

A Meta-Synthesis of Effective Practices and Outcomes in the Use of Manipulatives for Teaching Mathematics

Robert Jay N. Angco¹ and Lubella B. Angco, Cebu Technological University, Philippines

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Abstract

Use of manipulatives in mathematics classes has yielded compelling evidence of their value in improving several aspects of mathematics education. This report endeavored to gain a clearer picture of their benefits by using a meta-synthesis technique by integrating findings from 12 previous qualitative or mixed methods studies. They were evaluated using the Critical Appraisal Skills Programme and summarized using a PRISMA diagram. Content analysis identified five main themes: (a) activation of classroom engagement and participation, (b) building of confidence and motivation, (c) enhancement of mathematical proficiency, (d) availability and accessibility of varied resources as manipulative, and (e) teachers' competency and careful planning. A meta-theme that emerged was *Optimizing Mathematics Learning with Effective Classroom Resources through Manipulatives*. Thematic study revealed that strategically integrating manipulatives into classroom instruction has a favorable influence on student engagement, confidence, motivation, and mathematical ability. By incorporating manipulatives into mathematics instruction, educators may create inclusive, engaging, and dynamic learning environments that encourage deeper conceptual comprehension.

Keywords: Mathematics, manipulatives, meta-synthesis, systematic review

Rationale

The use of various kinds of manipulatives has become integral to primary classroom instruction, as teachers recognize their positive effects on student learning (Agodu, 2016; Vizzi, 2016). Research studies have demonstrated that integrating manipulatives into math education provides students with valuable visual aids to grasp abstract mathematical concepts effectively (Sulistyaningsih et al., 2017; Monte, 2021). Mathematics is acknowledged as vital to development, contributing significantly to various economic sectors and prioritized on many national educational agendas. Despite Filipino students' proficiency in knowledge acquisition, there is a concerning trend of poor performance in sessions demanding higher-order thinking skills. Mathematics is thus an essential part of school curricula, fostering critical thinking skills necessary across various subjects (Angco, 2021).

Mathematics education, particularly at the primary and secondary levels, is undergoing substantial changes, making teaching both challenging and exciting (Bungao-Abarquez, 2020). These levels are crucial because they set the foundation for students' future mathematical learning and growth. New insights and the rapidly changing cultural landscape necessitate innovative approaches to mathematics education. The National Council of Teachers of Mathematics endorses the use of manipulatives, citing both learning theory and considerable classroom data. Manipulatives help students transition from concrete experiences to abstract reasoning, thereby facilitating a deeper understanding of mathematical principles. Karten and Murawski (2020) emphasized that these tools provide tangible representations that bridge theoretical arithmetic with real-world applications, enhancing students' comprehension and retention. Research indicates that employing manipulatives benefits students in various ways, including improved recall of learned concepts and problem-solving skills (Kabel et al., 2021).

Manipulatives are commonly used tools in mathematics teaching and learning, benefiting students in inclusive classrooms by addressing diverse learning needs and styles (Bouck & Park, 2018;

¹ Corresponding author's email address: robertjay.angco@ctu.edu.ph

Muammar & Suhartina, 2018). These tools include objects such as base-ten blocks, fraction circles, algebra tiles, and geometric solids. By providing physical representations of abstract concepts, manipulatives make mathematics more accessible (Carboneau et al., 2020). By engaging students actively and fostering problem-solving skills, manipulatives cultivate critical thinking essential for independent learning (Liggett, 2017). Tjandra (2023) emphasized the benefits of manipulatives for students' grasp of mathematical ideas and active engagement in the learning process. Lange (2021) illustrated that manipulatives offer novel ways for students to interact with mathematical concepts, enriching their overall learning experience. Mathematics education, as Angco (2021) pointed out, involves more than rote calculations and formulas; it encompasses investigation, experimentation, and the development of problem-solving abilities essential at all educational levels. The purposeful use of manipulatives not only improves immediate learning outcomes, but also prepares children for lifelong mathematical thinking and application.

Hidayah et al. (2018) argued that employing manipulatives assists students to better observe and focus on their teacher. Because of the manipulatives, students' ability to think critically and memorize information increased. Their research study found that students were more enthusiastic to participate in learning activities, while other pupils stated that group learning and in-class demonstrations helped them better understand the material.

There is a dearth of studies conducted that synthesizes both teachers' and students' experiences using manipulatives in the classroom, particularly through meta-synthesis. The purpose of this study is to provide a thorough knowledge of how manipulatives are used in mathematics education. It integrates findings from various studies to examine how manipulatives impact student engagement, understanding, and retention of mathematical concepts. By synthesizing this information, the study seeks to offer evidence-based insights and practical recommendations for educators aiming to enhance their instructional practices through the effective use of manipulatives. Although new primary data is not presented in this paper, it offers valuable insights by combining findings from multiple sources, highlighting the overall effectiveness and benefits of manipulatives in education. This synthesis is crucial for understanding their broader impact and guiding future research and practice in the field.

Methodology

Design

In this study, meta-synthesis was employed to integrate findings from several qualitative and mixed-methods studies. This meta-synthesis aims to provide a comprehensive understanding of the use of manipulatives in mathematics education by conducting a systematic review of previous studies and integrating qualitative research findings into their major themes.

Search Strategy

Relevant research studies published between 2020 and 2024 were downloaded and analyzed. The software employed two descriptors or keywords: "manipulatives" and "mathematics teaching." These keywords were chosen to extract relevant articles. The screened data was then organized and filtered using a flow diagram in accordance with the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) 2020 principles. The Publish or Perish software was used to search Google Scholar, Semantic Scholar, Crossref, and OpenAlex for research on the use of manipulatives in mathematical education.

Selection Criteria

Selection criteria were adopted to guide the choice of articles for analysis; this was done in an effort to improve the quality and applicability of the themes that would emerge. Included studies were based on the following criteria: (a) the use of manipulatives, (b) utilization of qualitative or mixed-method designs, (c) written in English, (d) published between 2020 and 2024, and (e) meeting the

standards of the Critical Appraisal Skills Programme (CASP). Papers that met these criteria were then screened for inclusion.

The selection of research papers followed three stages using the PRISMA 2020 Flow Diagram: Identification, Screening, and Inclusion. During the Identification stage, 200 studies were found on Google Scholar, 100 studies on the Crossref database, and 50 studies on the OpenAlex database. A total of 450 studies were screened using the Publish or Perish software.

Figure 1 Search Strategy Using PRISMA

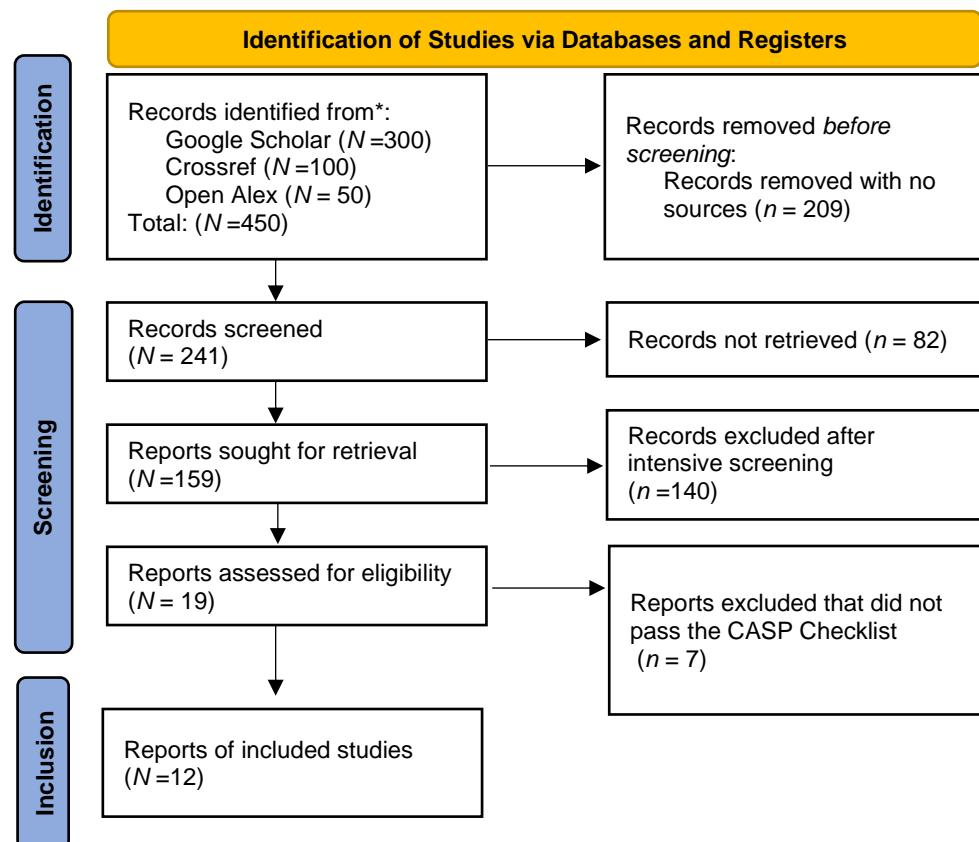


Table 1 Profile of Articles Selected about Use of Manipulatives in Teaching Mathematics

Seq. No.	Authors/Year	Setting	Nature	Initial Codes for the Use of Manipulatives
1	Chiphambo et al., 2020	South Africa	Mixed	<ul style="list-style-type: none"> • Enhance learners' mathematical proficiency • Active participation; engaging • Able to work as a team
2	Maboya et al., 2020	Africa	Qualitative	<ul style="list-style-type: none"> • Need to improve teacher mathematical content mastery, pedagogical content knowledge
3	Tan, 2020	Sydney, Australia	Qualitative	<ul style="list-style-type: none"> • Unsure of school expectations • Engaging but distracting • Educate teachers on how to use manipulatives
4	Cole, 2020	London, England	Qualitative	<ul style="list-style-type: none"> • Builds confidence • Manipulatives as a tool for sense-making
5	Gresham, 2021	Florida, USA	Mixed	<ul style="list-style-type: none"> • Offered opportunities for active engagement with manipulatives • Used physical manipulatives to teach mathematical concepts in elementary school could reduce math anxiety. • Need math confidence in using manipulatives

6	Schumacher, 2021	Not identified	Qualitative	<ul style="list-style-type: none"> Developed self-confidence Deeper level thinking Mathematical language evolves
7	Quane, 2022	Australia	Mixed	<ul style="list-style-type: none"> Reluctant to use Fosters positive attitude towards math
8	Ünlü, 2022	Not identified	Mixed	<ul style="list-style-type: none"> Choosing of appropriate manipulatives Should know when/how to use manipulatives Feedback is important
9	Butcher, 2023	Bahamas	Qualitative	<ul style="list-style-type: none"> Manipulatives can be beneficial or harmful, depends on how they are used in the classroom Overreliance Practice makes perfect Grades get better when using manipulatives
10	McMahon, 2023	Florida, USA	Qualitative	<ul style="list-style-type: none"> Improved confidence in math Increased performance
11	Tjandra, 2023	Jakarta, Indonesia	Mixed	<ul style="list-style-type: none"> Effectively supports students with diverse needs A need to provide sufficient resources Training on usage of manipulatives effectively
12	Jagnandan & Jagnandan, 2024	Georgetown, Guyana	Mixed	<ul style="list-style-type: none"> Positive mindset Eagerness Motivated

Content analysis of the 12 studies about use of manipulatives in mathematics instruction was conducted to generate initial codes. Table 1 presents a profile these studies' titles, origin, and accompanying authors, as well as the codes that were generated and utilized for thematic analysis. This study synthesizes current knowledge to provide evidence-based insights and useful recommendations for educators, even if no new primary data is presented.

Results and Discussion

Table 1 displays the codes generated that were used in the search for themes. These general codes were analyzed and categorized, resulting in five themes and one meta-theme. The identified themes are: (a) activation of classroom engagement and participation, (b) building confidence and motivation, (c) enhancement of mathematical proficiency, (d) availability and accessibility of varied resources/manipulatives, and (e) teacher competency and careful planning.

Success Using Manipulatives in Teaching Mathematics

This section highlights positive and successful experiences of using manipulatives in teaching mathematics. The themes discussed under this category are as follows:

Theme 1: Activation of Classroom Engagement and Participation

To make mathematics more engaging and entertaining for students, teachers must physically engage students in hands-on activities (Allen, 2007). Engaging students in learning is a challenging task that requires collaboration between teachers and students to prevent disengagement and maintain interest. Wong et al. (2022) defined learning engagement as comprising active participation in educational activities, emotional support, and effective communication with both peers and educators.

The effective use of manipulatives, combined with engagement and problem-solving, can help students understand abstract mathematical concepts (Monte, 2021). Manipulatives allow students to interact with mathematics at a deeper level, promoting hands-on learning, discussion, and cooperation. Studies have found that manipulatives can be an effective educational tool for problem-solving exercises, with results showing improved student motivation, involvement, and excitement throughout the learning process (Kwon & Capraro, 2023).

The increased use of manipulatives in the classroom has greatly enhanced engagement and involvement by translating abstract concepts into real experiences, especially for students who find mathematics challenging. Golafshani (2013) noted that manipulatives helped differentiate instruction, respond to different learning styles, and promote deeper conceptual knowledge. This hands-on approach not only makes learning more interactive and dynamic, but also encourages collaborative learning, enhancing students' communication and teamwork abilities. The positive influence on student motivation and instructor enthusiasm fosters a dynamic classroom atmosphere that promotes active learning and inclusive education.

Moreover, manipulatives provide a tangible connection to mathematical concepts, helping students visualize and internalize notions that might otherwise be abstract and difficult to grasp. This connection supports a more comprehensive understanding and better retention of material. By incorporating manipulatives into their teaching strategies, educators can create an environment where students are more likely to participate actively and maintain sustained interest in the class. Therefore, they are a crucial tool in making mathematics accessible, engaging, and enjoyable, ultimately leading to a more productive and inclusive educational experience.

Theme 2: Building of Confidence and Motivation

Teachers' attitudes towards manipulatives in the classroom are linked to their perceptions of the effectiveness of these tools in helping students learn. According to research, teachers who employ manipulatives in the classroom have higher levels of confidence, which leads to less anxiety about mathematics (Vinson 2001). This confidence can be conveyed to students, resulting in an improved learning atmosphere.

Motivation is a powerful force in academic achievement (Unamba & Njoku, 2023). Goracke (2009) found that using manipulatives significantly boosted student attitudes towards math lessons. As students gained confidence in their understanding of the content, their attitudes towards math improved. Additional research has indicated that pupils become more engaged and motivated to learn when manipulatives are used effectively. This engagement is crucial for fostering a positive learning experience.

Using manipulatives in mathematics lessons can lead to a positive attitude towards the subject. Studies by Enki (2014), Gürbüz (2007), and Yağcı (2010) supported this view, showing that students developed a more favorable attitude towards math when manipulatives were part of the instruction. These tools help demystify complex concepts, making them more accessible and less intimidating, which in turn boosts student confidence.

Manipulatives not only enhance understanding but also contribute to a sense of accomplishment. As students successfully manipulate and solve problems with these tools, they build self-assurance in their mathematical abilities. This increased confidence can translate to greater motivation and persistence in tackling challenging problems, further reinforcing a positive cycle of learning and achievement. Moreover, when students see tangible progress through their use of manipulatives, their intrinsic motivation increases. They are more likely to participate actively in class, take on more complex tasks, and exhibit a sustained interest in mathematics. This positive feedback loop, where confidence builds motivation and motivation fosters further confidence, is essential for long-term academic success.

Finally, the strategic use of manipulatives in the mathematics classroom not only supports conceptual understanding but also plays a crucial role in building student confidence and motivation. This holistic approach to teaching mathematics can lead to more enthusiastic attitudes towards learning and improved student outcomes.

Theme 3: Enhancement of Mathematical Proficiency

The strategic use of manipulatives in the classroom significantly enhances mathematical aptitude. Tools such as blocks, counters, and geometric shapes serve as powerful instruments for transforming abstract mathematical concepts into concrete, participatory experiences. Teachers use these hands-

on resources to create an engaging and accessible environment that caters to diverse learning styles and promotes a deeper comprehension of the concepts. This not only improves students' problem-solving skills and conceptual understanding, but it also fosters a positive attitude toward mathematics, building a foundation for academic success and future application of mathematical skills.

Research supports the notion that manipulatives contribute to increased math achievement (Boggan et al., 2010). Liggett (2017) suggested that using mathematical manipulatives to solve problems can significantly improve students' learning and development. Furthermore, Allen (2007) found that students who used manipulatives demonstrated higher achievement levels, enhanced understanding, and developed a more favorable attitude towards mathematical concepts with which they had previously struggled.

Hidayah et al. (2018) discovered that including manipulatives with a topic, combined with some questions, significantly improved students' knowledge of the concept followed a predetermined written plan. This implied that strategically using manipulatives in lesson planning can lead to better learning results. Furthermore, Donovan and Alibali's (2021) research found that individuals who used manipulatives as mathematical tools had better scores than those who did not use them. This emphasizes the significance of viewing manipulatives as necessary rather than optional instruments for mathematical learning.

Finally, the utilization of manipulatives in mathematics instruction not only facilitated conceptual understanding, but also promoted higher levels of achievement, problem-solving skills, and positive attitudes towards the subject. By strategically incorporating manipulatives into lesson plans and instructional practices, educators can enhance students' mathematical proficiency and set them on a pathway towards academic success.

Challenges and Limitations of Using Manipulatives in Teaching Mathematics

This section highlights challenges experienced when utilizing manipulatives to teach mathematics. The following themes from this category are covered below.

Theme 4: Availability and Accessibility of Varied Manipulative Resources

Ensuring the availability of necessary tools is essential for teachers to effectively support student learning, particularly in mathematics, where concepts can be abstract. Research highlights the utility of specific tools called manipulatives in this regard.

The availability of manipulatives is crucial for their utilization in classrooms. Teachers may face challenges in implementing manipulatives if these resources are not readily accessible. Agodu (2016) underscores this point, noting that teachers often resort to improvisation by using everyday materials such as buttons, spools, shapes, colors, and bean-sticks to create simple yet effective manipulative projects. This improvisation allows teachers to provide hands-on experiences despite resource constraints, emphasizing the importance of ingenuity in instructional practices.

Furthermore, Gaylo and Dales (2017) underline the need of teachers to use a variety of instructional methods and approaches to improve students' learning outcomes, particularly when dealing with low accomplishment in mathematics. This includes using a variety of manipulative resources to accommodate different learning styles and promote deeper knowledge of mathematical ideas.

Educators may create a more inclusive and engaging learning environment that meets the requirements of various learners and promotes mathematical proficiency by guaranteeing the availability and accessibility of a variety of manipulative resources. This emphasizes the significance of resource management and novel teaching practices in supporting effective mathematics instruction.

Theme 5: Teachers' Competency and Careful Planning

The effective utilization of manipulatives in the classroom demands careful planning and competency from teachers. While manipulatives offer immense benefits to students, it is crucial that they are used correctly to facilitate meaningful learning experiences. Learners must not only engage

with manipulatives but also comprehend the mathematical concepts being taught (Unamba & Njoku, 2023). Selecting appropriate math manipulatives tailored to students' needs and aligned with the goals of mathematical programs is paramount (Boggan et al., 2010).

Maboya (2014) defines manipulatives as tangible or visual objects explicitly designed to represent mathematical ideas, concepts, or techniques. The decision to incorporate manipulatives in primary school mathematics classrooms, and the manner in which they are used, depends on various factors including teachers' expertise in mathematics, their perceptions of classroom practice, and the perceived importance of manipulatives. Teachers play a pivotal role in effectively utilizing manipulatives to facilitate students' understanding of mathematical concepts (Hidayah et al., 2018). However, the utilization of manipulatives may be hindered by common misconceptions among mathematics teachers.

Teacher professional development emerges as a critical component when assessing the correct use of manipulatives during instruction. The level of instructional support provided by teachers, when utilizing manipulatives, significantly impacts student learning and math achievement. Research suggests that high levels of instructional assistance are more effective than low levels when incorporating manipulatives into instruction (Laski et al., 2015). Active mentoring of students allows for error correction, detailed explanations, and constructive feedback. It is imperative for teachers to explicitly elucidate the relationship between a manipulative and a mathematical concept, especially for young learners who may struggle to make connections without explicit guidance.

Teachers' competency and careful planning are indispensable for the effective integration of manipulatives into mathematics instruction. By prioritizing professional development and employing instructional strategies that promote meaningful engagement with manipulatives, educators can harness the full potential of these tools to enhance students' mathematical understanding and achievement.

Meta-Theme: Optimizing Math Learning with Effective Classroom Resources through Manipulatives

The use of manipulatives in mathematics education is an effective method for improving learning by offering tangible and interactive resources that aid conceptual understanding. Manipulatives such as blocks, counters, and geometric shapes provide children with a hands-on method to exploring abstract mathematical topics, bridging the gap between tangible and abstract reasoning. This practical engagement helps students build a deeper understanding of mathematical ideas since they can physically manipulate objects to envision and solve difficulties. By introducing manipulatives into classroom education, educators may create an engaging and dynamic learning environment that accommodates different learning styles and encourages active involvement.

The incorporation of manipulatives into mathematics classrooms is a critical meta-theme for improving learning outcomes through effective classroom tools. Manipulatives promote conceptual understanding, problem-solving abilities, and differentiated instruction by turning abstract concepts into tangibles and allowing for interactive investigation. As educators look for new ways to improve mathematics education, manipulatives emerge as an indispensable resource that may revolutionize the learning experience, making mathematics more engaging, accessible, and pleasant for students of all ages.

Conclusion and Recommendations

This meta-synthesis of studies on the use of manipulatives in mathematics education found persuasive evidence that they are effective in improving different areas of mathematical education. Thematic analysis revealed that strategically integrating manipulatives into classroom instruction has a favorable influence on student engagement, confidence, motivation, and mathematical ability. These findings highlight the value of using manipulatives as dynamic tools to bridge the gap between abstract mathematical notions and concrete, practical learning. It is recommended that educators prioritize the use of manipulatives in their teaching approaches in order to maximize mathematical learning. To achieve success, teachers may need professional development to improve their ability to

use manipulatives effectively and integrate them into well-planned lesson structures. Furthermore, efforts should be made to increase the availability and accessibility of a wide range of manipulative tools, allowing educators to meet the different requirements and learning styles of their pupils.

By incorporating manipulatives into mathematics instruction, educators may create inclusive, engaging, and dynamic learning environments that encourage deeper conceptual comprehension and academic performance. Finally, the results of this meta-synthesis show how beneficial manipulatives are when teaching mathematics. Further empirical studies involving primary data collection through classroom experiments and observations are nevertheless necessary. To further confirm the usefulness of manipulatives and investigate their potential uses in instructional strategies, future research studies should further test these findings in authentic learning environments. Such empirical research will help improve instructional tactics to optimize the benefits of manipulatives, and offer verifiable proof of their influence on student learning outcomes.

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Conflict of Interest

The authors declare that they have no conflicts of interest related to this research. There has been no financial, personal, or professional involvement with any organization or entity that could be perceived to influence or bias the outcomes of this study.

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