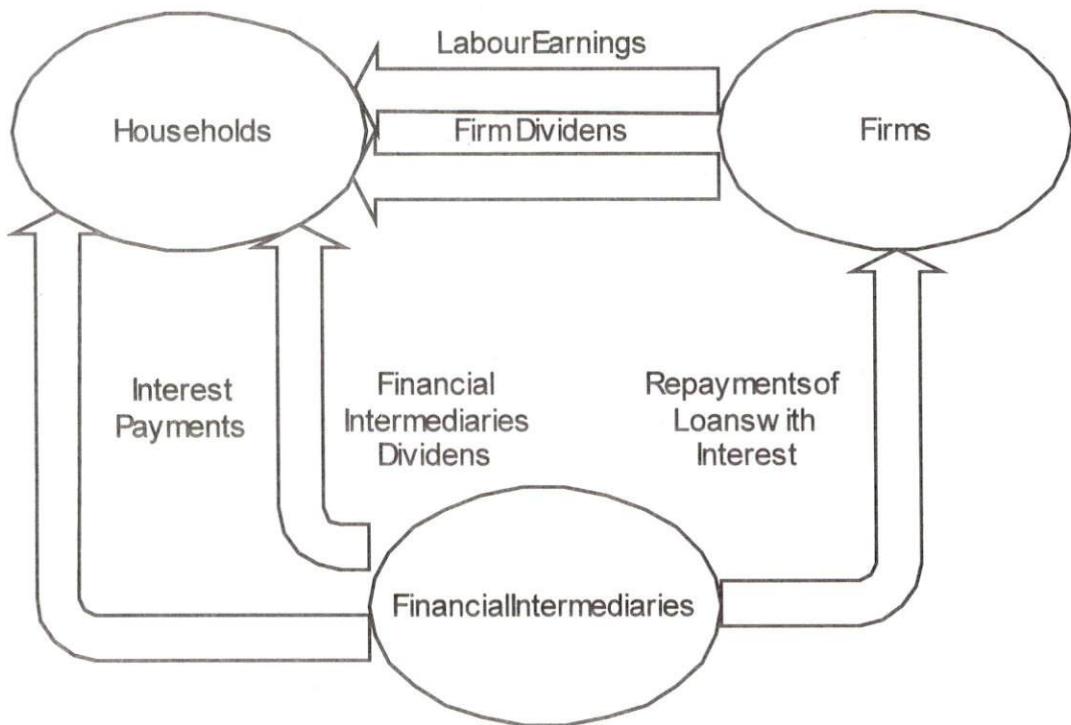
In models 1 and 3 the output responses are also the same and less than in the previous two models due to the relatively lower increase in employment. The response of output in model 4 is the lowest among all models.

## 6. Concluding Remarks

Models of limited participation can produce sufficiently strong liquidity effects such that nominal interest rates respond negatively to positive monetary shocks. By itself, however, limited participation cannot generate persistence in these responses. The foregoing analysis has sought to account for such persistence by introducing the idea of habit persistence in household behaviour. The results are promising, indicating that a combination of limited participation and habit persistence in consumption may be an effective way of generating serially correlated movements in nominal interest rates. The degree of serial correlation could be increased further by extending the model in two main ways. The first is to increase further the extent of habit persistence in consumption to more than two periods. The second is to strengthen the effect of habit persistence in leisure by abandoning the assumption that utility is linear in leisure.

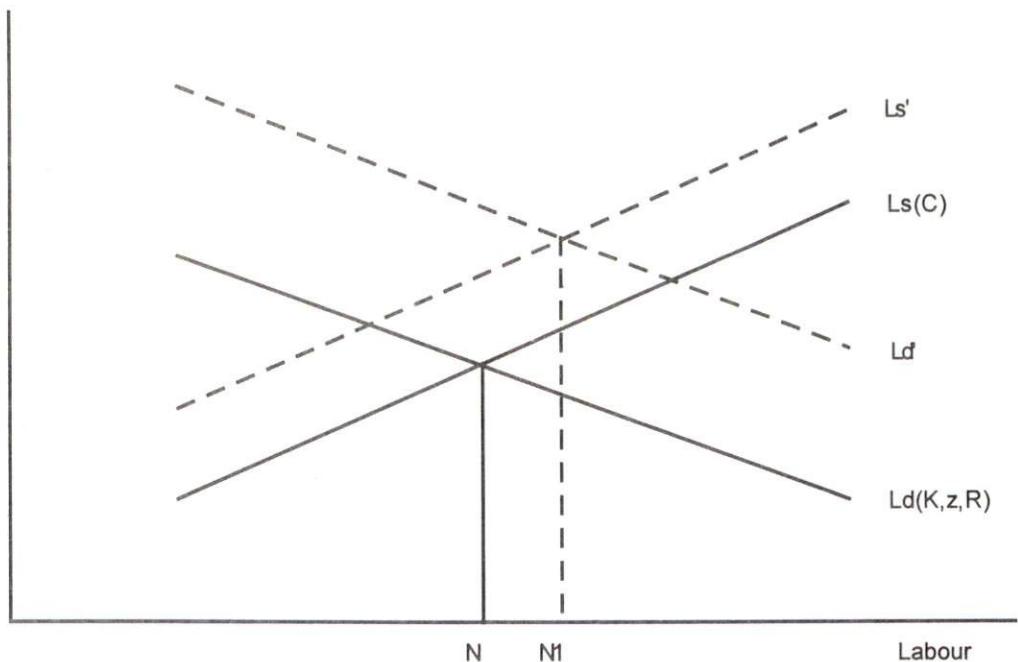


**Figure 1** Cash Flow in the Model Economies: Cash Flow Back to Firms.

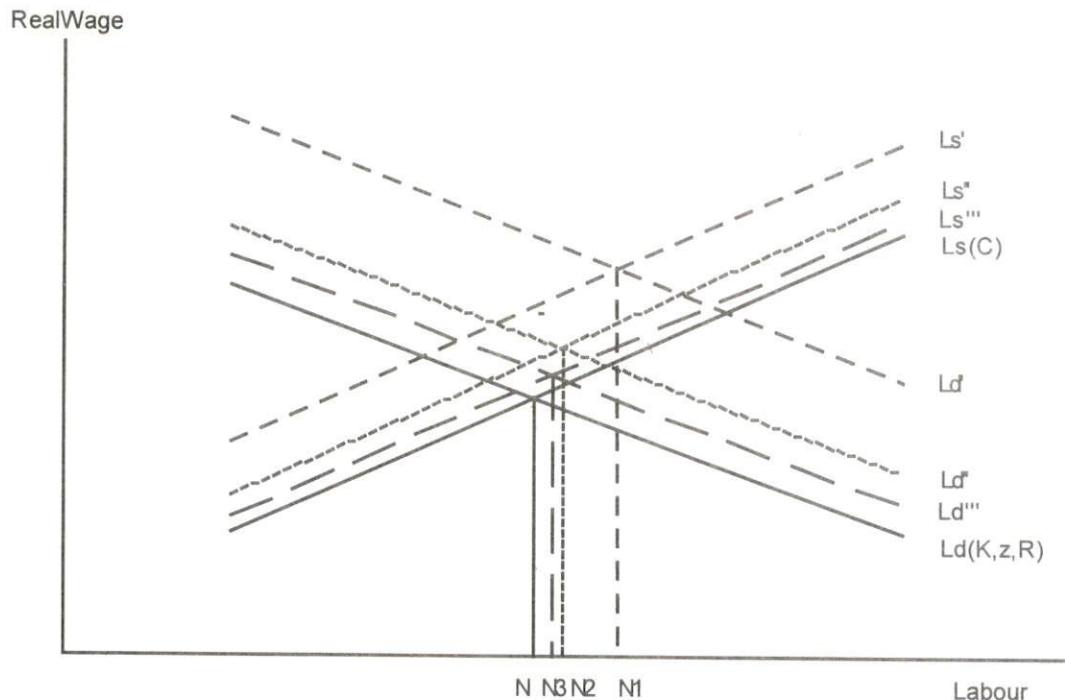


**Figure 2** Cash Flow in the Model Economies: Cash Flow Back to Households.

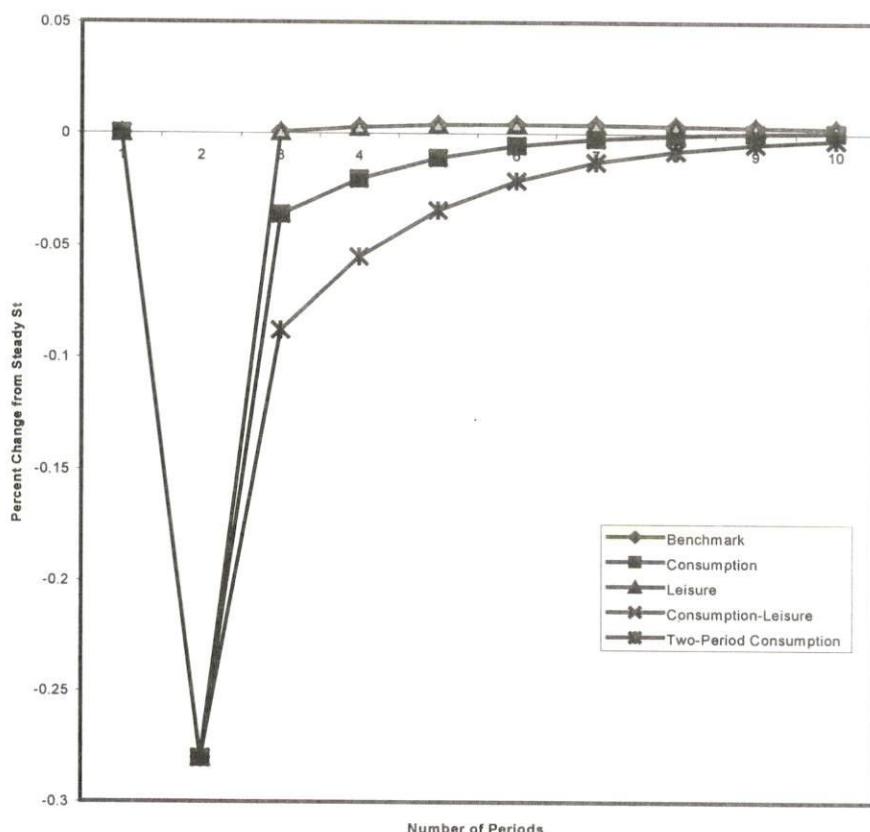
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**Figure 3** Equilibrium Response to a Monetary Shock in the Benchmark Model.



**Figure 4** Equilibrium Response to a Monetary Shock in the Habit Persistence Model.



**Figure 5** Nominal Interest Rate Response to a Monetary Shock.

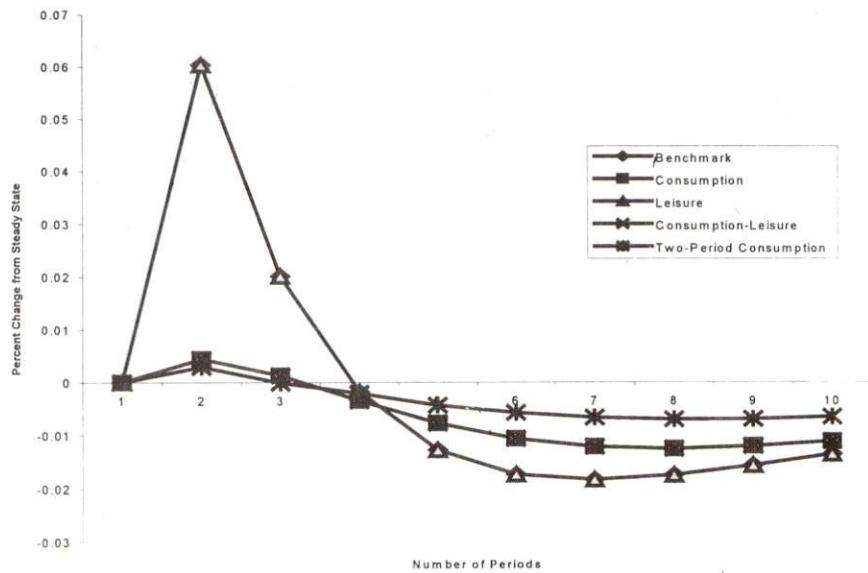


Figure 6 Consumption Response to a Monetary Shock.

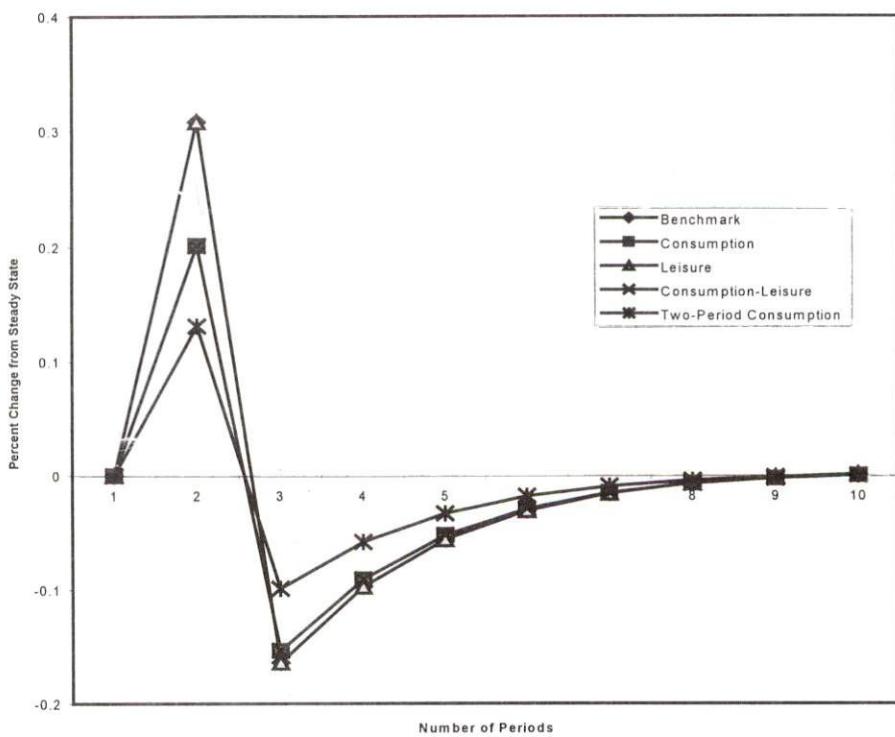
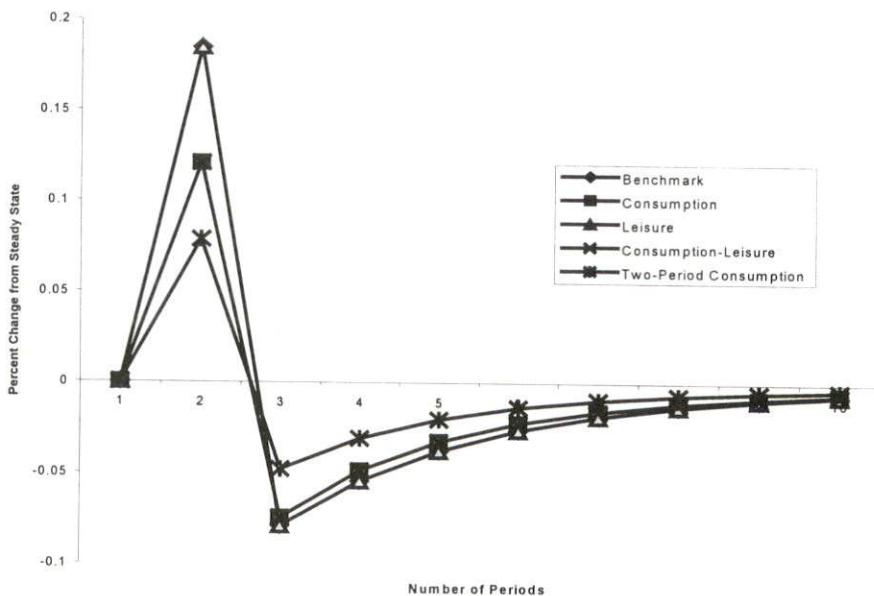


Figure 7 Employment Response to a Monetary Shock



**Figure 8** Output Response to a Monetary Shock.

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