Tracking of DOA of GPS Signals Using GPS/INS Integration Approach

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Abstract

This article introduces a modified method for tracking of direction-of-arrival (DOA) of GPS signals via GPS/INS integration approach. The proposed method uses the well-known complementary features of GPS and inertia navigation system (INS) to combat with the interferences embedded in the received GPS signals. Simulation results show that the method reduces tracking error, significantly.

Keywords: Direction-of-Arrival tracking, Kalman filter

Introduction

Typically, algorithms for tracking of the DOA for a particular GPS satellite shall properly handle the GPS-signal interferences, which can be complicated. Ignoring these interferences will lead to a significant DOA estimation error [1-5]. The proposed concept however, can cope with this problem by incorporating the additional DOA information obtained from the inertia navigation system (INS) [6]. The INS data is usually fused with GPS data which is known as GPS/INS integration (see Fig. 1).

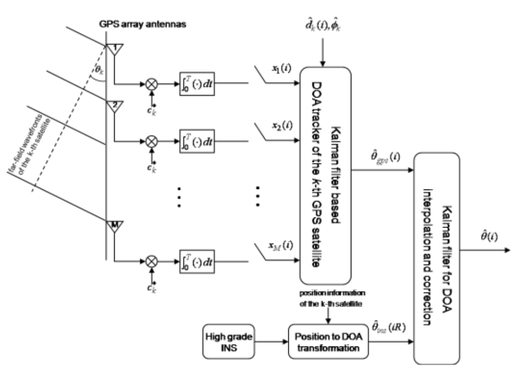

Fig. 1: GPS/INS integration architecture for DOA tracking.

The uniform linear array (ULA) consisting of *M* equally spaced GPS antennas followed by simple Kalman filter-based DOA tracker, is combined with INS based DOA tracker via another Kalman filter.

This article is organized as follows. In section II, models and assumptions for DOA tracking problem are defined. In section III, the GPS/INS integration strategy and the corresponding Kalman filter formulation are given. Section IV discusses simulation results. Finally, section V concludes this article.

Signal Models

Consider a uniform linear array (ULA) with an *M*-element array with half-wavelength ($\lambda/2$) spaced antenna elements. Thus, the received signal at the *m-th* antenna may be expressed as

$$y_m(t) = \sum_{k=1}^{K} s_k(t) e^{j(m-1)\pi \sin\theta_k(t) + \phi_k} + n_k(t) \qquad (1)$$

where K is the number of sources (GPS satellites in view), $\theta_k(t)$ is the azimuthal DOA of k-th sources, ϕ_k is the carrier phase and is uniformly distributed in [0, 2π], $s_k(t) = \sum_{i=-\infty}^{\infty} d_k(i) \cdot c_k(t - iT - \tau_k)$ is the transmitted data, $c_k(t)$ denotes the spreading sequence waveform, $n(t)$ is the additive white Gaussian noise, τ and T are the time delay and the symbol interval, respectively. The received signal may be written in a vector form a

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta}(t)) \cdot \mathbf{S}(t) + \mathbf{n}(t) \qquad (2)$$

where

- $\mathbf{y}(t) = [y_1(t), y_2(t), \cdots, y_M(t)]^T$ is an $M \times 1$ vector of the received signal at time, t.
- $\boldsymbol{\theta}(t) = [\theta_1(t), \theta_2(t), \cdots, \theta_K(t)]^T$ is the source DOA parameter vector.
- $\mathbf{A}(\boldsymbol{\theta}(t))$ is the array response matrix and is determined by the DOA of signals. The *k-th* column of $\mathbf{A}(\boldsymbol{\theta}(t))$ is defined as the array response vector associated with the *k-th* source, $\mathbf{a}(\theta_k(t))$ which is given by $[1, e^{-j\pi \sin\theta_k(t)}, \ldots, e^{-j\pi(M-1)\sin\theta_k(t)}]^T$.
- $\mathbf{S}(t) = \mathrm{diag}[s_1(t)e^{\phi_1}, \ldots, s_K(t)e^{\phi_K}]$.
- $\mathbf{n}(t) = [n_1(t), \ldots, n_M(t)]^T$ is an $M \times 1$ additive noise vector, which is assumed to be spatially and temporally white Gaussian.

The received signals are despread by the users' own spreading sequences for each of the antenna elements. Therefore, the *k-th* users' despread and sampled array vector signal, $x_k(i)$ may be expressed as

$$y_k(i) = d_k(i)\mathbf{a}(\theta_k(i))e^{j\phi_k} + \sum_{l=1, l\neq k}^{K} \psi_{kl} d_l(i)\mathbf{a}(\theta_l(i))e^{j\phi_l} + n_k(i) \qquad (3)$$

where the cross-correlation of the spread sequence, ψ_{kl} can be defined as $\psi_{kl} \equiv \int_0^T c_k(t)c_l(t)dt$ when the time delay of all users are zero and $\mathbf{n}_k(i)$ is the despread noise signal vector whose covariance matrix is $\sigma_n^2\mathbf{I}$. The outputs are then multiplied by a conjugate of the transmitted data estimate $\hat{d}_k^*(i)$ and carrier phase estimate $e^{-j\hat{\phi}_k}$ to remove the effect

of $d_k(i)$ and ϕ_k. With the assumption that $\hat{d}_k^*(i) = d_k(i)$ and $\hat{\phi}_k = \phi_k$, $y_k(i)\hat{d}_k^*(i)e^{-j\phi_k}$ can be written as

$$\begin{aligned}\tilde{y}_k(i) &= y_k(i)\hat{d}_k^*(i)e^{-j\hat{\phi}_k} \\ &= \mathbf{a}(\theta_k(i)) + \\ &\quad \hat{d}_k^*(i)e^{-j\hat{\phi}_k} \sum_{l=1, l\neq k}^{K} \psi_{kl} d_l(i)\mathbf{a}(\theta_l(i))e^{j\phi_l} + \\ &\quad \hat{d}_k^*(i)e^{-j\hat{\phi}_k}\mathbf{n}_k(i) \\ &= \mathbf{a}(\theta_k(i)) + \boldsymbol{\alpha}_k(i) + \tilde{\mathbf{n}}_k(i)\end{aligned} \qquad (4)$$

where $\boldsymbol{\alpha}_k(i)$ is a vector containing the interference of the spread signals of the *k-th* satellite, and $\tilde{\mathbf{n}}_k(i)$ represents the noise vector with the same statistical properties as that of $\mathbf{n}_k(i)$.

DOA Tracking Procedure

As depicted in Fig. 1, there are two steps to be carried out in order to complete the proposed DOA estimation procedure. Firstly, the DOA is tracked via the array of GPS antennas using an extended Kalman filter; in this step the interferences from other GPS satellites will not be considered. Then, additional information of the DOA is fused with the DOA estimated of the first step. The detail of these two steps is given below.

Step 1: *DOA tracking with extended Kalman filter*

In order to track the DOA of the k-th satellite, $\theta_k(i)$, an extended Kalman filter (EKF) is used. The state-space and observation models of the Kalman filtering algorithm for this DOA tracking problem can be formulated as follows:

State space model: For simplicity, the index k indicating the satellite number is omitted. The $\theta(i)$ is modeled as a random walk process as:

$$\mathbf{x}_{i+1} = \mathbf{A}\mathbf{x}_i + \mathbf{w}_i$$
$$\begin{bmatrix} \theta_{gps}(i+1) \\ \dot{\theta}_{gps}(i+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \theta_{gps}(i) \\ \dot{\theta}_{gps}(i) \end{bmatrix} + \begin{bmatrix} 0 \\ w_{\theta_{gps}}(i) \end{bmatrix}. \quad (5)$$

Observation model: The cross-correlation (interference) term, $\boldsymbol{\alpha}(i)$, contributed the despread signals is ignored. A simple observation can be written as:

$$\mathbf{y}_i = \mathbf{h}(\mathbf{x}_i) + \mathbf{v}_i$$
$$\begin{bmatrix} R\{\tilde{y}_1(i)\} \\ R\{\tilde{y}_2(i)\} \\ \vdots \\ R\{\tilde{y}_M(i)\} \\ I\{\tilde{y}_1(i)\} \\ I\{\tilde{y}_2(i)\} \\ \vdots \\ I\{\tilde{y}_M(i)\} \end{bmatrix} = \begin{bmatrix} \cos(0) \\ \cos(-1\pi\sin\theta_{gps}(i)) \\ \vdots \\ \cos(-(M-1)\pi\sin\theta_{gps}(i)) \\ \sin(0) \\ \sin(-1\pi\sin\theta_{gps}(i)) \\ \vdots \\ \sin(-(M-1)\pi\sin\theta_{gps}(i)) \end{bmatrix} + \begin{bmatrix} R\{\tilde{n}_1(i)\} \\ R\{\tilde{n}_2(i)\} \\ \vdots \\ R\{\tilde{n}_M(i)\} \\ I\{\tilde{n}_1(i)\} \\ I\{\tilde{n}_2(i)\} \\ \vdots \\ I\{\tilde{n}_M(i)\} \end{bmatrix} \quad (6)$$

where $R\{\tilde{y}_m(i)\}$ and $I\{\tilde{y}_m(i)\}$ are real and imaginary of $\tilde{y}_m(i)$ at the *m-th* antenna, and, $R\{\tilde{n}_m(i)\}$ and $I\{\tilde{n}_m(i)\}$ are real and imaginary of $\tilde{n}_m(i)$ at the *m-th* antenna. $\mathbf{h}(\cdot)$ is the nonlinear mapping between $\mathbf{y}_i$ and $\mathbf{x}_i$.

The EKF requires the linearized observation matrix (Jacobian matrix) which can be obtained by

$$\mathbf{H}(i) = \frac{\partial}{\partial \mathbf{x}_i}\mathbf{h}(\mathbf{x}_i)\bigg|_{\mathbf{x}_i=\hat{\mathbf{x}}_i^-} \quad (7)$$

which is

$$\begin{bmatrix} 0 & 0 \\ 1\pi\cos\hat{\theta}_{gps}^-(i)\sin(-1\pi\sin\hat{\theta}_{gps}^-(i)) & 0 \\ \vdots & \vdots \\ (M-1)\pi\cos\hat{\theta}_{gps}^-(i)\sin(-(M-1)\pi\sin\hat{\theta}_{gps}^-(i)) & 0 \\ 0 & 0 \\ -1\pi\cos\hat{\theta}_{gps}^-(i)\sin(-1\pi\sin\hat{\theta}_{gps}^-(i)) & 0 \\ \vdots & \vdots \\ -(M-1)\pi\cos\hat{\theta}_{gps}^-(i)\sin(-(M-1)\pi\sin\hat{\theta}_{gps}^-(i)) & 0 \end{bmatrix}$$

The linearized observation model is then

$$\mathbf{y}_i = \mathbf{H}(i) \cdot \mathbf{x}_i + \mathbf{v}_i \qquad (8)$$

Since the cross-correlation matrix is unknown and ignored, the bias errors embedded in the output estimates, $\hat{\theta}(i)$.

Step 2: *GPS/INS Integration for DOA Tracking*

The outputs from the array-GPS DOA tracker, $\hat{\theta}_{gps}(i)$ will be fused with the DOA estimates obtained from the INS unit $\hat{\theta}_{ins}(iR)$ via an additional Kalman filter. This approach is comparable to the well-known loosely-coupled GPS/INS integration [6]-[8]. Assuming that the INS provides accurate position estimates for a sufficiently long period of time and the satellites' position are available to the INS so that INS can use this information to transform its position into DOA. The array-GPS DOA tracker outputs the DOA estimates at a much higher rate than that of the INS (~100-200 Hz), therefore the integration filter (Kalman filter) will complete its processing steps (prediction and correction loops) only when the DOA from INS is available otherwise it will only run its prediction loop. The outputs from the array-GPS DOA tracker are erroneous since the cross-correlations are not accounted for. However, by fusing the erroneous estimates with the DOA derived from high grade INS unit, the error can significantly be reduced.

The state-space model for data fusion is defined as

$$\begin{aligned} &\mathbf{x}_{i+1} = \mathbf{A}\mathbf{x}_i + \mathbf{w}_i \\ &\begin{bmatrix} \Delta\theta(i+1) \\ \Delta\dot{\theta}(i+1) \end{bmatrix} = \\ &\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta(i) \\ \Delta\dot{\theta}(i) \end{bmatrix} + \begin{bmatrix} 0 \\ w_{\Delta\theta}(i) \end{bmatrix}. \end{aligned} \qquad (8)$$

Since the difference between the reliable DOA obtained from INS and DOA estimates obtained from the array-GPS is used as the measurement, a simple linear observation model may be obtained as

$$\begin{aligned} &\mathbf{y}_i = \mathbf{H}(i) \cdot \mathbf{x}_i + \mathbf{v}_i \\ &\left[\theta_{ins}(i) - \theta_{gps}(i)\right] = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta(i) \\ \Delta\dot{\theta}(i) \end{bmatrix} + [v(i)] \end{aligned} \qquad (9)$$

The integrated DOA estimate, $\hat{\theta}(i)$ is obtained by (see also Fig. 2)

$$\hat{\theta}(i) = \theta_{gps}(i) + \Delta\hat{\theta}(i) . \qquad 10)$$

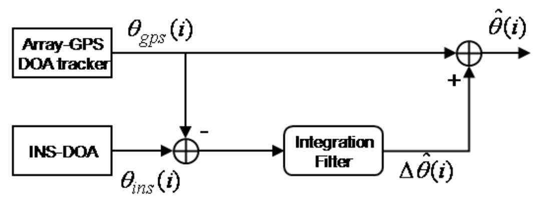

Fig. 2: GPS/INS integration scheme.

Time Alignment and Synchronization

In practice, the DOA estimates rate obtained from the GPS array system is higher than that from the INS system, thus the measurement times between both systems are most likely to be different. Therefore, time alignment (i.e., interpolation or extrapolation) of the DOA estimates of the two systems is needed (see Fig. 3). In real-time processing, a simple linear extrapolation can be used to obtain the GPS's estimated DOA at the time when INS produces its DOA estimate. The extrapolation can be computed using the following equations

$$\theta_{gps}(t_{ins}) = \theta_{gps}(t_{k-1}) + \frac{\theta_{gps}(t_{k-1}) - \theta_{gps}(t_{k-2})}{t_{k-1} - t_{k-2}}(t_{ins} - t_{k-1}) \quad (11)$$

where θ_{gps} is the DOA estimate obtained from array-GPS.

If the linear interpolation is applied rather than the extrapolation the following equation can be applied

$$\theta_{gps}(t_{ins}) = \frac{t_k - t_{ins}}{t_k - t_{k-1}}\theta_{gps}(t_{k-1}) + \frac{t_{ins} - t_{k-1}}{t_k - t_{k-1}}\theta_{gps}(t_k) \quad (12)$$

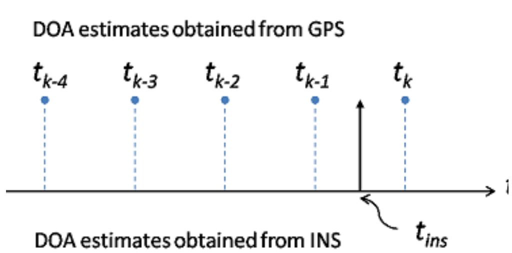

Fig. 3: Time alignment of estimates obtained from GPS and INS systems.

Simulation Results

For simulation purpose, the direction-of-arrivals of the GPS signals are generated randomly following the sinusoidal functions. The PN-sequences are used to generate the transmitted and received GPS signals. The following assumptions are also assumed in the simulations.

- 8 GPS linear array antennas are used.
- The data bits are known and can be removed from the received signal.
- Maximum of 12 GPS satellites are seen by all antennas.
- The array-GPS outputs DOA estimates 100 times faster than INS.
- High quality INS is available.
- Calibration and synchronization are perfect.

Fig. 4 and Fig. 5 show the DOA tracking ability of the array-GPS based DOA tracker with and without INS aiding. It can be seen (Fig. 5) that if the cross-correlation or interference term is ignored and not compensated for, the DOA estimation error is large. However, by using the proposed GPS/INS integration concept for DOA tracking, this error can be significantly reduced. It should be noted that the quality of the INS plays an important role in reducing the estimation error.

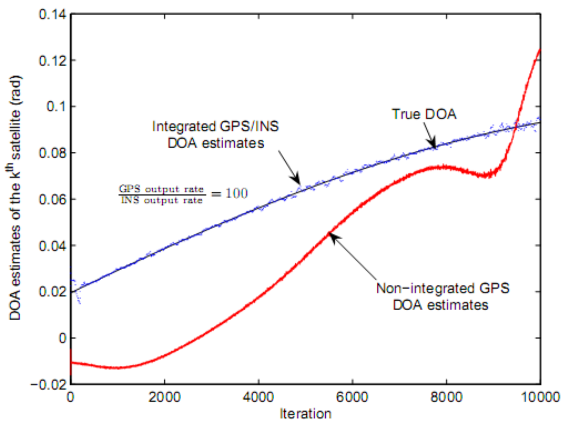

Fig. 4: DOA tracking via GPS/INS integration.

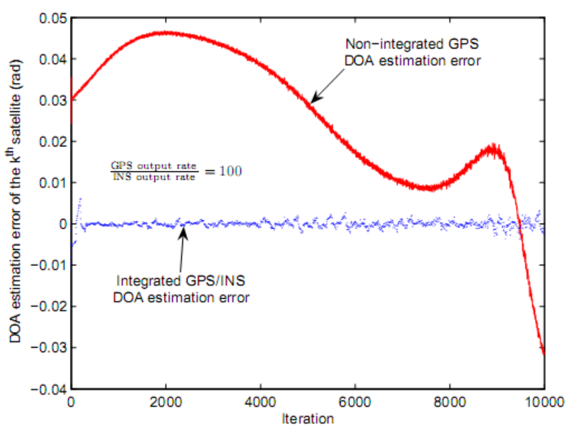

Fig. 5: Tracking errors via GPS/INS integration.

Conclusion

A formulation for tracking of DOA of GPS signals using GPS/INS integration is given. The proposed method does not need to account for the cross-correlation term. It is shown that an additional DOA information can greatly reduce the GPS-alone DOA estimation errors caused by the uncompensated cross-correlation of the despread GPS signals of other satellites.

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