

An Estimation of Portfolio's Value-at-Risk with Copula:
Empirical Evidence from Laos Securities Exchange
การประมาณมูลค่าความเสี่ยงของพอร์ตการลงทุนด้วยคอปูลา:
หลักฐานเชิงประจักษ์จากตลาดหลักทรัพย์ลาว

Krisada Khruachalee¹

Surang Boonyapongchai²

Pheng Her³

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Abstract

This research aims to examine the application of copula in estimating portfolio's VaR of the two most actively traded stocks (BECL and EDL-Gen) registered in the LSX which can describe the overall movement of the Laos stock market transactions during November 2019 to February 2024. The daily returns of both stocks were calculated to figure out the potential marginal distributions that were well describing the behavior of stock's return. The chi-square and Anderson-Darling tests would be applied to assess a goodness-of-fit between historical time series of daily return and the possible probability distributions which were normal, Student-t, log-normal, logistic, triangular, Gumbel, Fréchet, Weibull, generalized beta, and generalized extreme value distributions. The tested results were found that the

¹ Faculty of Business Administration, Siam Technology College

E-mail: krisadak@siamtechno.ac.th

² Faculty of Business Administration, Siam Technology College

E-mail: surangb@siamtechno.ac.th

³ Faculty of Economic and Tourism, Souphanouvong University, Luang Prabang, Lao PDR

E-mail: heping_econ@yahoo.com

Gumbel distribution, extreme value distributions type-I, was well suit to describe the behavior of each stock's returns. The method of maximum likelihood estimation was then employed to estimate parameters of the Gumbel distributions. In accordance with the variance-covariance matrix of both stocks, random samples from a multivariate normal distribution were then generated. The tail-independent Gaussian copula, which permits negative dependency and correlation matrix of the generated random samples were used to compute stock's returns. Based on equally weighted average of all individual estimated stock's returns held in the portfolio, the portfolio's returns were recalculated in a number of 1,000 times. Then, the portfolio VaRs based on normality assumption and copula method were estimated in according with 95%, 97.5% and 99% levels of confidence respectively. We found that the Gaussian copula VaR was marginally lower than the normality VaR for all given levels of confidence respectively. This can be implied that the Gaussian copula VaR was not consistently aligned with the conservative portfolio investment where investing in low-risk securities is prioritized. Even though, the portfolio based on low-volatility stocks was formulated, the Gaussian copula VaR was not favorable for risk-averse investors to measure the worse loss depending on the current position.

Keywords: *Risk Measures, Copula Functions, Value-at-Risk, Probability Distribution, Laos Securities Exchange*

บทคัดย่อ

งานวิจัยนี้มีวัตถุประสงค์เพื่อศึกษาการประยุกต์ใช้ฟังก์ชันคอปูลาในการประมาณมูลค่าความเสี่ยงของกลุ่มหลักทรัพย์การลงทุนของหุ้นที่มีการซื้อขายมากที่สุด 2 ตัว (BECL และ EDL-Gen) ที่จดทะเบียนในตลาดหลักทรัพย์ลาว ซึ่งสามารถอธิบายความเคลื่อนไหวโดยรวมของธุรกรรมในตลาดหุ้นระหว่างเดือนพฤศจิกายน 2562 ถึงกุมภาพันธ์ 2567 ได้ โดยผลตอบแทนรายวันของหุ้นทั้งสองจะถูกคำนวณเพื่อประเมินการแจกแจงความหนาแน่นตามขอบที่เป็นไปได้ที่ซึ่งใช้ในการอธิบายพฤติกรรมผลตอบแทนของหุ้นได้ดี นอกจากนี้ การทดสอบไคสแควร์และแอนเดอร์สัน-ดาร์ลิงจะถูกนำมาใช้เพื่อทดสอบภาวะสารูปสนธิ ระหว่างอนุกรมเวลาในอดีตของผลตอบแทนรายวันและการแจกแจงความน่าจะเป็นที่เป็นไปได้ ได้แก่ การแจกแจงแบบปกติ การแจกแจงแบบท การแจกแจงแบบล็อกนอร์มอล การแจกแจงแบบลอจิสติก การแจกแจงแบบสามเหลี่ยม การแจกแจงแบบกัมเบล การแจกแจงแบบพีรีเซต การแจกแจงแบบไวบูลล์ การแจกแจงแบบเบต้าทั่วไป และการแจกแจงแบบค่าสุดขีดวงนัยทั่วไป ซึ่งผลการทดสอบ พบว่า การแจกแจงแบบกัมเบล แบบค่าสุดขีด

วางนัยทั่วไป ชนิดที่ 1 มีความเหมาะสมในการอธิบายพฤติกรรมของผลตอบแทนของหุ้นแต่ละตัวได้เป็นอย่างดี จากนั้นจึงใช้วิธีการประมาณค่าความน่าจะเป็นสูงสุดเพื่อประมาณค่าพารามิเตอร์ของการแจกแจงแบบกัมเบล โดยตัวอย่างแบบสุ่มจากการแจกแจงแบบปกติหลายตัวแปรได้ถูกสร้างขึ้นจากเมทริกซ์ความแปรปรวน-ความแปรปรวนร่วมของหุ้นทั้งสอง ทั้งนี้ คอปูลาเกาส์เซียน ซึ่งถือได้ว่าเป็นการแจกแจงที่มีความเป็นอิสระจากส่วนท้ายและยังสามารถอธิบายความสัมพันธ์เชิงลบได้อีกด้วย ประกอบกับการใช้เมทริกซ์สหสัมพันธ์ของตัวอย่างสุ่มที่สร้างขึ้นเพื่อนำมาจำลองผลตอบแทนของหุ้นทั้งสอง จากการกำหนดค่าเฉลี่ยถ่วงน้ำหนักที่เท่ากันของผลตอบแทนของหุ้นโดยประมาณทั้งหมดที่อยู่ในกลุ่มหลักทรัพย์การลงทุน ผลตอบแทนของกลุ่มหลักทรัพย์การลงทุนจะถูกคำนวณใหม่จำนวน 1,000 ครั้ง จากนั้น ค่ามูลค่าความเสี่ยงของกลุ่มหลักทรัพย์ที่อิงตามสมมติฐานภาวะปกติและวิธีการของคอปูลาจะถูกประเมินตามระดับความเชื่อมั่น ร้อยละ 95.00, ร้อยละ 97.50 และ ร้อยละ 99.00 ตามลำดับ โดยทำให้พบว่า ค่ามูลค่าความเสี่ยงที่อิงตามคอปูลาเกาส์เซียนนั้นมีค่าต่ำกว่าค่ามูลค่าความเสี่ยงปกติเล็กน้อยสำหรับระดับความเชื่อมั่นที่กำหนดทั้งหมด ตามลำดับ ด้วยหลักฐานเชิงประจักษ์ดังกล่าวทำให้สามารถสรุปเป็นนัยได้ว่า ค่ามูลค่าความเสี่ยงที่อิงตามคอปูลาเกาส์เซียนไม่สอดคล้องกับการลงทุนในกลุ่มหลักทรัพย์การลงทุนแบบอนุรักษ์นิยมที่ให้ความสำคัญกับการลงทุนในหลักทรัพย์ที่มีความเสี่ยงต่ำ แม้ว่าจะมีการกำหนดกลุ่มหลักทรัพย์ที่อิงจากหุ้นที่มีความผันผวนต่ำ แต่ค่ามูลค่าความเสี่ยงที่อิงตามคอปูลาเกาส์เซียนก็ไม่เป็นผลดีต่อนักลงทุนที่ไม่ชอบความเสี่ยงในการวัดผลขาดทุนที่เฝ้าระวังโดยขึ้นอยู่กับสถานะปัจจุบัน

คำสำคัญ: มาตรวัดความเสี่ยง ฟังก์ชันคอปูลา มูลค่า ณ ความเสี่ยง การแจกแจงความน่าจะเป็น ตลาดหลักทรัพย์ลาว

Introduction

One of Southeast Asia's fastest-growing nations, Laos is seeing a surge in foreign investment, particularly from China. The Chinese government is making significant financial investments in the developing economies of Asia through the "One Belt One Road" initiative where Laos is unquestionably one of the project's primary beneficiaries in China. In addition, the Laos government has been taking steps to liberalize its economy and develop its financial markets. This includes reforms to improve transparency, regulatory frameworks, and investor protection, which can enhance confidence in the stock market. Therefore, Laos stock market investment is one of an alternative choice to leverage investors expected return. However, uncertainty of return, so called risk, on stock investment is the most stressful to general investors. Consequently, the essential components of investing are assessing stock market risk and creating risk management plans for the portfolio. The entire stock market is

generally impacted by a number of variables, such as recessions, interest rates, inflation, growth in the economy, and changes in currency values. Despite their challenging unpredictability, these factors increase the risk and volatility in the stock market. Interestingly, most investors do not seem to be bothered by the volatility in bull markets. Most investors do not realize that portfolio volatility alone will lower returns. Furthermore, as more and more individuals follow the herd, they allow their feelings to lead them to purchase more when prices are high and sell later when prices have decreased. Another prevalent issue is that investors who prioritize higher risk stocks in search of high rates of return. Generally speaking, riskier stocks are smaller and have weaker fundamentals than average stocks. In bull markets, the more speculative stocks typically lead the market higher, but in bear markets, they tend to fall. According to several anomalies in the stock market, stock exchanges and regulatory authorities impose risk management guidelines for market participants, including brokers, investment banks, and fund managers. These guidelines may cover areas such as risk assessment, margin requirements, and trading limits to mitigate excessive risk-taking. (Laos Securities Exchange, 2024)

Considering the overview of Laos investment scheme, even though the Lao Securities Exchange (LSX) would like to continuously increase the number of listed companies by relaxing the regulation on the foreign investors' holding of the LSX from 10 to 20 percent, there are currently 10 companies registered in the stock market where Banque Pour Le Commerce Exterieur Lao Public (BCEL), EDL-Generation Public Company (EDL-Gen) and Petroleum Trading Laos Public Company Limited (PTL) are top 3 stocks of the daily largest volume ranking.



Figure 1: Historical time series of LSX index during January 2019 – February 2024

Source: Laos Securities Exchange (2024)

Considering the trend of LSX index over past 5 years, Figure 1 provides an increasing trend of the LSX index where 16.92 points increased accounting to 1.59% raise for year-end 2023. This trend was contributed by the BCEL which is the most active stock cover 69.32% of total trading value, the EDL-Gen which is the second active stock cover 16.01% of total trading value and trading value of remaining listed companies cover 14.67% of total trading value. (Laos Securities Exchange, 2024)

In addition, about over a half of the stocks in the LSX are mostly traded by foreign investors where the proportion of total trading by foreign and domestic were 53.12% and 46.88% of total trading value respectively. As shown in Table 1, both foreign individual and institution investor can buy stock of PTL, SVN, PCD, LCC, MHTL, LAT, VCL and LALCO with no limitation (100%). On another hand, the total foreign investors can buy EDL-Gen's stock and BECL's stock in the ratio of not exceed 25% and 30% of the total share amount of a listed company respectively. (Laos Securities Exchange, 2024). Considering the occupation ratio of these stocks which is represented the proportion of foreign investor participation in trading, the trading volume of each individual stock is significantly lower when compared to stock markets in nearby regions.

Table 1 Listed companies on the LSX

No.	Issuer name	Foreign investment ratio (%)	Occupation ratio (%)
1	BCEL	30.0	9.9422
2	EDL-Gen	25.0	11.3775
3	SVN	100.0	4.0929
4	PTL	100.0	11.9789
5	MHTL	100.0	88.1687
6	VCL	100.0	0.8154
7	PDC	100.0	0.3441
8	LCC	100.0	9.8478
9	LAT	100.0	17.9629
10	LALCO	100.0	20.9923

Source: Laos Securities Exchange (2024)

This is the reason that Laos market volatility is likely low as there are a lot of low-frequency trading stocks traded in the market. Even though, the volatility in the Laos stock market is likely rigid, investors still need to have a protective strategy for speculating the market uncertainty. The investors must control the effects of systematic risk (so call “market risk”) on their investment portfolio because it is impossible to predict what will drive the market up or down. In order to implement any risk management strategies, the investors need to measure the risk level of their portfolio where the statistical measures of risk are significant elements of modern portfolio theory (MPT) and historical predictors of investment risk and volatility as suggested by Markowitz (1952).

Amidst a worsening global economic environment, gradual domestic reforms, and a significant devaluation of the national currency, the Lao economy is currently confronted with more difficulties. The investors especially foreign investors who look for an investment opportunity in an emerging market such as Laos should be well prepare for the risk management strategy. The measurement of loss realization is the crucial instrument that efficiently facilitate the investors to mitigate the risk and maximize return.

Literature Review

There are several general risk measures that are commonly used to assess the risk of a portfolio such as standard deviation which measures the dispersion of returns around the average return of a portfolio, Beta which measures the sensitivity of a portfolio's returns to changes in the overall market (Markowitz, 1952), Treynor ratio which measures the risk-adjusted return of a portfolio but uses beta as the measure of risk instead of standard deviation (Treynor, 1965), Sharp ratio which measures the risk-adjusted return of a portfolio by comparing the excess return (return above the risk-free rate) to the portfolio's standard deviation (Sharp, 1994) and Value at Risk (VaR) which measures the maximum potential loss of a portfolio over a specified time horizon at a given confidence level. Due to the flexibility of the VaR, it can be calculated for various types of risks, including market risk, credit risk, and operational risk. This flexibility makes it more suitable for analyzing different types of portfolios and financial instruments. It also provides an estimate of the worst-case scenario loss. Moreover, the VaR provides information not only about the expected loss but also the potential for extreme losses (tail risk). By considering different confidence levels, the VaR allows risk managers to assess the likelihood of experiencing large losses.

There are various methods in estimating the VaR, but the most common approaches are historical simulation, Monte Carlo simulation, variance-covariance approach with normal and Student's t distributions, AR-GARCH approach with normal and Student's t distributions, an application of quantile of non-normal distribution, and the extreme-value-theory approach. In order to calculate the VaR based on parametric approach (variance-covariance method), a normal distribution in returns is mandatorily required to assume. Moreover, there are two estimated factors which are an expected return and a standard deviation. When dealing with risk measurement issues where the distributions are well-known and accurately estimated, this approach works efficiently. However, this method is untrustworthy when there is a very small sample size.

With the mandatory requirement of a normal distribution in returns, we rarely find in the practical results because, as everyone knows, returns on the stock market are not normally distributed. Instead, we think of them as having fat tails where the extreme events happen more frequently than expected. But if we look at the distribution of stock market returns over different time frames then we will find that returns are not even monomodal that they do not just have one peak in the middle of the distribution as predicted by the normal distribution. As results, applying normality VaR would be bias in estimating the risk level regarding portfolio investment.

Furthermore, the correlation coefficient is typically used to determine how two or more assets' returns are interdependent. However, the correlation is most effective when applied to normal distributions, but financial market distributions are frequently non-normal. As we found from the works of Longin and Solnik (2001) and Ang and Chen (2002), they illustrated evidence that the return of assets is more highly correlated during the course of the market downturns which the risk of a portfolio could be higher than expected. The copula, therefore, has been applied to several areas of finance such as option pricing and portfolio VaR to handle asymmetric or skewed distributions. A multivariate uniform distribution is represented by a probability model called a copula, which considers at the relationship or dependence between numerous random variables. Regardless of the joint normality assumption, multivariate distribution modeling can be constructed with this dynamic tool (Nelsen, 1999). The application of copula with the VaR approach is practically an alternative choice for the portfolio risk measurement where the copula provides a multivariate joint distribution that combines the dependence between the variables and the marginal distribution. The works of Cherubini and Luciano (2001) can be used to validate the application of copula function in estimating the VaR and allocating the capital

where they estimated the marginal distribution using the historical empirical distribution and the Archimedean copula family. Their empirical results provide the effect of the fat-tailedness of the returns on the VaR trade-off which is shifted downwards by fat tails. In addition, Fortin and Kuzmics (2002) who determined the VaR of the portfolio using a linear combination of copula. Their model is estimated using bivariate data on a set of European stock indices. They found that the assumption of normal or student-t dependence is easily rejected in favor of an asymmetrically tail-dependent distribution. Recently, Khruachalee and Bodhisuwan (2019) estimated copula VaR and copula conditional VaR for the portfolio of SET, IDX and KLCI stock indices. They found that the copula conditional VaR provides a slightly higher risk level than the conditional VaR and VaR with the assumption of the multivariate normal distribution. Therefore, the use of Gaussian copula conditional VaR will be more reasonable for conservative investors. Moreover, there are several works applying copula with the VaR approach but measuring the dynamic risk level as copula VaR in a low frequency trading market is limited in the literatures especially in Laos securities market. This research aims to examine the application of copula in estimating portfolio's VaR of the low-frequency trading market which is represented by the two most actively traded stocks (BECL and EDL-Gen) registered in the LSX. These stocks can describe the overall movement of the Laos stock market transaction during November 2019 to February 2024 where the outbreak of Covid-19 still exists.

Materials and Methods

Data collection

The historical time series of daily closed price of Banque Pour Le Commerce Exterieur Lao Public (BCEL) and EDL-Generation Public Company (EDL-Gen), which they were 2 actively traded stocks in LSX between November 2019 to February 2024, were collected from the databases of Investors.com's website. In addition, the historical time series of daily closed price of each stock was transformed becoming a historical time series of daily return by applying the log transformation of return as follow.

$$R_{i,t} = \ln \left(\frac{P_{i,t+1}}{P_{i,t}} \right)$$

where $R_{i,t}$ denotes the rate of return of stock i at time t .

$P_{i,t}$ denotes the closed price of stock i at time t

$P_{i,t+1}$ denotes the closed price of stock i at time $t+1$

According to Markowitz (1952), the portfolio's return (R_p) is determined by the portfolio's weighted average of each individual expected return.

$$R_p = w_1 R_1 + w_2 R_2 + \cdots + w_N R_N = \sum_{i=1}^N w_i R_i$$

where w_i denotes the proportionate composition of a specific holding asset in a portfolio

R_i denotes the expected return of each individual asset.

N denotes the number of invested assets. ($N = 2$)

Accordingly, the expected return of the portfolio ($E(R_p)$) is defined as

$$E(R_p) = \sum_{i=1}^N w_i \mu_i$$

where μ_i denotes the mean return of each individual asset.

In addition, the portfolio's variance ($V(R_p)$) is described as

$$\begin{aligned} V(R_p) &= \sigma_p^2 = \sum_{i=1}^N w_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1, j \neq i}^N w_i w_j \sigma_{ij} \\ &= 2 \sum_{i=1}^N w_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j < i}^N w_i w_j \sigma_{ij} \end{aligned}$$

where σ_p^2 denotes the variance of the portfolio's return

σ_i^2 denotes the individual stock return variance

σ_{ij} denotes the covariance of stock return i and j

Analytical approach of VaR

VaR is a widely used measurement in the financial sector for estimating and controlling the portfolio's risk which represent the percentile of portfolio's return at the given level of confidence. Since the VaR is also defined as the statistical measurement of the maximum losses depends on the existing position, the predetermined probability distribution that certain loss will exceed the VaR is then defined as follow.

$$P(Loss > VaR) \leq 1 - \alpha$$

where α denotes the level of confidence.

$Loss$ denotes the level of loss.

With the probability distribution of the portfolio's return, the VaR can be derived by

$$P(R_p \leq R_p^-) = \int_{-\infty}^{R_p^-} f(R_p) dR_p = 1 - \alpha$$

where R_p^- represents the investment portfolio's greatest potential loss realization which can be defined as the quantile of the distribution where the threshold value with the predetermined probability of being exceeded is located.

The mathematical of Copula

Suppose the marginal distributions of a random vector $(R_1, R_2, \dots, R_d)$ are continuous which marginal cumulative distribution function (cdf) is $F_i(r) = P(R_i \leq r)$. Following the incorporation of each component, the random vector has uniformly distributed marginals.

$$(U_1, U_2, \dots, U_d) = (F_1(R_1), F_2(R_2), \dots, F_d(R_d))$$

The copula of $(R_1, R_2, \dots, R_d)$ is well defined by the joint cumulative distribution function of $(U_1, U_2, \dots, U_d)$ as

$$C(u_1, \dots, u_d) = P(U_1 \leq u_1, \dots, U_d \leq u_d)$$

This expression's contribution is to generate the multivariate probability distribution and create a pseudo-random sample which required sample can be illustrated as

$$(R_1, R_2, \dots, R_d) = (F_1^{-1}(U_1), F_2^{-1}(U_2), \dots, F_d^{-1}(U_d))$$

If F_i are assumed to be continuous distribution, the inversion of F_i^{-1} is straightforward which can be expressed as

$$C(u_1, \dots, u_d) = P(R_1 \leq F_1^{-1}(u_1), \dots, R_d \leq F_d^{-1}(u_d))$$

The mathematical derivation in details of these expression can be found in Hofert and Mächler (2011) and Nelsen (1999).

Research Findings

Fitting the stock return distributions

Determining a marginal probability distribution of each stock return (BECL and EDL-Gen), the maximum likelihood estimation method will be used to estimate the parameters of competed probability distributions which is well describing the time series of stock return where the competed distribution consists of normal, student-t, log-normal, logistic, triangular, Gumbel, Fréchet, Weibull, generalized beta, and generalized extreme value distributions. These distributions were widely discussed in the literatures for describing the stock return in several market. Then, the chi-square and Anderson-Darling tests will be engaged to assess the potential probability distributions that can be suitably describe the behavior of each stock return.

Fitted distribution of stock's return

By applying the historical time series of daily return of BCEL and EDL-Gen with the function *fitdistrplus* by Delignette-Muller and Cornu (2008) in R programming language by R Core Team (2024), we can estimate the parameters of all probability distributions that is well describing each stock's return. Then, the goodness-of-fit results as in Table 2 provided by the chi-square and Anderson-Darling tests will be used to evaluate the most fitted distribution. We can indicate that the Gumbel distribution outperformed in describing the return of both stocks. Therefore, the Gumbel distributions which were determined by

$$\text{BCEL} \sim \text{Gumbell}(-0.008941, 0.028171)$$

$$\text{EDL-Gen} \sim \text{Gumbell}(-0.011027, 0.027163)$$

Table 2 P-value of the goodness of fit test of the BCEL and EDL-Gen stocks

No.	Distribution	BCEL		EDL-Gen	
		Chi-square	Anderson-darling	Chi-square	Anderson-darling
1	Normal	0.516	0.645	0.520	0.556
2	Student-t	0.560	0.541	0.529	0.506
3	Log-normal	0.432	0.428	0.456	0.470
4	Logistic	0.427	0.366	0.462	0.349
5	Triangular	0.288	0.351	0.218	0.378
6	Gumbel	0.617	0.718	0.661	0.677
7	Fréchet	0.179	0.269	0.227	0.381
8	Weibull	0.189	0.270	0.122	0.232
9	Generalized beta	0.373	0.453	0.338	0.448
10	Generalized extreme value	0.331	0.508	0.361	0.562

were used in modelling the multivariate distribution where the Gaussian Copula, which also considers tail independent and is applicable for negative dependence, will be employed. Table 3 indicates the parameter α and β of the Gumbel distribution that best describe each stock's return where the α and β are considered as location and scale parameters respectively. In addition, we

also applied the *plotdist* function by Delignette-Muller and Dutang (2015) to provide the empirical and theoretical density plots for each daily stock return as illustrated in Figure 2-3.

Table 3 The estimated parameter α and β which determine the location and scale of the Gumbel distribution of each stock return.

Stock symbol	Parameters	
	Location (α)	Scale (β)
BECL	-0.008941	0.028171
EDL-Gen	-0.011027	0.027163

Based on the results of the goodness of fit test, it can be surely confirmed that the stock's return does not follow normal distribution as mentioned in various classical financial theories. In addition, each stock returns do not just have only one peak in the middle of the distribution. Thus, the normal distribution may not appropriate in predicting the possible value. As results, estimating the copula VaR with the appropriated marginal distribution would lead to the right track of investors in determining the portfolio's risk level.

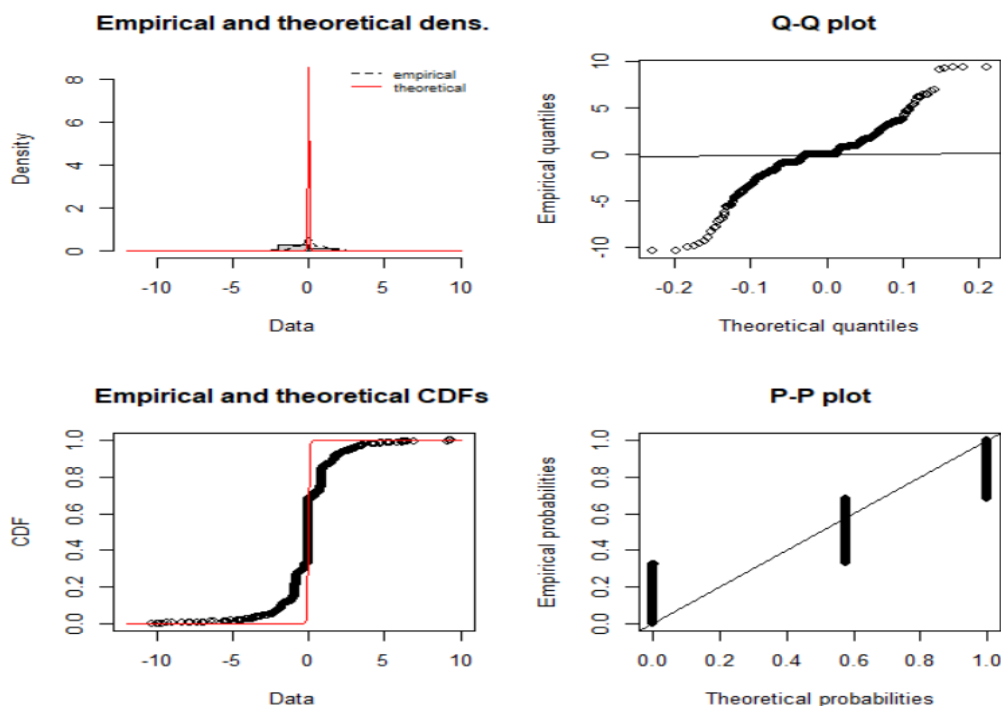


Figure 2 Empirical and theoretical density of BECL's return

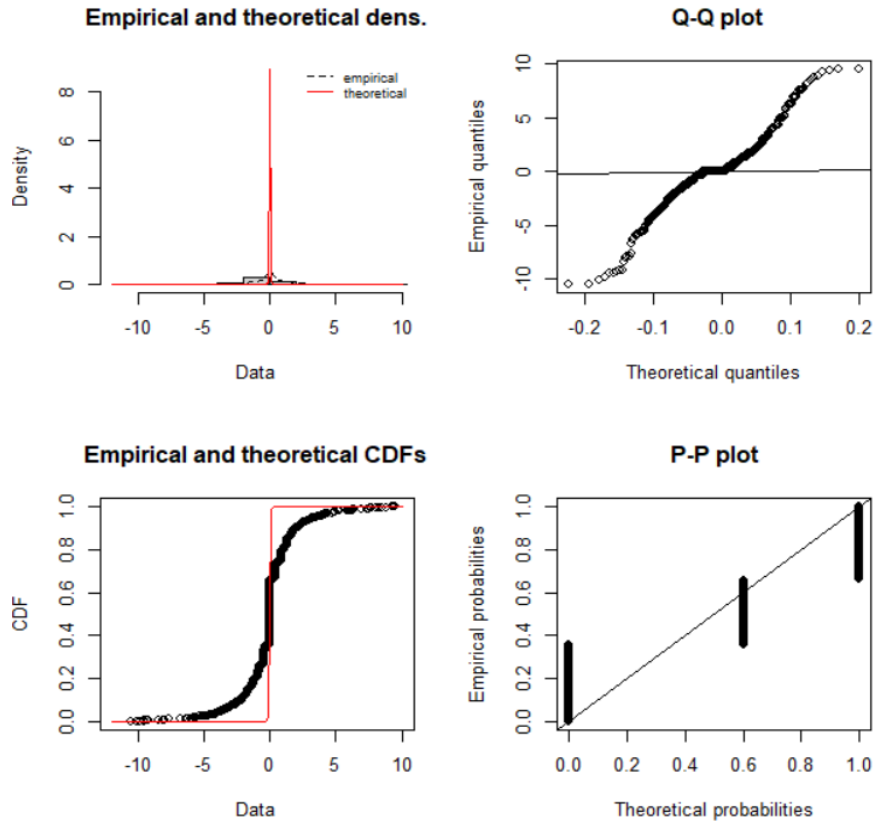


Figure 3 Empirical and theoretical density of EDL-Gen's return

The simulation for VaR

By randomly select 1,000 random variables from a standard normal distribution where each variable mutually shares their characters through the variance-covariance matrix, the vector of cumulative probability $(\mathbf{u})$, based on the random sample, will be generated by the application of function "*pnorm*" of Venables and Ripley (2002) in R package.

With the implementation of Gaussian copula and the correlation matrix $(\boldsymbol{\rho})$ between the daily return of BCEL and EDL-Gen, the simulated rate of return will be generated based on the inversion of joint cumulative distribution function of the Gumbel distribution as follow.

$$C(u_{BCEL}, u_{EDL-Gen}; \boldsymbol{\rho}) = \Phi_{\boldsymbol{\rho}}(\Phi^{-1}(u_{BCEL}), \Phi^{-1}(u_{EDL-Gen}))$$

$$\text{where } u_{BCEL} = F_{BCEL}(R_{BCEL}) \text{ and } u_{EDL-Gen} = F_{EDL-Gen}(R_{EDL-Gen})$$

Φ_{ρ} is a normal cumulative joint distribution function of a multivariate Gaussian distribution with mean vector zero and $n \times n$ correlation matrix ρ .

Φ^{-1} is a probability density function whose values are inversed from normal cumulative distribution.

ρ is the correlation matrix between variable $\Phi^{-1}(R_{BCEL})$ and $\Phi^{-1}(R_{EDL-Gen})$.

Then, the portfolio's return (R_p) according to the equally weighted average of each expected returns ($w_i = 0.5$) are recalculated in a number of 1,000 times. As results, there are 1,000 simulated values of the portfolio's return. This process was suggested by Hofert, Kojadinovic, Maechler, and Yan (2020), Ivan and Jun (2010), and Jun (2007).

With the 1,000 simulated values of the portfolio's return, the function *fitdistrplus* by Delignette-Muller and Cornu (2008) in R programing was implemented again to estimate the best fitted distribution of portfolio's return. We found that the Gumbel distribution was also well describing the portfolio's return where location and scale parameters can be estimated as following expression.

$$R_p \sim \text{Gumbell}(-0.008132, 0.018635)$$

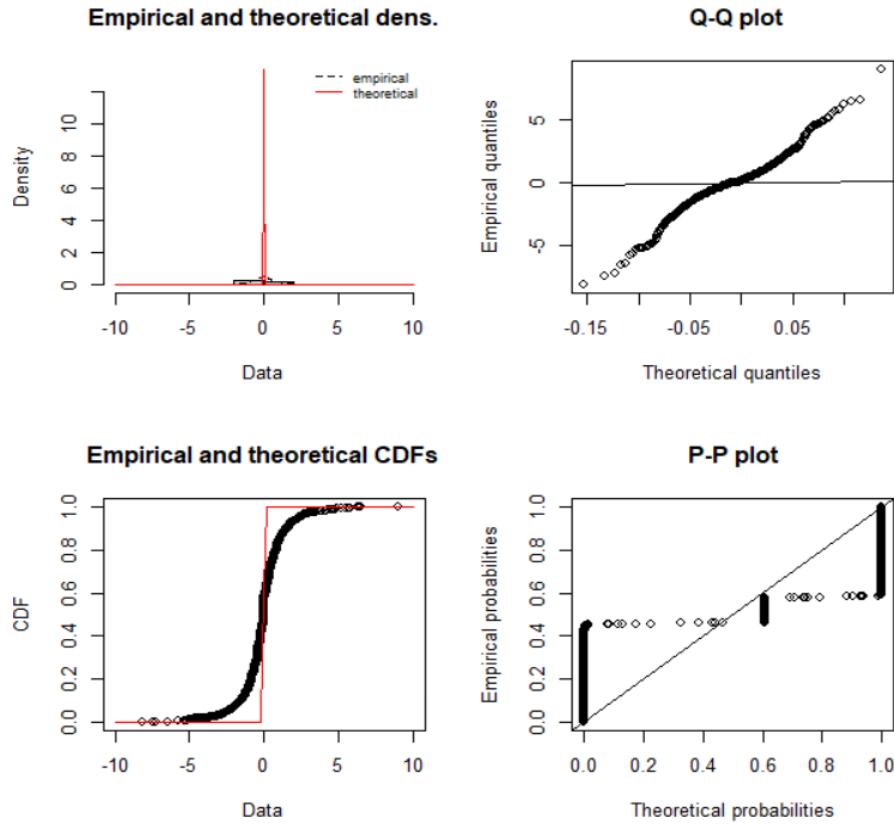


Figure 4 Empirical and theoretical density of portfolio's return (R_p)

In order to estimate the portfolio's copula VaR in according with 95%, 97.5% and 99% levels of confidence respectively, probability distribution of the portfolio's return was integrated based on the following expression.

$$P(R_p \leq VaR_p) = \int_{-\infty}^{VaR_p} f(R_p) dR_p = 1 - \alpha$$

$$\text{where } f(R_p) = \frac{1}{0.018635} e^{\left(\frac{r_p + 0.008132}{0.018635} + e^{\left(\frac{r_p + 0.008132}{0.018635} \right)} \right)}$$

Table 4 Comparison of the portfolio's risk of the two most actively traded stocks (BECL and EDL-Gen) relied on normality and Gaussian copula of VaR

Confidence level	Normality VaR	Gaussian Copula VaR
99.0 %	1.8136 %	1.7201%
97.5 %	1.6342%	1.5268%
95.0 %	1.2108%	1.0126%

Table 4 indicated that the Gaussian copula VaRs were slightly lower than the normality VaR for all given levels of confidence. This can be implied that the normality VaR was consistently aligned with a conservative portfolio investment where investing in low-risk securities is prioritized such as blue-ship stock, high dividend stock and big cap stock. Even though, the portfolio based on low-volatility stocks was formulated, the Gaussian copula VaR was not favorable for risk-averse to measure the worse loss.

Conclusion and Discussion

With the application of copula in estimating a portfolio's VaR for two actively traded stock registered in the LSX which can considerably describe the overall movement of Laos stock market transaction during November 2019 to February 2024, the value of Gaussian copula VaRs was slightly lower than the normality VaR for all given levels of confidence. These results do not align with Khruachalee and Bodhisuwan (2021) that examined the performance of copula VaR in Thai stock market where the normality VaR was insignificantly higher than the Gaussian copula VaRs. In addition, Barreto and Ishimura (2022) highlighted that traditional VaR models often underestimate risk compared to copula-based models, especially in portfolios with heavy tails and extreme dependencies. The copula models capture tail dependencies more effectively, leading to higher accuracy in risk estimation under non-normal conditions. However, the study of Khanthavit (2013) also found that the copula VaR's values were significantly lower than normality VaR for all given confident levels. Moreover, Wang, Lai, and Wu (2024) examined the dependence structures in international financial markets using mixed copula models. It reports scenarios where normality-based VaR models produced higher estimates than copula-based models, primarily because the normality assumption failed to account for the heavy tails and asymmetric dependencies present in the data. This anomaly may cause by the number of parameters

that was assumed in determining in the copula VaR. Since there were many estimated parameters, the estimation errors may raise. Then, they lead to the error of VaR estimation.

Although, we rarely found the evidences from Laos stock market but we assume that the volatility of stocks may cause the variability of the portfolio's VaR since it determines the market risk. Even though, the portfolio contained two actively traded stocks, comparing with Thai stock market there are insignificantly active. Therefore, the application of copula VaRs in Laos stock market may considerably suitable for investors who are less risk averse where the highly traded stocks are more favorable than low frequency traded. For those conservative investors who are literally look for safe stocks, they should apply the classical normality VaR for their portfolio's risk measure.

Consequently, the application of copula in modelling a multivariate distribution irrespective of the joint normality assumption with the VaR approach to determine the portfolio's risk level would favor for the investors who have less risk averse. As the estimated VaRs based on normality assumption were slightly higher than the copula VaR for all predetermined levels of confidence, the normality VaR would be considered as a conservative portfolio investment strategy for risk-averse. However, the conservative portfolio investment strategy does not indicate the best performance of portfolio management. On another hand, the accurately estimated value of VaR will provide an essential means of assessing better portfolio management.

Recommendations for Future Research

This research attempted to examine the application of copula for portfolio VaR with low-frequency trading stocks where 2 stocks registered in the Lao securities exchange were applied. However, based on the results of this findings, there are some recommendations that can be made to improve the effectiveness of this risk measure. Firstly, the equally weighted average of all individual return may not fully cover the risk's environment of the low-frequency trading portfolio. Therefore, the weight given to each stock can be determined by its market capitalization or trading volume. Secondly, since there are only 10 stocks registered in the Lao securities exchange, the portfolio of all 10 stocks could be possible for the overall risk measurement. Thirdly, back testing can also be applied in order to validate the accuracy of the VaR prediction based on this approach. Finally, exploring asymmetric copula models and empirical copulas could be applied to enhance the depth of analysis (Liebscher, 2008).

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