Making Mathematical Connections with Transformations Using Open Approach

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Abstract. *The purpose of this study was to investigate the mathematical connection making with transformations at elementary education level using open approach. It was a qualitative research study utilizing a natural setting. The researcher as a research instrument paid attention to the process and described the meaning from the collective observations of the mathematics classroom based on the problem solving as a teaching approach. The target group included the teachers and 24 fifth grade students at the Ban Chonnabot Community School, implementing a lesson study and open approach. The data were collected on the learning activities in the four problem situations on parallelograms of the students enrolled in 2010 academic year, the collaboratively designed research lesson, collaborative observing the research lesson, together with the teaching approach, through the recorded video, recorded audio, semi-structured interviews, field notes, the students' works, classroom observation forms, and the collaborative reflections on teaching practice. The results showed that the mathematical connection with cutting, folding, moving, connecting, and making the complete units for the students were the natural ways of thinking about transformations through the teaching approach. The concepts discovered were parts that were reshuffled, the cutting out to make a rectangle, the measurement of the base and height for further calculation, the class discussion of the solutions, the procedural knowledge of counting, measuring, cutting, moving, and base multiplied by height. Moreover, the transformations served as the mathematical connector to link the numbers and operations.*

Keywords: Mathematical Connections, Transformations, Open Approach

Introduction

The core curriculum of mathematics, according to the 2008 Thai basic education core curriculum, requires that the concepts of geometric transformation covering transformation, reflection, and rotation be studied at Mathayom Suksa1 2 (Ministry of Education, 2008). The content prescribed for each grade is as if it were a cake to be cut out in pieces before handing out to the waiting eaters (Rodjai, 2009). In most Thai classrooms, the teacher takes the role of lecturer on the mathematics content to deliver learning to the students. The outcomes of the students are the last priority (Inprasitha, 2007). The instruction rarely focuses on having the students to learn by themselves. The teachers have full power and a strong hold that they know the most and their knowledge is always correct. The students simply wait to obtain it and adjust themselves to fit the content, knowledge, and the teacher's way of teaching (National Education Commission, 2000). Most learning and teaching rely on the textbook exercises, each of whose problems has only one correct answer. They never allow students of different abilities to equally benefit from them. In an analysis of the mathematics textbooks used in Thailand, it was found that most of them were composed of exercises to practice mathematics skills, particularly the calculative ones or those based on rule or formula recapture (Inprasitha, 1997). The classroom and teaching approach of this kind would leave no space for the students either to learn by themselves or to develop their own capacity.

1 Mathyom Suksa is a Thai word referring to secondary education. Mathayom Suksa 2 is grade 8.

The students see mathematic topics as separate and fail to see that what has been learned in one domain could be applied in understanding other domains. The transformation should be able to get rid of this separation (Rubenstein &Thompson, 1995). Transformation serves as the mathematical connector to the wider spectrum of topics, as mathematics is not separate from the essence of knowledge (Coxford, 1995). Transformations in the school curriculum state that transformations supply a unifying concept, not only in the study of Euclidean geometry, but among many other branches of mathematics as well (Crowley, 1995; Brown, 1973). Thus, it is necessary that the preparation for learning about transformation be done with the students, since they are at an earlier grade in their primary education.

Mathematical connections are the linkage through activities to other concepts, such as basic competence on number and mathematics ability (Desforges, & Mitchell, 2000; Wright, Marthland, & Stafford, 2000). It is widely recognized that teaching for mathematics learning should be designed from problem situations (NCTM, 1989, p. 11). Appropriate activities designed for the inquiry should connect the concepts and procedures among the different topics or to other contents. This is problematic in mathematics instruction (Romberg, 1994). The students should be able to connect mathematic concepts learned from a unit to the next ones, making all the contents interrelated (Lampert, 2001, p. 179).

An open approach is an innovation that originated in Japan, aiming at development of a high level of mathematics thinking of the students. In this approach, every student could learn mathematics in a way that is appropriate to his/her ability, along with making their own decisions in the learning process. They could increase the quality of the process and outcomes concerning mathematics (Nohda, 2000). The approach stresses the fact that problem solution does not end at a particular answer. The approach to a problem is an important aspect. Classroom conditions affect the discussion of the concepts and various ideas of the students (Becker & Shimada, 1997). Instructional design for the classroom that emphasizes problem solving as a teaching approach starts with the presentation of a problem or task which involves the students in the problem situation. The teacher's problem posing is an important factor in making the problem problematic for the students (Isoda, 2010). The role of the teacher is the attempt to compile all the students' thinking at the stage of their independent problem solving. She has to recognize every thought proposed by students and to connect all the ideas from the whole class discussion, using the board to summarize the problem solving and to promote the students' learning. The board will show what the problem is, what ideas the students have come up with, and the like. The teacher tries to summarize the connections to encourage the students to generalize, and to identify the related mathematics rules and formula (Inprasitha, 2010). It is then necessary to make the classroom focus on problem solving, in the process of which the students learn to solve the problem by themselves. Thailand has adopted the approach and integrated it with the lesson study and uses the open approach as the teaching approach (Inprasitha, 2010).

This article illustrates how the students could be facilitated to make the mathematical connection by themselves by creating an open-ended problem situation using transformation as the mathematical connector. The researcher used the open approach as the teaching approach that stresses problem solving.

Research Objective

The objective of the research was to investigate the mathematical connection making with transformations at an elementary education level, using an open approach.

Conceptual Framework

The provision of mathematics instruction emphasizing on problem solving through the open approach, proposed by Inprasitha (2010), introduces four phases:

Phase 1: Posing Open-ended Problems – The teacher poses an open-ended problem through some material and have the students try to comprehend the problem.

Phase 2: Students' Self-learning through Problem Solving - The students learn by themselves from their attempt to solve the open-ended problem using various approaches. The teacher notes down the students' method of solving the problem.

Phase 3: Whole Class Discussion and Comparison - The students present the solutions of the open-ended problem and the teacher keeps paying attention to every idea and tries to connect the proposed ideas together.

Phase 4: Summarization Through Connecting Students' mathematical ideas that Emerged in the Classroom – The teacher tries to summarize and connect to facilitate the students in finding the mathematics generalization, rules, and formula. Afterwards, the students are guided to summarize the ideas presented on the board in their notebooks in their own words.

Research Design

The research employed a qualitative research methodology with a certain extent of longitudinal approach, paying attention to the process and meaning. The research approach assigned the researcher's roles as a learner who entered the school that has used the lesson study innovation and an open approach during the 2006-2010 academic years**. As a school participant, the researcher acted as a participant observer and used the collected information in designing the learning plans in collaboration with the** teachers, the students on teaching practice, the school's coordinator, the researchers, and the teachers. The instruction took place every Monday using the mathematics textbook published in Japan (Gakkohtosho Co., Publisher) as the starting point in designing the learning modules along with the Thai mathematics textbooks. The classroom observation was collaboratively carried out every Monday and Thursday. The researcher taught in the teaching experiment, as she had had direct experience with the reasoning and mathematics learning of the students. The Ban Chonnabot Community School, which served as the site for the research, is under the Teaching Professional Development Project and used the lesson study and the open approach.

The target group was composed of 24 Prathom Suksa[2] 5 students, aged 10 -11 years. Each class was to have the students engaged in the group activities. Each group consisted of 4-5 students. According to Piaget's intellectual development theory, children of this age are typically capable of using reasoning in their thinking. The process and reasoning in solving the problems still need concrete materials. This fact was also noted by Bruner's principle of learning through imagination. It is the stage in which the children solve the problem through their perception.

The researcher is a full time teacher who has taken part in and used the lesson study and open approach. The study took five years during the 2006-2010 academic years**.** The researcher took part in the stages of learning plan setting up, classroom observation, and reflection which was regularly done. She stayed in the classroom to learn details of the students' thinking processes and their reasoning.

Research Procedures

The research followed three main steps of lesson study: namely, learning plan design, classroom observation, and reflection, all of which were done in collaboration.

Stage 1: The collaborative learning plan construction was mutually carried out among the teachers who took part in the research, the researcher, teaching practice students of Faculty of Education, Khon Kaen University, the research team from the Center for Research in Mathematics Education, Khon Kaen University and the school coordinator. The researcher made notes from observing the collaborative learning plan setting up. Discussion issues were as follows: 1) the aim of the lesson; 2) the assignment or construction of the open-ended problems with details on the instruction; 3) order of the instructions; 4) materials and equipment; 5) time allowed for performing each instruction; 6) the anticipation of the students; 7) information on the students' thinking in the previous class; and 8) the material design relevant to the instructions.

2 Prathom Suksa is a Thai word referring to elementary education. Prathom Suksa is grade 5.

Stage 2: The collective classroom observation was done at this stage where the learning plans were implemented in the classroom taught by the teacher. The observers included the researcher, teaching practice students, members of the researcher team from the Center for Research in Mathematics Education and the school coordinator. The main focus of the observation was the four teaching steps of the framework suggested by Inprasitha (2010), not the teaching competence of the teacher.

The sequence of teaching episodes was composed of the phrases in which the representative from the lesson planning team or teaching agents, the students in the steps of problem solving, the classroom observers, including the teaching practice students, members of the research team from the Center for Research in Mathematics Education, and the school coordinator, all were the witnesses of the teaching episodes. The instruments for note taking included video recorder, audio recorder, still photo camera, and observation forms.

Stage 3: The collective learning reflection was conducted every Thursday. The reflectors included those who took part in setting up the learning plans. The reflection had made use of the observations noted on the observation forms leading to the improvement of the lesson plans to be used in the next step of the class teaching.

Data Collection and Analysis

This study centered on the engagement in the learning activities in the four problem situations on the parallelograms of the students enrolled in 2010 academic year, the collaboratively designed research lesson, collaborative observing the research lesson along the teaching approach through the recorded video, recorded audio, semi-structured interviews, field notes, the students' works, classroom observation forms, classroom observation forms, and the collaborative reflection on teaching practice. The interview questions were created by the Center for Research in Mathematics Education, Khon Kaen University.

For data analysis, the ethnographic approach uses the analysis of significant ways of thinking, the generation of meaning units, and the development of essence description of the problem solving as a teaching approach (Inprasitha, 2010).

Research Results

The mathematics classroom problem solving using an open approach covering four instructional steps on the area of a quadrilateral, took four teaching periods.

The problem situation on "How big am I" took one period in which the students explained the needed lengths to calculate the quadrilateral, starting from learning about the transformation.

The learning preparation was needed for the students to find the area of the quadrilateral before the next stages in which the students learned the reshuffling, the cuts to make a parallelogram, the measurement, and the multiplication of the base and the height.

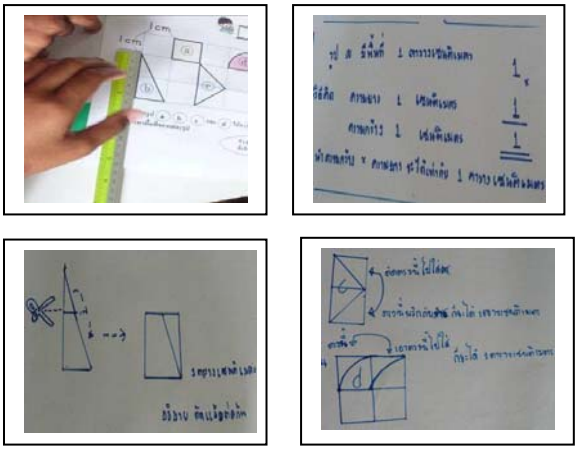

Figure 1. Learning how to learn transformations

The Problem Situation "How big am I?"

1. The teacher had the students measure each side of quadrilaterals a, b, and c.
2. The teacher had the students compare the area of quadrilaterals a, b, and c.
3. The teacher had the students write to explain how to find the area of quadrilaterals a, b, and c in as many ways as possible.

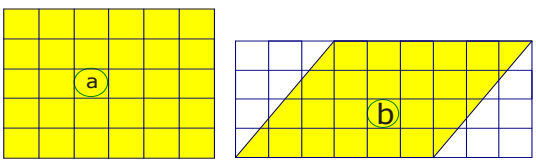

Figure b

Figure a

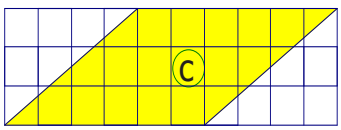

Figure c

Phase 1: Posing Open-ended Problems - The teacher poses an open-ended problem through some material and have the students try to comprehend the problem. The teacher had all the students recall the things learned using some guiding questions as follows:

Teacher: Yesterday, how did we find the area?

Students (whole class): By measuring length and width.

Teacher: Are there other ways? Students (whole class): By multiplying width and length.

Teacher: Now I will give you 3 figures. Let's take a look how these figures are like.

Phase 2 and 3. The class continued with the following mathematics concepts?

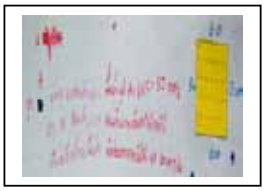

Figure a is 6 c.m. long and 5 c.m. wide. It's a rectangular.
The figure's area is 6x5=30 c.m.
squares or by counting the squares.
They are 30 equal size squares.

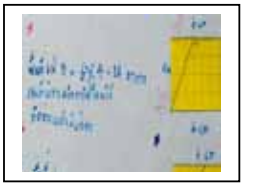

Figure b is 6 c.m. long and 4 c.m. wide. It is a quadrilateral.
Area of figure b=6x4=24 c.m.
squares or by cutting and replacing.

Student k: By cutting parts out to fix in other side to make it full.

Figure a is larger than figure b
from counting the small squares.
Figure b is smaller than figure a from folding.
Figure c is smaller than figure a
from folding

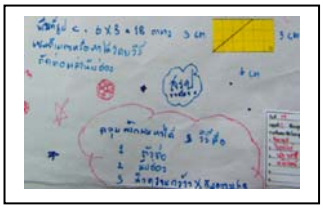

Figure c is 6 c.m. wide and 3 c.m. long. It is a quadrilateral.
Figure c has 6x3=18 c.m. squares.

By linking the arrow, it makes us know many inch squares the areas have.

Phase 4: Summarization through connecting students' mathematical ideas emerged in the classroom. Here the teacher and the whole class students collectively concluded the lessons learned.

The results are as follows:

Method 1 – counting the squares

Teacher: what kind of figure b and c are?

Students (whole class): parallelogram

Teacher: How could we find area of figure a?

Students (whole class): By multiplying the width and the length.

Teacher: How long and wide is it?

Teacher: In multiplying 5 cm. width with 6 cm. length, what do we get?

Students (whole class): 30 cm. squares.

Teacher: Besides multiplying, does anyone have any other way?

Student k: Yes, by counting the boxes.

Method 2 – cutting and connecting

Teacher: How about figure b?

How do we do?
Students (whole class): By cutting parts out to connect them in.

Teacher: Come and point it up please.

Student k: Right here, Mam.

Teacher: Then where to connect to?

Student k: Here, Mam.

Teacher: In cutting off and connecting in, what figure do we get?

Students (whole class): a rectangular.

Method 3 – using area finding formula
Teacher: Which do we need to find the area?
Students (whole class): the blue line.
Teacher: What is the blue line?
Students (whole class): The length.

Teacher: How about the green one?

Students (whole class): The width.
Teacher: How do we find the area?
Students (whole class): By multiplying the width and the length.
Teacher: Which lines we need to use to find the area?
Students (whole class): The green and the blue lines.

The teacher used the blackboard to conclude the students' thinking and used the supplementary material to facilitate the discussion on the students' ideas, such as counting, measuring, cutting, replacing, and base multiplied by height.

Figure 2. The teacher used the blackboard to conclude the students' thinking

Conclusion

The learning plans on cutting, folding, connecting, and making the complete unit for the students were the natural mode of thinking on transformation through the open approach – firstly the teacher presented the open-ended problem through the designed material to the students to make themselves understand them. The students were encouraged to recall the things learned so far, guided by the teacher's questioning. The second and third steps were carried out simultaneously. The concepts discovered were the parts' reshuffling, the cutting out to make a rectangular, the measurement of the base and height for further calculation, the class discussion on the solutions, the procedural knowledge of counting, measuring, cutting, moving, and base multiplied by height, all to compare the thinking modes for the students to learn together. Here the teacher and the whole class collectively concluded the lessons learned. The fourth step was the conclusion to connect the mathematics concepts of the students that occurred in the classroom, using additional material and blackboard. Moreover, the transformation served as the mathematical connector to link the number and operations using the area finding formula.

Discussion

The preparation of the students helped facilitate them in seeing the mathematical connections, as noted by Leak (1995), who pointed out that in connecting among the contents, the teacher has to realize what preparation the students need to reach the goal. The transformation is an effective concept to expand the mathematics curriculum. It effectively prepares the students to perform the task of connecting and supporting the tasks concerning figure, reasoning, problem solving, spatial sense, algebra, geometry, and statistics (Rubenstein &Thompson, 1995). The mathematics classroom that focuses on solving the problem through an open approach is a student-centered approach. It facilitates the students to learn by themselves to generate a variety of ideas (Isoda, 2010), and to be able to conclude the linkages of the mathematical concepts of the students occurring in the classroom (National Education Commission, 2000).

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